Effect of natural ventilation on transmission load of building external walls and optimization of insulation thickness

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Abstract
The effect of natural ventilation on yearly transmission load and optimum insulation thickness of different orientation external walls is studied in this paper. In addition, the effects of insulation location and life-cycle cost analysis (LCC) model on optimum insulation thickness are also investigated. The research is performed for four cities in hot summer and cold winter zone in China. Expanded Polystyrene (EPS) is selected as the insulation material. A FORTRAN code developed by an implicit finite difference method is applied to calculate yearly cooling and heating transmission loads. A LCC method, which is according with the reality, is applied to determine optimum insulation thickness. Results show that insulation location almost has no effect on total yearly transmission load while natural ventilation plays a significant role in reducing the yearly cooling transmission load for all orientation walls. Moreover, it is found that natural ventilation results in obvious decrease of optimum insulation thickness and different LCC models lead to significant distinction in optimum insulation thickness. Research indicates that the effect of orientation on optimum insulation thickness cannot be ignored. At last, a sensitivity analysis on optimum insulation thickness is carried out and the results show that insulation price is the most sensitive factor.

Key words: Optimum insulation thickness, Natural ventilation, Cooling and heating transmission loads, Life-cycle cost analysis (LCC), Sensitivity analysis

1. Introduction

Energy shortage and environmental pollution are the most important issues all over the world. In China, energy consumption is rapidly increasing because of its population increase, urbanization and growth in the living standard. Building energy consumption accounts for 25–40% of the total commercial energy consumption, which is mainly caused by space heating and cooling. The amount of building energy consumption will be rapidly increased in the future (ARCBEE, 2009). Therefore, the environmental pollution caused by building energy consumption is becoming more and more severe. Due to the limited energy-sources and environmental pollution, building energy saving has become compulsory and is one of the key programs in the 12th Five-Year Plan (2010-2015).

The application of thermal insulation to building external walls is one of the most effective ways to save building energy due to its significant effect in reducing the heat transfer. Therefore, the selection of a proper insulation material and determination of its optimum thickness are particularly important (Yu et al., 2009). In the last several years, fruitful results from determination of optimum insulation thickness for external walls have been obtained (Hasan, 1999, Al-Sanea and Zedan, 2002, Çomakli and Yuksel, 2003, Al-Khawaja, 2004, Al-Sanea, et al., 2005, Bolattürk, 2006, Mahlia, et al., 2007, Bolattürk, 2008, Yu, et al., 2009, Ucar and Balo, 2010, Daouas, 2011, Ozel, 2011, Kayfeci, et al., 2013, Ozel, 2013). The relevant information of them are summarized in Table 1, which shows the methodologies applied to the calculation of cooling and heating loads, the economic models, insulation materials as well as the main results.

From Table 1, we can know that the topic of optimum insulation thickness for external wall has been paid more attention for a long time because of its importance in building energy saving. Furthermore, it is clear that there are mainly two methods to be used to calculate yearly cooling or heating load in the previous literatures. One of the methods is the degree-days (or degree-hours) concept, and the other is the dynamic time-dependent model. The former, which is based on static conditions, is
simple and rough. While the latter is considered to be able to obtain highly accurate results on the determination of optimum insulation thickness due to its advantages of considering the transient heat transfer process of the wall in practice (Al-Sanea, et al., 2013, Ozel, 2013, 2014). In addition, most studies which used the dynamic time-dependent model in the literatures have been conducted mainly under steady-periodic conditions.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Country</th>
<th>Methodology</th>
<th>Economic Model</th>
<th>Life time of building/ insulation material (years)</th>
<th>Insulation material</th>
<th>Optimal thickness (mm)</th>
</tr>
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<tr>
<td>Hasan, 1999</td>
<td>Palestine</td>
<td>Degree-days</td>
<td>LCC</td>
<td>10/10</td>
<td>Polystyrene</td>
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<td></td>
<td></td>
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<td>Rock wool</td>
<td>20-57</td>
</tr>
<tr>
<td>Al-Sanea and Zedan, 2002</td>
<td>Saudi Arabia</td>
<td>implicit finite-volume method (IFVM)</td>
<td>LCC</td>
<td>30/30</td>
<td>Molded polystyrene</td>
<td>93</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>Injected polystyrene</td>
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<td>Glass fiber</td>
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<td>Rock wool</td>
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<td>Polystyrene board</td>
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<td>Styropor</td>
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<td>Qatar</td>
<td>Solar-air degree days</td>
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<td>Fiberglass</td>
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<td>Al-Sanea et al., 2005</td>
<td>Saudi Arabia</td>
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<td>LCC</td>
<td>30/30</td>
<td>Expanded polystyrene</td>
<td>48-160</td>
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<td>Yu et al., 2009</td>
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<td>20/20</td>
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<td>60-180/30-240</td>
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<tr>
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<td></td>
<td>Perlite polyvinyl chloride</td>
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<td>Ucar and Balo, 2010</td>
<td>Turkey</td>
<td>Degree-days</td>
<td>P1-P2</td>
<td>10/10</td>
<td>Polystyrene</td>
<td>60-400</td>
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<td>Rock wool</td>
<td>60-400</td>
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<td>Daouas, 2011</td>
<td>Tunisia</td>
<td>*CFFT</td>
<td>LCC</td>
<td>30/30</td>
<td>Expanded polystyrene</td>
<td>101-117</td>
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<td>Ozel, 2011</td>
<td>Turkey</td>
<td>Implicit finite-difference Method (IFDM)</td>
<td>LCC</td>
<td>10/10</td>
<td>Extruded polystyrene</td>
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<td>Kayfeci et al., 2013</td>
<td>Turkey</td>
<td>Cooling degree-hours Formula</td>
<td>LCC</td>
<td>10/10</td>
<td>Glass wool</td>
<td>32-41</td>
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<td>Ozel, 2013</td>
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<td></td>
<td></td>
<td>Expanded polystyrene</td>
<td>136-160</td>
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</table>

