Computer Simulation of Goniophotometric Curves for Fabrics

Part 2: Aggregated Elliptic Cylinder Model Subjected to Parallel Incident Light Flux Parallel to the Filament Axis

By Nobuo Muto*, Koji Nihira**, Tsuneyo Tsuboi***, Toshihiko Arai****, and Toshihiro Gunji*****,
Members, TMSJ

*Faculty of Engineering, Tokai University, Kanagawa

**Faculty of Engineering, Meiji University, Kanagawa

***Faculty of Engineering, Aichi Institute of Technology, Aichi

****Faculty of Engineering, Kanagawa Institute of Technology, Kanagawa

*****Faculty of Home Economics, Otsuma Women's University, Tokyo

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Abstract

A previous paper^[1] described the simulation of specular reflected light curves by use of a fabric model. The simulation treated the case of a parallel light flux projected perpendicular to the filament axis in the fabric, and the model consisted of fine elliptic columns arranged side by side on the surface of an elliptic cylinder.

In the present paper, the authors present a computer simulation of specular reflected light curves in which a parallel light flux is applied parallel to the filament axis of the fabric, using a new filament model. The new model consists of a number of thin elliptic cylinders of various minor axis lengths. Distribution of the minor axis length obeys two different rules: elliptic distribution, and normal distribution.

As with the method described in a previous paper,^[1] the present study aims to simulate the specular reflection curves of fabrics.

1. Introduction

Generally speaking, in two-dimensional variable-angle photometry of fabrics illuminated by parallel light fluxes, it can be assumed that the planes of photometry will be at various angles to the filament axis. In the previous paper,^[1] however, two specific cases were considered: planes of photometry being perpendicular to the filaments of the fabric, and parallel to the same. Furthermore, a computer simulation was made of specular reflected light curves obtained with the axis of the incident parallel light flux being perpendicular to the filament axis of the fabric. The model used was that of fine elliptic columns arranged side by side on the surface of an elliptic cylinder.

In contrast to the previous paper,^[1] the present computer simulation assumes that the plane of photometry containing both the axis of the incident parallel light flux and the axis of photodetector was parallel to the filament axis in the fabric. The models are arranged the basic-models, that are elliptic cylinder surfaces models having various k values (k is half the minor axis of the

elliptic cylinder surface). The arrangement methods are two types. The first type of arrangement consisted of simple elliptic cylinders having elliptically distributed k values in an attempt to simulate the three-dimensional shape of fabric. The second type of arrangement consisted of simple elliptic cylinders of various k values with the k values obeying normal distribution.

2. Models and Calculation of Specular Reflected Light Curves

Generally speaking, in two-dimensional photometry of fabrics illuminated by a parallel light flux as shown in Fig. 1 the plane of photometry is parallel to the filling if the plane is perpendicular to warps. Consequently, simulated specular reflected light curves of fabrics for general incident angle can be derived by combining two specular reflected light components.

Taking a piece of fabric which has been cut at right-angles to the warp, the topmost layer of each warp yarn can be simulated by of a few dozen single filaments lying

Fig. 1 Incident light and fabric (I).

on an ellipse and that the topmost layer of each filling fiber consists of several dozen single filaments curved elliptically. Furthermore, although the cross-sections of the single filaments take various shapes, for simplicity it was assumed that fine elliptic columns are arranged side by side on the surface of an elliptic cylinder.

In this model, specular reflection in two-dimensional photometry occurs on infinitesimally small surface elements perpendicular to the plane of photometry. Such an aggregation of surfaces can either be of a continuous nature or, as in fabrics woven from twisted yarns for example, an aggregation of infinitesimally small discrete surface elements. In the latter case, it is realistic to consider the infinitesimally small surfaces as forming a population of normal distribution with respect to the k value. Furthermore, in the case of fabrics of irregular geometric pattern, the k value representing the surface roughness of the fabric the variance of the normal distribution is expected to be relatively small.

Next, let us consider a parallel light flux incident upon a smooth and simple fabric model. In a two-dimensional photometry, the photometry plane containing the axes of incident and reflected light beams can either be perpendicular or parallel to the filament axis, as shown in Fig. 2. In a situation where the incident parallel light flux is perpendicular to the filament axis, the only surfaces contributing to specular reflection in the direction of light

Fig. 2 Incident light and fabric (II).

reception are those perpendicular to the plane of photometry, as shown in Fig. 2. The reason for this is that reflection from other surfaces is not in the direction of photodetector. Consequently, in such a situation simulation of specular reflected light curves of fabrics can be made by using the model shown in Fig. 2(b). The previous paper⁽¹⁾ described such a simulation. In a situation where the parallel light flux is parallel to the filament axis, the only surface giving rise to specular reflection are those perpendicular to the plane of photodetector, shown as B in Fig. 2. Consequently, simulation of the specular reflected light curves for this situation can be made by using a model consisting of an aggregate of simple elliptic cylinder surfaces having different k values, as shown in Fig. 2(d).

Further for a better simulation of real fabric surfaces specular reflected light curves were calculated for an aggregate of multiple elliptic cylinders having different k values obeying the normal distribution.

