Conjugate heat transfer in a total heat exchanger with cross-corrugated triangular ducts in laminar flow

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Abstract
Membrane-based total heat exchanger is a device to recover both sensible heat and moisture from exhaust air stream from a building. Heat and mass transfer intensification has been undertaken by using a structure of cross-corrugated triangular ducts. Conjugate heat under laminar flow regime in this total heat exchanger are investigated. Contrary to the traditional methods of assuming a uniform temperature or a uniform heat flux boundary condition, in this study, the real boundary conditions on the exchanger surfaces are obtained by the numerical solution of the coupled equations that govern the transfer of momentum and energy in the two air streams and in the membrane materials. The naturally formed heat boundary conditions are then used to calculate the cyclic mean Nusselt numbers along the exchanger ducts. The data are compared with those results under uniform temperature and uniform heat flux boundary conditions. The heat transfer will change with different thermal wall boundary conditions. Synergy principle is applied to reveal the Nusselt number variation with various thermal wall boundary conditions.

Key words: Conjugate heat transfer, Cross-corrugated triangular ducts, Synergy principle, Membrane, Total heat recover

1. Introduction

Conditioning ventilation air typically constitutes 20% to 40% of the thermal load for commercial buildings and can be even higher in buildings that require 100% outdoor air to meet ventilation standards (Zhang, 2012). It is well known that energy recovery devices could save a large fraction of the thermal load, because heat and humidity would be recovered from the exhaust stream in winter, and excess heat and moisture would be transferred to the exhaust in order to cool and dehumidify the incoming air (fresh) in summer. With energy recovery devices, the efficiency of existing HVAC systems can also be improved because otherwise fresh air needs to be dehumidified by cooling coil through condensation followed by a re-heating process, which is very energy intensive (Liang, 2014).

Membrane-based total heat exchanger has attracted much attention to fulfill this task (Zhang and Jiang, 1999; Kister and Cussler, 2002; Nasif, et al., 2010; Mahmud, et al., 2010). The device is just like an air-to-air parallel plate sensible heat exchanger. But in place of traditional metal heat exchange plates, hydrophilic membranes, which can transfer both heat and moisture simultaneously, are used as the heat and mass transfer media. The device has many virtues like it is stationary, compact, and easy to construct. However, practical application until now is still scarce. The reason is that heat and mass transfer in the unit is slow, which limits their market penetrations.

To intensify heat and mass transfer, in this study, a novel duct structure, cross-corrugated triangular ducts, is used to augment heat and mass transfer in air side. The structure is shown in Fig.1. It has similar geometry as chevron plates used in traditionally heat exchangers (primary surface heat exchangers). But the cross section is triangular other than sinusoidal. Triangular cross sections are naturally formed by corrugations of ultra-thin materials like paper, plastic film,
tinsel, and hydrophilic membrane, which are increasingly used in air conditioning industries, due to their superiorities in weight-lightness, cheapness, and abilities in selective transfer. Literature review found that though heat transfer (Biomerius, et al., 1999; Han, et al., 2010; Liu and Wu, 2013; Zhang and Che, 2011) and mass transfer (Tzanetakis, et al., 2004; Hall, et al., 2001; Scott and Lobato, 2003) in chevron plate heat exchangers have been investigated by various investigators, cross corrugated triangular ducts are less fully studied.

![Fig.1. Schematic of a cross-corrugated triangular duct heat and mass exchanger.](image)

Of the limited number of studies with this structure, the boundary conditions are assumed either as uniform temperature or uniform heat flux boundary conditions (Zhang, 2005a, 2005b; Zhang, 2007a, 2007b; Zhang and Chen, 2011). That assumption may hold for common metal heat exchangers. However, for a total heat exchanger, the heat and mass transfer in the ducts are closely coupled with the membranes. A uniform temperature or a uniform heat flux boundary condition is not justified. This is a conjugate heat mass transfer problem. To address this problem, in this study, the heat transfer under real boundary conditions will be considered. This is a naturally formed boundary condition resulted from the interactions between the two flows, and the membrane.

