Study on the Effect of Free Convection on Effective Thermal Conductivity of Unidirectionally Oriented Fiber Assemblies

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Abstract

The effective thermal conductivity of a unidirectionally oriented fiber assembly heated from a lower surface is measured by an experimental apparatus based on the steady comparative method, and the free convective heat transfer generating in a fiber assembly is estimated experimentally. The following results are obtained:

1. The effective thermal conductivity of a fiber assembly heated from a lower surface is higher than that heated from an upper surface in the region of the lower volumetric ratio.

2. The critical modified Rayleigh's number to generate the free convection is about 0.18 for polyester fibers and 0.19 for cotton fibers. The free convective heat transfer is given by

(i) for polyester fibers;

\[ Nu^* = 1.21 \times Ra^{0.113} \quad (Ra^* > 0.18) \]

\[ Nu^* = 1 \quad (Ra^* \leq 0.18) \]

(ii) for cotton fibers;

\[ Nu^* = 1.20 \times Ra^{0.112} \quad (Ra^* > 0.19) \]

\[ Nu^* = 1 \quad (Ra^* \leq 0.19) \]

where, \( Nu^* \) and \( Ra^* \) are the modified Nusselt's number and the modified Rayleigh's number, respectively.

1. Introduction

The heat transfer of a complex material constituted of solid-and gas phases as a fiber assembly is different from that of a single solid material, and is complicated by radiative and convective heat transfer together with conductive heat transfer. The heat transfer is so important to estimate the property of the thermal insulation that this has been studied by many investigators \(^\text{[2]}\). The authors have attempted to estimate the effective thermal conductivity of a fiber assembly based on the heat transfer model considering the radiative heat transfer \(^\text{[1]}\). However, as the convection depends upon the ambient temperature and the structure of the assembly, the author et al. \(^\text{[2]}\) discussed the convection in a fiber assembly put between two parallel plates. However, the contribution of the convection to the heat transfer of a fiber assembly may change according to its circumstances, it can't be said that our study was sufficient to estimate the convective heat transfer.

From this point of view the effective thermal conductivity of a unidirectionally oriented fiber assembly in an air-tight vessel heated from a lower surface is measured in this study, and the magnitude of the convection component, its temperature dependency and the critical point are searched experimentally. The unidirectional fiber assembly studied in this paper is arrayed its fiber axes perpendicular to the heat flow.

2. Experiment

2.1 Experimental apparatus and method

The effective thermal conductivity of a fiber assembly in this paper is measured by the experimental apparatus used in our previous study \(^\text{[1]}\) based on the steady comparative method. The scheme of the apparatus is shown in Fig. 1. This is made of three parts, the experimental, the measuring and the controlling part.

In the experimental part, \( 1 \) is a cylindrical sample chamber, \( 2 \) a disc standard sample, \( 3 \) a heating or cooling plate, \( 4 \) a copper plate and \( 5 \) is the insulator wounded around the apparatus. In the measuring part, the temperature of the upper plate \( T_{V} \) and that of the lower plate \( T_{L} \) is the temperature of the upper surface of a standard sample, and the temperature of the lower plate \( T_{S} \) are measured by copper-constantan thermo-couples. The controlling part regulates the temperature \( T_{V} \) and \( T_{L} \).

The effective thermal conductivity \( \lambda_e \) (W/m-K) of a fiber assembly is measured according to the following procedure. Firstly, a sample is packed in the sample chamber \( 1 \), and the
temperatures of the upper and lower plates, $T_r$ and $T_L$, are regulated by the temperatures prescribed in Table 1 with the controlling part. When these temperatures have reached a steady state, the output of each thermo-couple is read and converted to temperature. Based on the principle of the comparative method, the effective thermal conductivity is determined by substituting the temperature to the following equation.

$$\lambda_e = \frac{A_e}{A_g} \left( \frac{T_r - T_L}{T_r - T_V} \right)$$

(1)

where $A_e$ and $A_g$ are the heights of the sample chamber and the standard sample respectively, and $\lambda_g$ is the thermal conductivity ($\text{W/(m-K)}$) of the standard sample. This value is measured beforehand by the apparatus mentioned above by heating distilled water from the upper surface. This can be represented in the temperature region of this study as

$$\lambda_e = 0.826(1 + 0.001 t_m)$$

(2)

where $t_m$ is the mean temperature (°C) of the distilled standard sample, and is determined by

$$t_m = \frac{1}{2} (T_L + T_T) - 273.15$$

(3)

The mean temperature of the experimental sample $t_s$ (°C) is determined by

$$t_s = \frac{1}{2} (T_r + T_L) - 273.15$$

(4)

2.2 Sample and method for packing

The experimental materials used are made of card slivers of polyester (mean diameter: 11.3 μm, mean fiber length: 38 mm, fiber density: 1.36 g/cm³) and of cotton (14.2 μm, 30 mm, 1.54 g/cm³), and are drafted to the prescribed volumetric ratio (0.001–0.05). The thin films obtained are arranged parallel in the sample chamber and piled up to the height of the chamber.

