Continuous Control of Sliver Thickness

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Abstract

This is an attempt to approximate the characteristics of two major types of servo-draft systems—the open-loop system and the closed-loop system—by comparatively simple mathematical models consisting of linear transfer functions. The effectiveness of servo-draft systems in terms of the frequency response of the mathematical models has been investigated with the following results:

1. Sinusoidal variations in input sliver thickness cannot be effectively attenuated by the servo-draft system if the frequency of such variations exceeds a certain limit. This limit frequency is fixed by the time constants of the system components.

2. The open-loop servo-draft system is much higher in limit frequency than the closed-loop system if their time constants are equal. In other words, the open-loop system can attenuate sliver irregularities of higher frequencies than the closed-loop system can.

3. In most servo-draft systems there are frequency ranges in which the normalized amplitude of input thickness variations is magnified, rather than attenuated.

4. The open-loop and closed-loop systems can be combined into one servo-draft system to make use of the advantages of the two systems. The characteristics of such a combined system are easily predictable from the characteristics of the two constituent systems.

Introduction

The term "servo-draft system" refers to any drawing frame capable of continuously measuring the thickness of slivers and changing the draft ratio automatically in response to the measured thickness signal so as to produce uniform output slivers.

Servo-draft systems are classified into two major categories. One is the open-loop system in which sliver thickness is measured before the sliver enters the drafting zone, the draft ratio being changed in proportion to the deviation of the input sliver thickness from a set value. Since the results of this correction are not detected at the output end, no feedback loop forms in this system. Hence the name "open-loop."

The other is the closed-loop system in which sliver thickness is detected after the drafting zone. This system is based on the usual feedback control principles. Figures 1 and 2 show the basic schematic diagrams of the two systems.

The solid lines with arrow heads indicate the paths along which the sliver thickness signal is transmitted. Function of s written in each square space in Figures 1 and 2 is the linear transfer function chosen to approximate the characteristics of the particular element.
These transfer functions are discussed in section 2.

In sections 3 we express, by using the mathematical models developed in section 2, the effectiveness of the servo-draft systems in terms of the reducing ratio \( A(u) \) which is essentially the overall transfer function between the input and the output sliver thickness.

Section 4 shows how various parameters in the mathematical model for each servo-draft system are adjusted to give the best control results. Section 5 discusses the relative advantages of the open-loop system and the closed-loop system in terms of the reducing ratio \( A(u) \).

1. Symbols and Assumptions

To obtain comparatively simple mathematical models which approximate the performance of the servo-draft systems, let it be assumed that each element of the schematic diagrams shown in Figures 1 and 2 can be approximated by a simple linear transfer function, such as a first order lag or a time delay element. The mathematical models are based on the assumptions:

1. That the measuring element is approximated by a constant \( K_m \). That is to say, the measuring element measures the variations in the input sliver thickness accurately without any attenuation or time lag.

2. That the final control element which consists of a speed-regulating mechanism for the drafting rollers is approximated by a first order lag.

3. That the computing element of the open-loop system (Fig. 1) is approximated by a constant \( K_r \) and a time delay element \( e^{-Ls} \) which compensates for the time \( L \) which the sliver takes to travel from the detecting element to the drafting zone. (Most of existing open-loop servo-draft systems are equipped with such time delay units.)

Since the closed-loop servo-draft systems are feedback control systems, the computing element can take many different forms. Here, however, we consider only two of the possible forms: proportional plus integrating action \( K_c \left( \frac{1}{1 + T_i s} \right) \) and a first order lag \( \frac{K_c}{1 + T_b s} \).

4. That the draft ratio is the ratio of the thickness of the sliver entering the drafting zone to the thickness of the sliver leaving the same zone, and is a function of time. It is also assumed that the draft ratio is completely controlled by the final control element, with no slip between the draft roller surface and the sliver. It is assumed further that sliver thickness does not change in the servo-draft system except in the drafting zone. The following notations and symbols are used in the discussion hereinafter. Draft ratio \( d \) in Table 1 is defined by the following equation:

\[
d = \frac{y}{x} \quad \ldots \quad (1)
\]

\[
d_0 = \frac{y_0}{x_0} \quad \ldots \quad (2)
\]

By substituting the relations in Table 1 into equations (1) and (2) and neglecting a small term, we get:

\[
\begin{align*}
x_1 & = \frac{y_1}{x_1} - d_1 \quad \ldots \quad (3) \\
x_0 & = \frac{y_0}{x_0} - d_0
\end{align*}
\]

Equation (3) endorses that the drafting zone can be approximated by a simple addition point in the block diagrams, which will be discussed in the next section, if the variables are changed from \( x, y, d \) to \( x_1, y_1, d_1 \).