2.1 The case of elliptic cylinder surface

The major and minor axis of the elliptic cylinders are taken as 2 and $2k$ respectively, as shown in Fig. 3. Each

Fig. 3 The simple elliptic cylinder model.

elliptic cylinder is connected to the neighbouring elliptic cylinder by a plane passing through their center axis, but it is assumed that the reflected light from this plane is zero.

In the present simulation, the center of the photometry plane coincides the center of the elliptic cylinder and the plane is perpendicular to the axis of the cylinder. Consequently, the simulation can be made on the basis of two-dimensional photometry. The following definitions, therefore, assume two-dimensionality. Specular reflected light curves were calculated for parallel light flux striking an elliptic cylinder surface at an incident angle of Φ , taking the axis of the elliptic cylinder as 1.

The equation of the ellipse shown in Fig. 4 is

Fig. 4 Light beam projected on elliptic cylinder.

$$x^2 + \frac{y^2}{k^2} = 1 \quad \dots\dots\dots(1)$$

The gradient angle γ_n of a normal line on point P_n is

$$\gamma_n = \tan^{-1} \frac{kx_n}{(1-x_n^2)^{\frac{1}{2}}} \dots\dots\dots(2)$$

Receiving angle δ_n is

$$\delta_n = \Phi + 2 \gamma_n \dots\dots\dots(3)$$

Since the length of the cylinder is 1, the solid angle $\Delta\omega_n$ of specular reflected light formed by receiving angles δ_n and δ_{n+1} is

$$\Delta\omega_n = \delta_{n+1} - \delta_n \dots\dots\dots(4)$$

Taking dl as the infinitesimal length on elliptic column, the area ΔS of infinitesimal surface is

$$\Delta S = 1 \cdot dl \dots\dots\dots(5)$$

while, dl/dx is

$$\frac{dl}{dx} = (1 + \frac{k^2x^2}{1-x^2})^{\frac{1}{2}} \dots\dots\dots(6)$$

Therefore, the peripheral length l varies by eq. (6) when evenly spaced points x_n, x_{n+1} are conceived on the x coordinate.

The incident angle ϕ_n to the infinitesimally small surface ΔS at point P_n is

$$\phi_n = \Phi + \gamma_n = \frac{1}{2} (\Phi + \delta_n) \dots\dots\dots(7)$$

Since the incident parallel flux is a constant, the luminous flux is assumed equal to unity. The specular reflected luminous intensity $\Delta I(\delta_n)$ of infinitesimally small surface ΔS at point P_n then is

$$\Delta I(\delta_n) = f(\phi, n) \cdot \frac{1}{\Delta\omega_n} \cdot \frac{dx}{dl} \cdot \cos\phi_n \dots\dots\dots(8)$$

where $f(\phi, n)$ is Fresnel's coefficient.

Equation (8) gives the specular reflected luminous intensity at every point on the elliptic column from -1 to 1 on the x coordinate. At times reflected light is intercepted by the adjoining elliptic column producing secondary reflected light. The secondary reflection, however, is neglected in the present study for being very weak.

The reflected light at point P_1 on the ellipsoid I in Fig. 5 can be expressed by the polynomial below.

Fig. 5 Secondary reflection.

$$y = x \tan(\frac{\pi}{2} - \delta) + y_1 - x_1 \tan(\frac{\pi}{2} - \delta) \dots\dots\dots(9)$$

Furthermore, the equation for the ellipsoid II in Fig. 5 is

$$(x-z)^2 + \frac{y^2}{k^2} = 1 \dots\dots\dots(10)$$

Eq. (11) below is obtained by substituting eq. (9) into eq. (10).

$$Ax^2+Bx+C=0 \dots\dots\dots(11)$$

When the discriminant of eq. (11) is positive, the specular reflected light from ellipsoid I strikes against ellipsoid II to produce a secondary reflection of zero strength as assumed above.

Intensity of specular reflected light is derived as described above. While the axis of reflected light does not coincide with the center of photometry (point 0) the diameter of the ellipse is far smaller than the radius of photometry. For this reason the line of reflected light is assumed to intersect point 0.

2.2 Calculation of the goniophotometric curves for the group of simple elliptic columns (I)

The group of elliptic columns shown in Fig. 6 are

Fig. 6 Group of elliptical columns having elliptically distributed height

assumed to have elliptically distributed k values that obey the formula

$$w^2 + \frac{k^2}{k_{max}^2} = 1 \dots\dots\dots(12)$$

where w is the x coordinate. Solving eq. (12) for k , eq. (13) below is obtained.

$$k_n = (1 - w_n^2)^{\frac{1}{2}} \dots\dots\dots(13)$$

The goniophotometric curve for the group of columns is obtained by taking the summation of $\Delta I(\delta_n)$.

2.3 Calculation of the goniophotometric curves for the group of simple elliptic columns (II)

The equation of normal distribution is

$$y = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma} \exp(-\frac{u^2}{2\sigma^2}) \dots\dots\dots(14)$$

where σ is the standard deviation and u is x coordinate.