**Nomenclature**

- $A_{cyc}$: surface area of a flow cycle ($m^2$)
- $A_{ci}$: cross-sectional area at inlet ($m^2$)
- $c_p$: specific heat of fluid ($kJ kg^{-1} K^{-1}$)
- $D_h$: hydrodynamic diameter (m)
- $D_{ma}$: moisture diffusivity in dry air ($m^2/s$)
- $f$: friction factor
- $H$: changnel height (mm)
- $h$: heat flux ($kW/m^2$)
- $h_{ct}$: convective heat transfer coefficient ($kW m^{-2} K^{-1}$)
- $Nu$: Nusselt number
- $P$: pressure (Pa)
- $Sc$: Schmidt number
- $Sh$: Sherwood number
- $Re$: Reynolds number
- $T$: temperature (K)
- $u$: velocity (m/s)
- $V_a$: air flow rate ($m^3/s$)
- $V_{cyc}$: volume of a flow cycle ($m^3$)
2. Numerical method and equations

2.1. Governing Equations

Laminar flow is assumed. The general form of the mass continuity equation in the duct is shown below as

\[ \frac{\partial u_i}{\partial x_i} = 0 \]  

where \( u_i \) is the flow velocity component (m/s).

For fully developed laminar flow in ducts, the Navier-Stokes equations reduce to

\[ \rho \frac{\partial}{\partial x_j} \left( u_i u_j \right) = - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \]  

where \( P \) and \( \mu \) are pressure (Pa) and dynamic viscosity (Pa·s), respectively.

For fully incompressible flow, the energy transport equation is simplified as

\[ \frac{\partial}{\partial x_i} \left( \rho c_p u_i T - \lambda \frac{\partial T}{\partial x_i} \right) = 0 \]  

The hydraulic diameter of the channel is defined as

\[ D_h = \frac{4V_{cyc}}{A_{cyc}} \]  

where \( V_{cyc} \) and \( A_{cyc} \) are the volume and the surface area of a cycle in the channel, respectively.

The Reynolds number, Re, is
\[
\text{Re} = \frac{D_\text{h} u_m D_\text{h}}{\mu_n}
\]

where \( u_m \) is the area-weighted mean velocity in the inlet (m/s).

The heat transfer coefficient is evaluated from

\[
h = \frac{c_p \rho u_m A_{ci}}{A_{\text{cyc}} \Delta T_m}
\]

where \( c_p \) is the specific heat of fluid, kJ/(kg K); \( A_{ci} \) is the cross-sectional area at inlet or outlet, (m\(^2\)); \( T_i \) and \( T_o \) are fluid temperature at inlet and outlet, respectively (K); \( \Delta T \) is the logarithmic temperature difference between the wall and the fluid, which is calculated by

\[
\Delta T_m = \psi \left( \frac{T_i - T_w}{T_o - T_w} \right)
\]

where \( T_w \) represent the mean surface temperature. \( \psi \) is correction factor and adopted 0.96 in this study.

The Nusselt number is defined as

\[
Nu = \frac{hD_\text{h}}{\lambda}
\]

Previous studies have used heat and mass transfer analogy to get mass transfer coefficients from heat transfer correlations. According to Chilton-Colburn analogy, the relationship between \( Sh \) and \( Nu \) is

\[
Nu = Sh \cdot Le^{1/3}
\]

where

\[
Le = \frac{Pr}{Sc}
\]

\[
Pr = \frac{c_p \mu}{\lambda}
\]

\[
Sc = \frac{\mu}{\rho D_{va}}
\]

The cycle-average friction factor is calculated by

\[
f = \frac{(P_i - P_o)}{\rho u_m^2 L_{\text{cyc}}}
\]

where \( L_{\text{cyc}} \) is the length of a cycle, (m); \( P_i \) and \( P_o \) are pressure at inlet and outlet of a cycle, respectively, (Pa).

2.2 Synergy principle

Guo et.al (2005) defined a synergy angle \( \beta \) as the included angle of temperature gradient and velocity vector. And they deducted Nu as a function of \( Re \) and \( Pr \) and \( \cos \theta \)

\[
Nu_x = Re_x Pr \int_0^1 (\overline{U} \cdot \overline{V}) d\overline{y}
\]

and

\[
\overline{U} \cdot \overline{V} = \left| \overline{U} \right| \left| \overline{V} \right| \cos \beta
\]
where $\bar{U}$ is dimensionless velocity, $\nabla T$ is dimensionless temperature, $\beta$ is synergy angle.