The volumetric ratio $\xi$ of the experimental sample is defined as

$$\xi = \frac{M_s}{\rho V_s}$$

(5)

where $V_s$ is the apparent volume (= volume of the sample chamber), $M_s$ the mass of the fiber assembly, and $\rho$ is the density of the structural fiber.

2.3 Qualification of experimental apparatus

The effective thermal conductivity of the experimental apparatus when occupied only by air is measured to get Fig. 2, in which the ordinate represents Nusselt's number $N_u$ and the abscissa Grashof's number $G_r$. These non-dimensional numbers are defined by

$$N_u = \frac{\alpha l}{\lambda_e}$$

$$G_r = \frac{g \beta V_s (T_L - T_T)}{\nu^3}$$

where $\lambda_e$ is the thermal conductivity of air, $\alpha$ its heat transfer coefficient (W/m²-K), $\beta$ the coefficient of its cubical expansion (1/K), $\nu$ the coefficient of its kinematic viscosity (m²/s), $l$ the thickness of air layer (m), and $g$ is the gravity (m²/s).

![Schematic diagram of the experimental apparatus](image)

**Fig. 1 Schematic diagram of the experimental apparatus**
Marks ○ represent measured values obtained in this study. Solid or dashed lines show reference curves\(^4\).

Fig. 2 shows that our results agree pretty well with the results (solid line)\(^4\) obtained for general fluids with a little larger values than S. Sugawara et al results (dashed dotted line)\(^9\). However, the whole tendency is quite similar to the reference.

3. Results and Consideration

3.1 When no convection

The effective thermal conductivity of a unidirectionally oriented fiber assembly measured when the convective heat transfer doesn’t arise, namely when the sample is heated from the upper surface and cooled from the lower surface (hereafter called upper heating) is given in Fig. 3 for polyester fibers, and in Fig. 4 for cotton fibers. The ordinate there represents the effective thermal conductivity \(\lambda_{\text{eff}}\) (W/(m•K)) with normal scale, the abscissa the volumetric ratio \(\xi\) with logarithmic scale, and solid lines show the results at each mean temperature of the sample \(t_s\) (°C) from 20°C to 80°C (70°C: for the case of cotton) every 10°C.

These figures show that \(\lambda_{\text{eff}}\) decreases with the increase of mean temperature when \(\xi\) is small, and reaches the minimum at \(\xi = 0.015-0.03\). After \(\xi\) exceeds this point \(\lambda_{\text{eff}}\) rises a little with increase of \(\xi\). It is considered in our previous papers\(^1\), that this is because the radiative heat transfer between the fiber surfaces and the heating or the cooling surface largely contributes to the heat transfer of the fiber assembly when the volumetric ratio is small. Therefore, \(\lambda_{\text{eff}}\) under this condition consists of the sum of the components of conductive and radiative heat transfer. These are calculated based on our model analysis\(^1\) and are shown as dotted lines in Figs. 3 and 4. They are the values at \(t_s = 20\) and 80°C (cotton: 70°C), and the values at other temperatures lie between the two curves.

3.2 When heated from lower surface.

The effective thermal conductivity of the unidirectionally oriented fiber assembly when the sample is heated from the lower surface and cooled from the upper surface (hereafter called lower heating) is given in Figs. 5 and 6 for polyester fibers when \(t_s = 40°C\) (313.15 K) and 80°C (353.15 K) respectively and in Fig. 7 for cotton fibers when \(t_s = 40°C\) (313.15 K). Marks ○ represent measured values when lower
heating, solid lines represent the results when upper heating shown in Figs. 3 and 4. Fine dotted lines represent the conductive component calculated according to the model analysis.

Figs. 5-7 show that the effective thermal conductivity $\lambda_{eL}$ is greater than $\lambda_{eV}$ in the region of small volumetric ratio at every $t_1$. Namely, the change of air density arises based on the temperature difference and the convective heat transfer in the fiber assembly occurs. The critical point of convection occurrence is near to $\xi \approx 0.01$.