The goniophotometric curves for the above distribution eq. (14) are obtained as follows:

- (1) The goniophotometric curve of each simple elliptic column is obtained by varying k from 0.1 to 1.0 in eqs. (6) and (8).

Fig. 7 Simplified flow chart for simple elliptic cylinder model.

Fig. 9 Simplified flow chart for the columns having height obeying normal distribution.

Fig. 8 Simplified flow chart for the column group shown in Fig. 6.

- (2) The greatest value of k is determined and the number of elliptic columns is weighted by normal distribution.
- (3) The goniophotometric curves of aggregated simple elliptic columns distributed in normal distribution are obtained by taking the sum of respective values of specular reflected light for each receiving angle.

3. Computer Simulation

A simplified flowchart for the simulation of the goniophotometric curve for the simple elliptic column model is shown in Fig. 7. Figures 8 and 9, respectively show the simplified flowcharts for the group of elliptic cylinders having k values obeying both elliptic and normal distributions.

4. Results of Calculation and Discussion

Figures 10 and 11 show the simulated specular reflected light curves for a simple elliptic cylinder surface, at 45° and 60° angles of incidence respectively. At 45° angle of incidence, the maximum value of the specular reflected light curves becomes smaller and the spread becomes wider as k value increases. When the k value is small, the receiving angle that gives the highest specular re-

Fig. 10 Calculated specular reflected light curves I.

Fig. 11 Calculated specular reflected light curves II.

flected light curve value is close to the angle Φ of specular reflection for the normal plane shown in Fig. 4. However, as the k value increases the maximum occurs at an angle greater than the angle Φ of specular reflection in normal plane. The same applies to the case of angle of incidence. Refractive index was taken to be 1.5 throughout the present study.

Figure 12 shows the specular reflected light curves for

Fig. 12 Calculated specular reflected light curves III.

the group of elliptic cylinders shown in Fig. 4 having elliptically distributed k values and when the incidence angle is 45° . The peak value of the specular reflected light curves is large when the maximum k value $k_{\max}$ is small,

but as the $k_{\max}$ value becomes greater, the curve peak becomes low and it tends to occur at larger receiving angle. As shown in Fig. 13, the curves for the incidence angle of

Fig. 13 Calculated specular reflected light curves IV.

$\Phi = 60^\circ$ is similar in trend to the case of $\Phi = 45^\circ$.

The reason for the increasing peak height in specular reflected light intensity with decreasing k value is that the number of incremental surface elements close to the normal plane increases as the elliptic cylinders become flatter under decreasing k .

Figure 14 shows the specular reflected light curves for

Fig. 14 Calculated specular reflected light curves V.

light beam 45° in incidence angle projected on the group of elliptic cylinders having normally distributed k_n values. The curve consists of a normal distribution curve, the maximum value of which has been made equal to the number of simple elliptic cylinders having a k value of 0.3. The peak of the specular reflected light curves becomes taller as the standard deviation σ decreases. However, σ has little effect on the receiving angle where the peak specular reflected light intensity occurs.

Figures 15 and 16 show the specular reflected light curves consisting of normal distribution curves for the group of elliptic columns with the k values obeying normal distribution having its peak at $k = 0.5$ and $k = 0.7$. The $k = 0.7$ curves are taller in peak and wider in spread than the $k = 0.5$ curves. The curves shown in Figs. 17, 18, and 19 are similar to those in Fig. 15 and 16 except that the incident light angle was 60° . The curves show trend similar to those for 45° incidence angle.

Fig. 15 Calculated specular reflected light curves VI.

Fig. 19 Calculated specular reflected light curves X.

Fig. 16 Calculated specular reflected light curves VII.

Fig. 17 Calculated specular reflected light curves VIII.

Fig. 18 Calculated specular reflected light curves IX.

5. Conclusion

Specular reflected light curves for fabrics were simulated for the case in which the plane containing the axes of incident light flux and of the receiving surface was parallel to the axis of the filaments. The models used were made closer to the real three-dimensional nature of woven fabrics than the model used previously.^[1] In a two-dimensional photometry, specular reflected light from the topmost layer of the fabric is the sum of specular reflected light from infinitesimally small surface elements that are perpendicular to the plane of photometry. When the plane of photometry is parallel to the filament axis, it is estimated that the infinitesimally small surfaces elements responsible for specular reflection are arranged discretely. However, in the present study the reflecting planes were approximated by a continuous surface of infinitesimal width. Furthermore, the wavy cross-section of the fabric was approximated by ellipse. Consequently, when the photometry plane is parallel to the filament axis the fabric surface was approximated by thin elliptic cylinders. This paper describes the computer simulation of specular reflected light curves for two cases: one being elliptic cylinders having elliptically distributed height and the second elliptic cylinders having height that obeys normal distribution.

The present simulation is believed to be useful in future analysis of specular reflected light curves in real fabrics.

Reference

[1] Muto N., Arai T., Nihira K., Tsuboi, T. and Gunji, T.; *J. Text. Mach. Soc. Japan*, English Edition, **31**, No. 4, p. 86 (1985).