\( X_1(s), Y_1(s), Z_1(s), D_1(s) \) : Laplace transforms of \( x_1, y_1, z_1, d_1 \).

\( L \) : Time required to shift the sliver from the detecting element to the drafting zone, or vice versa

\( K_m \) : Proportional gain of measuring element

\( K_c \) : Proportional gain of computing element

\( T_i \) : Integrating time

\( G_c(s) \) : Transfer function of computing element

\( G_c(s) = e^{-Ls} K_c \) (Open-loop system) \ldots (4)

\( G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \) (Closed-loop system with proportional plus integrating action) \ldots (5)

\( G_c(s) = K_c \left(1 + \frac{1}{T_b s} \right) \) (Closed-loop system with first order lag) \ldots (6)

\( K_r \) : Proportional gain of final control element

\( T_a \) : Time constant of final control element

\( G_a(s) \) : Transfer function of final control element

\( G_a(s) = \frac{K_a}{1 + T_a s} \) \ldots (7)

\( L' \) : Time delay of time element in the open-loop system

<table>
<thead>
<tr>
<th>Sliver thickness</th>
<th>Mean value</th>
<th>Deviation from the mean value</th>
<th>Sum</th>
<th>Normalized deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>At time the sliver leaves the drafting zone</td>
<td>( x_1(t) )</td>
<td>( x_1(t) - x_1(t) )</td>
<td>( x_1(t)/x_1(t) )</td>
<td></td>
</tr>
<tr>
<td>At time the sliver enters the drafting zone</td>
<td>( y_0(t) )</td>
<td>( y_0(t) - y_0(t) )</td>
<td>( y_0(t)/y_0(t) )</td>
<td></td>
</tr>
<tr>
<td>In the detecting element</td>
<td>( z_0(t) )</td>
<td>( z_0(t) - z_0(t) )</td>
<td>( z_0(t)/z_0(t) )</td>
<td></td>
</tr>
<tr>
<td>Draft ratio</td>
<td>( d_0 )</td>
<td>( d_0(t) - d_0(t) )</td>
<td>( d_0(t)/d_0(t) )</td>
<td></td>
</tr>
</tbody>
</table>
ω: Angular frequency of sinusoidal variations in sliver thickness

To normalize various time constants in terms of $T_0$, we define several new constants as follows:

$$\gamma = \frac{L'}{T_0}, \quad \delta = \frac{L - L'}{T_0}$$

(8) \sim

$$\delta = \frac{L}{T_0}, \quad \alpha = \frac{T_s}{T_0}, \quad \beta = \frac{T_s}{T_0}$$

(13)

$u$: Frequency variable defined as $u = \omega T_0$ ...... (14)

$s$: Variable of Laplace transforms

$$s = j\omega \cdot j \frac{u}{T_0}$$

(15)

$A(u)$: Reducing ratio defined as

$$A(u) = \frac{X_1(s)}{y_0}$$

where $s$ is as given in equation (15).

On the preceding assumptions, the schematic diagrams of servo-draft systems shown in Figures 1 and 2 can be reduced to the block diagrams shown in Figures 3 and 4.

2. Reducing Ratio

To make effective use of the linear mathematical models developed in the preceding section, it is helpful if we define a measure of effectiveness of the servo-draft systems in relation to the models. The effectiveness of the servo-draft systems may be evaluated from many different points of view. Here, however, we evaluate their effectiveness in terms of their frequency response, which is the relation between the amplitude of input and output sliver thickness for sinusoidal variations.

The reducing ratio $A(u)$ is defined in equation (16) as the ratio of the normalized amplitudes (see Table 1) of the sinusoidal thickness variations in the input sliver to that of the output sliver. In linear systems such as the mathematical models shown in Figures 3 and 4, $A(u)$ depends only on the various time constants in the servo-draft systems but not on the amplitudes of thickness variations. Because $A(u)$ is defined with the use of the normalized amplitudes $y_1/y_0$ and $x_1/x_0$ (which latter is the ratio of the amplitudes of thickness variations to the mean thickness), it is independent of the mean draft ratio $d_0$.

Note that the definition of $A(u)$ by equation (16) is valid both for the open-loop and closed-loop systems, since in the block diagram in Figure 3, $|Z_1(s)|$ is always equal to $|Y_1(s)|$ and $|Y_1(s)|$ can be used to represent the input amplitude in place of $|Z_1(s)|$.