The convective component of the effective thermal conductivity $\lambda_{eV}$ (W/(m-K)) is obtained by

$$\lambda_{eV} = \lambda_{eL} - \lambda_{L}$$

Fig. 8 summarises the experimental curves at $t_1 = 20$–
80°C (cotton: 70°C) every 10°C when lower heating for polyester fibers and Fig. 9 for cotton fibers. They show at every mean temperature the same tendency as in the case of Figs. 5-7. On each volumetric ratio, \( \lambda_{el} \) is obtained from Eq. (7) and \( \lambda_{ev} \) of Figs. 3 and 4.

3.3 Temperature dependency when lower heating
The temperature dependency of the effective thermal conductivity when lower heating is shown in Fig. 10 for polyester fiber, and that in Fig. 11 for cotton fibers. The ordinate there represents the effective thermal conductivity \( \lambda_{el} \) and \( \lambda_{ev} \), the abscissa the mean temperature of the sample \( t_s \) both with normal scale. Solid and dotted lines show the cases when lower and when upper heating respectively. Each value is read from experimental curves of Figs. 3, 4, 8 and 9 for \( \xi = 0.0015, 0.0025, 0.005 \) and 0.01.

These figures show that \( \lambda_{el} \) when lower heating increases with the rise of \( t_s \) similarly to upper heating. However, the difference between both heating explain little even though \( t_s \) is going high. This may be because, as the convection depends upon various properties, the temperature dependency of the effective thermal conductivity is affected by the compound dependency of various properties. \( \lambda_{el} \) shows convex tendency to the axis of the temperature with the raise of \( t_s \) when the volumetric ratio is small. This shows that both upper and lower heatings are affected to the same extent by the radiative heat transfer.

3.4 Estimation of convective heat transfer
The convective component thermal conductivity of the fiber assembly \( \lambda_{ev} \) is calculated with eq. (7). However, because there are many parameters governing the convection, it is estimated by the non-dimensional numbers as shown.

Fig. 9 Summary of experimental curves at mean sample temperature 20°C 70°C for cotton fiber assemblies heated from the lower surface.

Fig. 10 Temperature dependency of effective thermal conductivity for polyester fibers

Fig. 11 The temperature dependency of the effective thermal conductivity for the cotton fiber
in eq. (6). Therefore, we define the non-dimensional numbers by applying the definition for a packed bed given by T. Masuoka\textsuperscript{21}, and attempt to estimate the convective component.

In general, the magnitude of the convective heat transfer is estimated based on the non-dimensional Nusselt’s number defined from the ratio to the conductive heat transfer. Therefore, in this paper the modified Nusselt’s number $N_u*$ is defined as

$$N_u^* = \frac{\alpha T}{\lambda_{cd}}$$  \hspace{1cm} (8)

where $\lambda_{cd}$ is the conductive component of the effective thermal conductivity (W/(mK)). The numerical value of this component is equal to the value obtained in our previously papers\textsuperscript{11} (the fine dotted lines in Figs. 3-7), and $\alpha_T$ is the apparent heat transfer coefficient of the fiber assembly and is defined as

$$\alpha = \frac{q(T_L - T_v)}{\Delta T}$$  \hspace{1cm} (9)

where $q$ is the component of the heat flux based on the conduction and convection. Further, if the notion for the effective thermal conductivity is introduced to the heat flux component, the following relation is given:

$$q = (\lambda_{cd} + \lambda_{tor}) \cdot \frac{T_L - T_v}{\ell}$$  \hspace{1cm} (10)

Therefore, from eqs. (8)-(10), the modified Nusselt’s number in this study is given as

$$N_u^* = \frac{\lambda_{tor} + \lambda_{cd}}{\lambda_{cd}}$$  \hspace{1cm} (11)

For a general packed bed, the effective thermal conductivity when the convection doesn’t occur is used\textsuperscript{22} for the value corresponding to $\lambda_{cd}$. However, it is thought in adequate to include the radiative component, because the effect of the radiative heat transfer in the fiber assembly is great and the radiation depends on the surface properties of the heating and cooling plates. Therefore in this paper the conductive component obtained by the model analysis is used.

In general, Grashof’s number defined as eq. (6) or Rayleigh’s number is used for the non-dimensional number prescribing the convective heat transfer. But here, the modified Rayleigh’s number containing the permeability $R_a*$ defined by the following equation\textsuperscript{6} is used, because it is affected by the permeability based on the Darcy’s law for the packed bed.

$$R_a^* = \frac{\rho C_p \ell K(T_L - T_v)}{\nu K}$$  \hspace{1cm} (12)

where $C_p$ is the specific heat (kJ/(kg·K)) of air and $\rho_a$ its density (kg/m$^3$).