*CFFT: Complex Finite Fourier Transform.

It is well known that natural ventilation can significantly reduce cooling load in summer and transient season and therefore it is considered to be an effective energy saving measure. The effect of natural ventilation on total yearly transmission load and optimum insulation thickness of external wall is a very important topic which is worthy of investigation. However, this important issue has not yet been thoroughly analyzed in the above literatures. Therefore, in this work, total yearly transmission load and optimum insulation thickness of external wall under the condition of natural ventilation will be investigated in detail.
Moreover, Table 1 also indicates the economic analysis methods which are commonly used to determine the optimum insulation thickness are life-cycle cost analysis (LCC) and P1-P2 method. In all previous studies, the lifetime of insulation material is deemed to be equal to that of building either in LCC or in P1-P2 method. But this is not in accordance with the actual situation. For example, in China, the lifetime of building is considerably larger than that of insulation material (GB 50068-2001). For this reason, this study focuses on the calculation of the optimum insulation thickness by fully considering the different lifetime between building and insulation material.

Additionally, it is quite clear in Table 1 that the optimum insulation thicknesses are very different because of the diverse climatic conditions, the distinct nature of the insulation materials, the different level of economic development, and so on. The results obtained in the literatures cannot be directly applied in engineering without thinking about their limiting conditions for application. Therefore, it is very meaningful for determining the optimum insulation thickness which is suitable for the actual situation of the region.

As far as the thermal insulation of building external walls is concerned, another interesting issue is the effect of insulation location on cooling and heating transmission loads and optimum insulation thickness. Several studies on this issue have been reported (Al-Regib and Zubair, 1995, Ozel and Pihtili, 2007, Kolaitis, et al., 2013, Ozel, 2014). Al-Regib and Zubair (1995) investigated the transient heat transfer through insulated walls for three different insulation locations. The results showed that cooling load for building was smaller for outside insulation than for inside insulation. Ozel and Pihtili (2007) determined optimum location and distribution of insulation in a wall. According to their results, the best thermal performance was obtained when each one of three equal pieces insulation layers was placed to the inside, middle and outside of the wall respectively. Besides, various wall orientations didn’t have a noticeable effect on the location of insulation. Kolaitis, et al. (2013) performed a comparative assessment of internal versus external thermal insulation systems for energy efficient retrofitting of residential buildings. Ozel (2014) explored the effect of insulation location on the heat transfer characteristics of building walls and the optimization of insulation thickness. He drew a conclusion that insulation location didn’t affect the yearly load and optimum insulation thickness. Their results proved that both external and internal thermal insulation could significantly reduce the total energy requirements, on average, external insulation outperformed the internal insulation by 8%.

In view of the above reasons, the main objectives of the present study are to investigate the effects of natural ventilation and insulation location on yearly cooling and heating transmission loads and optimum insulation thickness individually and collaboratively, and to study the effect of different LCC models on optimum insulation thickness. The investigation is carried out in hot summer and cold winter zone of China. Four typical cities such as Chengdu, Changsha, Hefei and Shanghai are selected because of their different climatic conditions. Meanwhile, EPS is chosen as the insulation material due to its extensive application. Therefore, it is very meaningful for determining the optimum insulation thickness which is suitable for the actual situation of the region.

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1. A FORTRAN code developed by an implicit finite-difference method is utilized to calculate yearly cooling and heating transmission loads of different orientation walls for the four cities under the condition of natural ventilation.