For a three dimensional problem, $\beta$ has the form as (Jin et al, 2013; Saha et al, 2014)

$$\beta = \arccos \left( \frac{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z}}{|\bar{U}| |\nabla T|} \right)$$  \hspace{1cm} (16)

If $\beta = 0^\circ$, means temperature gradient and velocity vector direction are coincident, then convective heat transfer coefficient maximize. And if $\beta = 90^\circ$, the fluid flow has no contribution to heat transfer. The fully developed inner duct flow has higher Nusselt number under uniform heat flux wall boundary condition than under uniform temperature condition. It can be explained as $\beta$ is lower in former case. In this study, the synergy angle is used to interpret convective heat transfer extent under various thermal wall boundary conditions.

2.3. Solution method

A non-staggered mesh structure is used, and the size near wall is smaller than in duct. The meshes on the outside walls of a computational block are shown in Fig.2. This graph only depicts a part of the adoptive meshes, to get an amplified view of the mesh structure. Totally there are ten cycles in $x$ and $z$ direction and two layers of flow channels in $y$ direction.

![Fig.2. Part of the mesh structure.](image)

Boundary conditions are defined. Non-slip velocity wall conditions are assumed. In the interior of the core, the fresh air and exhaust air exchange moisture through the membrane layer. Thus the temperature on membrane surfaces are neither uniform temperatures nor uniform heat fluxes. That means the temperature varies along the membrane surfaces. Due to the small thickness in membrane (100 μm, thermal conductivity 0.127 W/(m·K), temperature differences between the two sides of a membrane are rather small. Favre (2003) found the temperature difference is in the order of 10-4°C. Zhang (2000, 2007a) explains this as heat released during adsorption (fresh air side, high humidity) is balanced by heat absorbed on the desorption side(exhaust air side, high humidity). Thus for heat transfer condition on the two sides of the same membrane, there has

$$\frac{\partial T_m}{\partial n} \bigg|_{(x,y,z)=(x_{m1},y_{m1},z_{m1})} = \frac{\partial T_e}{\partial n} \bigg|_{(x,y,z)=(x_{m2},y_{m2},z_{m2})}$$  \hspace{1cm} (17)

where subscript “m” refers to membrane, and “1” and “2” refer to fresh side and exhaust side at the same point of membrane, respectively.

For the top layer and the bottom layer membranes in the computational domain, uniform wall temperature boundary conditions are assumed, and the values are selected as 298K. In the interior, the boundary conditions on membranes are
naturally formed by the coupling between the fresh air and the exhaust air. A real exchanger usually has dozens of flow channels, which is difficult to simulate directly. So only two neighboring channels are considered for computation. At the inlet, velocity is set to uniform and norm to the inlet face, and the temperature for fresh air and exhaust air are 308K and 298K respectively, which corresponds to summer condition. The inlet humidity ratio in fresh air and exhaust air are set to 0.024kg/kg and 0.011kg/kg respectively. The adopted model dimensions and boundary conditions are summarized in Table 1.

The governing equations are solved by using standard finite difference methods that employ control-volume based discretization techniques along with a pressure-correction algorithm. The N-S equations are solved by SIMPLEC scheme, while the convective term in the energy equation is solved by first-order upwind implicit approximation, and the diffusive term is by second-order central difference scheme.

The grid independency test has been done. The calculations were primarily carried out with three different grid densities, 791951, 1197212 and 9775468 nodes. The channel fully developed periodic mean pressure drop and temperature change for the two fine grids are almost the same and 4.6% higher than that for the coarse grid. For the finest grids, 9775468, the solution time is very long, which is hard to use practically. Based on the above experience, which establishes the grid independency, the final calculations are performed for the 1197212 grids and the results obtained in this paper refer to the grid geometry mentioned above.