In evaluating the effectiveness of the servo-draft systems in terms of the reducing ratio $A(u)$, the following properties of $A(u)$ should be born in mind: $A(u) = 0$ indicates that the servo-draft system completely eliminates sinusoidal thickness variations at this frequency. $A(u) = 1$ means that the normalized amplitude of sinusoidal thickness variations is not subject to any change in the servo-draft system. $A(u) > 1$ means that the normalized amplitude is magnified.

It follows, then, that $A(u)$ should be close to zero in the frequency range in which sliver thickness variations are desired attenuated, and that the peak value of $A(u)$ should not far exceed 1.

Reducing ratios for the block diagrams shown in Figures 3 and 4 can be easily calculated with the following results:

$A(u)$ for the open-loop system is:

$$A(u) = \frac{X_1(s)}{Y_1(s)} = 1 + \frac{K_f e^{-j\omega u}}{1 + ju}$$

where $K_f = K_m K_c K_a$ ...... (17)

$A(u)$ for the closed-loop system with proportional plus integrating action is:

$$A(u) = \frac{X_1(s)}{Y_1(s)} = \frac{1}{1 + K_d (1 + ju) e^{-j\omega u}}$$

where $K_d = K_m K_c K_a (x/d_0)$ ......... (17)'

![Fig. 3. Block diagram of the open-loop servo-draft system.](image)

![Fig. 4. Block diagram of the closed-loop system.](image)
$A(u)$ for the closed-loop system with first order lag is:

$$A(u) = \left| \frac{X_1(s)}{x_0} \right| \frac{Y_1(s)}{y_0} = \frac{1}{1+K_d (1+j\mu)(1+j\beta u)}$$

where $K_d$ is as given in equation (18).

### 3. Optimum Values for Various Parameters

Before we can compare the three different systems in terms of $A(u)$ functions (equations (17)-(19)), it is necessary to get the optimum values of the undetermined parameters $K_f$, $\gamma$, $\alpha$, $\beta$ and $K_d$.

#### 3-1. Open-Loop System

Two parameters, $K_f$ and $\gamma$, in equation (17) must be determined in such a way as to give an optimum $A(u)$ curve.

$K_f = 1$ is obviously the best value for $K_f$, since it makes $A(u) = 0$ for $u = 0$. In other words, a stepwise change in input sliver thickness leaves no permanent deviation in output sliver thickness. $T_o$ find the best value for $\gamma$, substitute into equation (17) $K_f = 1$ and also the various values of $\gamma$ and plot the results of such substitution on a semi-log diagram. (See Fig. 5.)

This figure shows that the $A(u)$ curve for $\gamma = -0.6$ gives low $A(u)$ values in the lower frequencies and not an unreasonably high peak value (about 1.12) in the intermediate frequency range. $\gamma = -0.6$, then, seems to be the best value.

#### 3-2. Closed-Loop System with Proportional Plus Integrating Action

Equation (18) leaves two parameters, $\alpha$ and $K_d$, for determination. Since this is a feedback control system, the conventional feedback theories can be used to discover the best $\alpha$ and $K_d$ values. The criterion used here chooses $\alpha$ which satisfies

$$\alpha = 1/u_n$$

where $u_n$ is the smallest of the $u$'s that satisfies $A(u) = 1$.

Once $\alpha$ is determined, $K_f$ is determined by a graphical method in such a way as to make the maximum value of $A(u)$ a reasonable magnitude, e.g., 1.1. Table 2 shows the values of $\alpha$ and $K_d$ thus calculated for the three different values of $\delta$.

The curves (B), (C) and (D) in Figure 6 are obtained by substituting values in Table 2 into equation (18).

#### 3-3. Closed-Loop System with First Order Lag

Equation (19) leaves two parameters, $\beta$ and $K_d$, for determination. Since there is no integrating element in the feedback loop of this servo-draft system, a stepwise change in input sliver thickness leaves a permanent error in output sliver thickness. The relation between the $K_d$ value and the magnitude $x_\infty$ of the permanent error in output sliver thickness is given by the relation

$$\frac{x_\infty}{y_\infty} = \frac{1}{1+K_d}$$

where $y_\infty$ is the magnitude of the stepwise change in the input sliver thickness. Equation (22) shows that to make the permanent error $x_\infty$ small, $K_d$ should be made large.

However, $K_d$ should not be too large, because it would tend to make the maximum value of $A(u)$ large and the servo-draft system more unstable.