Further, the air permeability $K$ is given under the condition of this study by

(i) for polyester fiber

$$K = 1.37 \times 10^{-11} \cdot \xi^{0.11} (1 - \xi)^8$$

(ii) for cotton fiber

$$K = 1.04 \times 10^{-11} \cdot \xi^{0.22} (1 - \xi)^8$$  \hspace{1cm} (14)

The results obtained are shown in Figs. 12, for polyester fibers and in Fig. 13 for cotton fibers. The ordinate represents the modified Nusselt’s number, the abscissa the modified Rayleigh’s number, both with logarithmic scale. Further, because the dependency of $R_a*$ on the sample mean temperature is less than 5%, and that on $N_u*$ is less than 10%, the arithmetic mean of the non-dimensional numbers from 20°C to 50°C or from 50°C to 80°C (or 70°C) is used as the experimental values.

![Fig. 12 Non-dimensional estimation of free convective heat transfer in polyester fiber assemblies](image)

![Fig. 13 Non-dimensional estimation of free convective heat transfer in cotton fiber assemblies](image)
These figures show that the convective heat transfer of the fiber assembly is much smaller than that of air and the existence of fibers represses the convection. Even though the experimental scattering is found, the relation between \( N_u^* \) representing the magnitude of the convection and \( R_a^* \) prescribing that magnitude can be regarded as linear on the logarithmic graph. Therefore, the relation between \( N_u^* \) and \( R_a^* \) is approximated to linear with the least square method, and is obtained:

(i) for polyester fiber

\[
N_u^* = 1.21 R_a^{0.113} \quad (R_a^* > 0.18)
\]
\[
N_u^* = 1 \quad (R_a^* \leq 0.18)
\]  

(ii) for cotton fiber

\[
N_u^* = 1.20 R_a^{0.112} \quad (R_a^* > 0.19)
\]
\[
N_u^* = 1 \quad (R_a^* \leq 0.19)
\]

These equations are the representation of the free convective heat transfer in the fiber assembly obtained under the experimental condition of this paper. However, as \( R_a^* \) is in the narrow region of 0.01 to 2, it is thought necessary to experiment in a much broader region of \( R_a^* \) in order to obtain more exact representation.

The critical point of convection occurrence is \( R_a^* = 0.18 \) for polyester fibers, and \( R_a^* = 0.19 \) for cotton fibers. The critical point for the fiber assembly is smaller than this value. This may be because, first the absolute difference between the regions of the volumetric ratio in this paper the region is less than 0.05 and it is about ten times for general packed bed, second the difference of the shape of the packed material constituting both assembly. In other words, the difference of the property of air permeability exists due to the difference of the pore shape and the size formed by each packed material. Namely, even though the volumetric ratios are the same, pores exist in the fiber assembly along the air flow and these pores are subdivided by fine fibers. In contrast to this, in the packed bed pores are concentrated and flow passages wind complicatedly.

The effect of the difference of fiber kinds to convection does not exist as clarified by Figs. 12, 13 and eqs. (15), (16). This may be because the property and the size of the structural fibers are contained in the non-dimensional number and the effect of the difference is difficult to appear due to the small volumetric ratio.

4. Conclusion

The contribution of the convective heat transfer to the heat transfer property of the unidirectionally oriented fiber assembly of polyesters and cotton fibers is studied experimentally when the fiber assembly is heated from the lower surface. Furthermore the estimation based on the non-dimensional numbers is attempted by applying the theoretical results of the conductive heat transfer according to the model analysis in our previous papers. The following matters are clarified.

(1) The effective thermal conductivity of the fiber assembly heated from the lower surface has the same qualitative tendency as that heated from the upper surface even the effective thermal conductivity of the lower heating is higher than that of the upper heating in the region of the lower volumetric ratio.

(2) The free convective heat transfer is given by

(i) for polyester fibers,

\[
N_u^* = 1.21 R_a^{0.113} \quad (R_a^* > 0.18)
\]
\[
N_u^* = 1 \quad (R_a^* \leq 0.18)
\]

(ii) for cotton fibers,

\[
N_u^* = 1.20 R_a^{0.112} \quad (R_a^* > 0.19)
\]
\[
N_u^* = 1 \quad (R_a^* \leq 0.19)
\]

where, \( N_u^* \) and \( R_a^* \) are the modified Nusselt's number and the Rayleigh's number, respectively.

(3) The critical modified Rayleigh's number to generate the free convection is about 0.18 for polyester fibers and 0.19 for cotton fibers.

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References