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3. Results and discussions
3.1 Model validation

For validation of this model, the experimental data from reference (Scott and Lobato, 2003) are compared with the simulation result. The geometry of the cross-corrugated duct selected in reference (Scott and Lobato, 2003) is: \( H=1 \text{mm} \), \( W=2 \text{mm} \), \( L_{cyc}=2 \text{mm} \), \( \theta=90^\circ \), \( \gamma=90^\circ \). The data is given in the form of \( Sh \) in this reference. So the calculated \( Nu \) in this study is converted to \( Sh \) through Chilton-Colburn analogy. The maximum deviation is below 20%.
Fig.3. Comparisons of calculated Sherwood numbers with those from literature.

3.2 Flow distribution

Figures 4 shows the velocity contours and vectors in the y-z plane at the middle section of a flow channel, for Re=100. To give an amplified view, only 3 cycles in the inlet is given. As seen from this figure, the flow pattern in the upper half of the channel is similar as that in a right triangular duct. But in the lower troughs, fluid become impinge on the frontal walls and thus leads to a re-circulation or swirl flow. This flow pattern will be periodically fully developed after 3-5 cycles in the flow direction, as depicted many times by Zhang (2005a, 2005b, 2005c). To give the velocity vectors here, is to get a clear comparison between temperature contours in the flow channel and verify the synergy angles’ values. Because this flow is mainly caused by forced convection, the effect of temperature difference has on fluid flow is neglected. So the thermal boundary conditions on the membrane won’t affect the velocity distribution. Fig.4 is valid for uniform wall temperature, uniform heat flux and conjugated boundary conditions.

Fig.4. Velocity contours (m/s) in the y-z plane at the middle section of a flow channel, Re=100.

3.3 Temperature distribution and synergy angle

Figures 5-7 are dimensionless temperature contours and cosine values of synergy angle under various thermal wall boundary conditions. Both the dimensionless temperature and cosine values of synergy angle ranges from 0 to 1, thus the same legend is used for them in each figure. Fig.5 represents the case of uniform heat flux boundary condition on the membrane surface. It can be seen that $\cos \beta$ approximates to 1 near the wall surface of the valleys. It means the velocity direction is coincident with temperature gradient in these locations, which can enhance heat transfer. However, as depicted in Fig.4, the velocity magnitude in the lower troughs is far less than in the upper half of the flow channel. So, the inlet velocity should be as large as possible if the operating pressure drop allowed. Bring these figures for comparison, the temperature contours have same trend for uniform heat flux boundary condition and conjugated wall boundary condition. For value of $\cos \beta$ approaching to 1, the case of uniform heat flux boundary condition accounts for largest area, while the case of uniform wall temperature condition accounts for smallest area. This means velocity and temperature have best synergism effect for flow channels in uniform heat flux boundary condition. The synergism effect for conjugated wall boundary condition lies between uniform heat flux boundary condition and uniform wall
temperature condition. Detailed value of volume-averaged $\cos\beta$ will be discussed in later paragraph.

Figure 8 depicts the dimensionless temperature contours on the membrane surface under coupled wall boundary condition. Location $z=0$ represents the fresh air inlet and location $x=0$ represents the exhaust air inlet. It can be seen that the temperature contours are also periodical along the flow direction. The highest and lowest temperature value shown in the peaks and troughs are actually in the locations of this membrane contacting with neighboring membranes. The heat transfer is fully developed after 6 cycles along both the fresh air flow direction and exhaust air flow direction. So only the cells in thermally fully developed regions can be used for Nusslet number calculation.

Fig. 5. Dimensionless temperature and $\cos\beta$ contours at the middle section of a flow channel, for uniform heat flux boundary condition and Re=100 (a) Temperature (b) $\cos\beta$.

Fig. 6. Dimensionless temperature and $\cos\beta$ contours at the middle section of a flow channel, for uniform wall temperature boundary condition and Re=100 (a) Temperature (b) $\cos\beta$. 
Fig. 7. Dimensionless temperature and \( \cos \beta \) contours at the middle section of a flow channel, for conjugated wall boundary condition and \( \text{Re}=100 \) (a) Temperature (b) \( \cos \beta \).

Fig. 8. Dimensionless temperature contours on the membrane surface, for conjugated boundary condition, \( \text{Re}=100 \).