Therefore, a trial-and-error method has been used to get the best values for $K_d$ and $\beta$. The method is, first, to assign an arbitrary value such as 10 to $K_d$, then obtain $\beta$ by finding the $\beta$ value.
that makes the absolute value of the loop transfer function \(-20\) db at \(-180^\circ\) phase angle. If such \(\beta\) does not exist, or if the \(\beta\) value obtained is unreasonably large, the whole process has to be repeated with a reduced value of \(K_a\). The best \(K_a\) and \(\beta\) values obtained in this manner for \(\delta = 1\) are \(K_a = 10\) and \(\beta = 100\).

Curve (A) in Figure 6 is obtained by substituting \(\delta = 1\), \(K_a = 10\) and \(\beta = 100\) into equation (19).

4. Open-Loop System and Closed-Loop System Compared

In the light of the information obtained in the preceding sections, a comparison, in terms of \(A(u)\) function, is made of the three different systems — the open-loop system, the closed-loop system with proportional plus integrating action and the closed-loop system with first order lag. The \(A(u)\) curve for \(\gamma = 0.6\) in Figure 5 is re-plotted, as curve (E), in Figure 6. Curves (A), (B) and (E) in Figure 6, then, represent the reducing ratio \(A(u)\) of the three systems mentioned above when they have identical time delay \(L\) and time constant \(Ta\) (See Figs. 1 and 2) and when the parameters of each system are set to optimum values.

Figure 6 shows that the open-loop system is superior to the closed-loop system, in that its \(A(u)\) function remains small in a much wider frequency range than the closed-loop system does. In other words, the open-loop system can attenuate sliver thickness variations at higher frequencies than the closed-loop system. However, there is an inherent drawback in the open-loop system: it cannot correct the errors that arise from changes in component characteristics during long-term operations. This drawback can be remedied by using simultaneously, in one servo-draft system, both the open-loop and closed-loop features.

Such a combined system has two detecting elements — before and after the drafting zone — and the final control element responds to the sum of the two signals from the two detecting elements, as shown in Figure 7. It can be easily shown that the reducing ratio \(A(u)\) of this combined system is the product of the reducing ratios of the open-loop part and the closed-loop part. The dotted line in Figure 6 is the product of curves (B) and (E), and represents the reducing ratio of the combined system which consists of the open-loop and closed-loop systems whose reducing ratios are given by curves (B) and (E), respectively. It is considered that, in such a scheme, the open-loop part works primarily in the higher frequency range and the closed-loop part primarily in the lower frequency range.

Figure 6 shows also that curves (A) and (B) are almost identical except in the intermediate frequency range, and even therein the difference is minor. This means that a properly designed closed-loop system with a first order lag is comparable in effectiveness with the closed-loop system having proportional plus integrating action. This may suggest a way to simplify servo-draft systems.

Conclusions

The results of the preceding discussions may be summed up thus:

1. The open-loop system (see the block diagram in Figure 3) gives the best control results.
when the parameters satisfy the equations

\[ \gamma = \frac{L' - L}{T_a} = -0.6 \quad \ldots \ldots (23) \]

\[ K_f = K_wK_rK_x = 1 \quad \ldots \ldots (24) \]

(2) A properly designed closed-loop system having a first order lag as the computing element is comparable in effectiveness with a closed-loop system with proportional plus integrating action.

(3) For identical time delay \( L \) and time constant \( T_a \), the open-loop system is superior to the closed-loop system, in that the former is effective in a far wider frequency range.

(4) Open loop servo-draft systems have the inherent drawback of not being able to correct the errors in output sliver thickness which arise from changes in component characteristics during operations in a mill. This drawback can be eliminated if the closed-loop feature is added to the open-loop system as shown in Figure 7.

(5) In most servo-draft systems there are frequency ranges in which sinusoidal variations in input sliver thickness are magnified, rather than attenuated. It is, therefore, necessary that servo-draft systems be so designed as to prevent the major frequencies of input sliver irregularities from falling within these frequency ranges and to make the maximum value of \( A(u) \) not very much larger than 1.

(6) In the open-loop system, time delay \( L \) between the detecting element and the drafting zone can be completely compensated for, at least in theory, by the time delay mechanism in the computing element. Therefore, the frequency range in which \( A(u) \) is small widens without limit as the time constant \( T_a \) of the final control element is decreased.

In the closed-loop system, both \( L \) and \( T_a \) affect the width of the frequency range in which \( A(u) \) is small, therefore, it is made \( L \) and \( T_a \) desirable to small.

These investigations, made with simple mathematical models, have given us a rough idea of how servo-draft systems work, and will guide us in planning our future experiments. The authors hope, in the light of these results, to revise mathematical models to make them more accurate as further experimental data become available.

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