3.4 Nusselt numbers and friction factors

Figure 9 shows thermally fully developed Nusselt numbers of cross-corrugated triangular duct. For comparison, the data for apex angle of 60° calculated by Zhang (2005b) is also plotted in Fig.9. The subscript H means all the membrane surface is set as uniform heat flux boundary conditions. If the boundary conditions are under uniform temperature, the Nusselt numbers are defined as \( \text{Nu}_T \). If the boundary conditions are under coupled wall heat boundary condition, they are \( \text{Nu}_F \). It can be seen from Fig.9, \( \text{Nu}_H \) is highest among the applied boundary conditions in the full calculated \( \text{Re} \) range. It is 1.14 to 1.26 times of \( \text{Nu}_T \) and 1.35 to 1.50 times of \( \text{Nu}_F \).

For fully developed inner duct flow, \( \text{Nu}_H \) is always higher than \( \text{Nu}_T \) but little study gave the reason. Guo, et al. (2005) explains this problem using synergy principle. In this study, the volume-averaged cosine values of synergy angle under various wall boundary conditions in thermally fully developed cells are calculated and plotted in Fig.10. As the trend depicted in Figs 4-6, the volume-averaged \( \cos \beta \) is largest for uniform heat flux boundary condition. And it is lowest for uniform temperature boundary condition. That means \( \cos \beta \) under coupled wall boundary condition lies between the other two wall boundary conditions. Thus from equations (14) and (15), it can be deducted that \( \text{Nu}_C \) is lower than \( \text{Nu}_H \) and higher than \( \text{Nu}_T \). The Nusselt number under same wall boundary condition is higher for 90° apex.
angle than 60º apex angle. This is due to larger dead zones existing in 60º apex angle duct.

![Figure 9](image1.png)

Fig.9. Fully developed $Nu$ versus $Re$ under various wall boundary conditions.

![Figure 10](image2.png)

Fig.10. Fully developed volume-averaged $cos\beta$ versus $Re$ under various wall boundary conditions.

![Figure 11](image3.png)

Fig.11. Variations of fully developed cyclic mean friction factor versus $Re$.

Figure 11 shows variations of fully developed cyclic mean friction factor versus $Re$. Data from reference Zhang (2005b).
(2005b) is also given for comparison and validation. Generally the cross-corrugated triangular ducts with apex angle of 90° has friction factor of 1.3-1.5 times of those with apex angle of 60°. The correlations used to estimate the fully developed cyclic mean friction factors and cyclic mean Nusselt numbers are given below. They are suitable for cross-corrugated triangular ducts with apex angle of 90° in the Reynolds number range of 100-500.

\[
\begin{align*}
  f &= 8.473 \times \text{Re}^{-0.406} \\
  Nu_H &= 1.299 \times \text{Re}^{0.335} \times Pr^{0.333} \\
  Nu_C &= 0.796 \times \text{Re}^{0.392} \times Pr^{0.333} \\
  Nu_T &= 0.641 \times \text{Re}^{0.401} \times Pr^{0.333}
\end{align*}
\]

4. Conclusions

Conjugate heat and mass transfer in a total heat exchanger, which uses cross corrugated triangular duct structure to augment air side heat and mass transfer, is investigated here. Through experimental and numerical analysis, following results can be found:

(1) For the cross-corrugated triangular ducts, both the flow and the heat and mass transfer demonstrate cyclic patterns. At the entrance region, the cyclically mean values of friction factor, Nusselt and Sherwood numbers decrease rapidly along the flow. After 5-6 cycles, both the fluid flow and heat and mass transfer become fully developed. After the entry regions, the cyclic mean friction factors, Nusselt and Sherwood numbers come to stable values.

(2) The conjugated boundary conditions are neither uniform temperature nor uniform heat flux boundary conditions. This naturally formed boundary condition have effect on the Nusselt and Sherwood numbers. The synergy angle in the cross-corrugated triangular duct is calculated to evaluate the synergy performance of velocity vector and temperature gradient. And the cosine values of synergy angle calculated under these boundary conditions are compared. The result show that Nusselt number under naturally formed boundary condition lies between uniform heat flux and uniform temperature boundary conditions.

(3) Compared to flow duct with 60° apex angle, cross-corrugated triangular duct with 90° apex angle has enhancement on Nut by 10% to 60%, but accompanied with a friction factor increasing by 27% to 44%.

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