2. LCC model over a building lifetime of 50 years and EPS lifetime of 20 years is applied to optimize insulation thicknesses. Moreover, the effect of LCC model on optimum insulation thickness is investigated.

3. The effects of insulation location and natural ventilation individually and collaboratively on total yearly transmission load of the external walls and optimum insulation thickness are studied.

4. Sensitivity analysis of optimum insulation thickness following the change of four uncertain factors is performed.

2. Physical and mathematical description

2.1 Wall structure

The investigation is carried out for a typical external wall of building. The structures of uninsulated and insulated walls are shown in Figs. 1(a), (b) and (c), respectively. The uninsulated wall consists of 20mm external cement plaster, 240mm brick and 20mm internal cement plaster. The external insulated wall is made up of 20mm external cement plaster, insulation material, 240mm brick and 20mm internal cement plaster. The internal insulated wall is composed of 20mm external cement plaster, 240mm brick, insulation material and 20mm internal cement plaster.
2.2 Mathematical model

(1) Governing equation

A composite wall structure, which consists of M parallel layers with different thickness and physical properties, is illustrated in Fig. 2.

\[
\rho_j c_j \frac{\partial T_j}{\partial t} = k_j \frac{\partial^2 T_j}{\partial x^2} \quad j = 1, 2, \ldots, M
\]  

where \( x \) is the space coordinate, \( t \) is the time, \( T_j \) is the temperature, \( \rho_j, c_j \) and \( k_j \) are the density, the specific heat and the thermal conductivity of the \( j \)-th layer, respectively.

(2) Boundary conditions

The boundary conditions at the outdoor and indoor wall surfaces are respectively given as follows:

\[
\text{outdoor:} \quad -k_1 \left( \frac{\partial T_1}{\partial x} \right)_{x=0} = h_0 \left( T_e(t) - T_{x=0} \right) \quad (2)
\]

\[
\text{indoor:} \quad -k_M \left( \frac{\partial T_M}{\partial x} \right)_{x=L} = h_i \left( T_{x=L} - T_i \right) \quad (3)
\]
where, $h_0$ and $h_1$ are the combined (convective and radiative) heat transfer coefficients at the outdoor and the indoor wall surfaces, respectively. $T_i$ is the indoor air temperature, $T_e$ is the sol-air temperature including the effect of solar radiation on the outdoor temperatures and is expressed as (Lu, 2007):

$$T_i(t) = T_0(t) + \frac{\alpha I_t}{h_0}$$

(4)

where, $T_0$ is the outdoor air temperature, $I_t$ and $\alpha$ denote the total solar radiation and the solar absorptivity of the outdoor wall surface, respectively.

(3) Initial conditions

The following arbitrary uniform temperature field is assumed as initial condition:

$$T_j(x,0) = T_{init} \quad j = 1,2,...,M$$

(5)

where $T_{init}$ is the initial temperature.

(4) Calculation of transmission load

The instantaneous transmission load of the external wall can be written as follows:

$$q_i = h_i (T_{x,L} - T_i)$$

(6)

where $T_{x,L}$ is the inner surface temperature of the composite wall.

The daily cooling (positive) or heating transmission load (negative) ($Q_i$) can be calculated from $q_i$ by integration over a 24-h period as below:

$$Q_i = \int_0^{24} q_i dt$$

(7)

It is worth noticing that $q_i$ are summed up separately for cooling and for heating.

Consequently, the yearly cooling transmission load can be calculated from daily cooling transmission load which are added over the cooling season. Similarly, the yearly heating transmission load can be obtained from daily heating transmission load over heating season. Therefore, the total yearly transmission load equals the sum of absolute values of yearly cooling and heating transmission loads.

2.3 Calculation conditions

2.3.1 Climatic conditions of the four cities

The climatic conditions of the four selected cities are given in Table 2. It is seen from Table 2 that their climatic conditions are different from each other. Changsha occupies the highest mean outdoor temperature in the hottest month July while Hefei possesses the lowest one in the coldest month January. Moreover, for all the four cities, the heating degree-days (HDD18) are about 10 times higher than the cooling degree-days (CDD26), which indicates that in the hot summer and cold winter zone, the requirement of heating is much more than cooling over the whole year. In other words, the heating load dominates over that for cooling in the total yearly load. In addition, the method used to determine the periods of cooling and heating seasons is the same as that in reference (Huang, 2006). It is seen that Changsha possesses the longest cooling period while the longest heating period appears in Hefei.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Climate characteristics of the four selected cities.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Chengdu</td>
</tr>
<tr>
<td>Monthly mean outdoor temperatures in July(°C)</td>
<td>25.8</td>
</tr>
<tr>
<td>Monthly mean outdoor temperatures in January(°C)</td>
<td>5.8</td>
</tr>
<tr>
<td>CDD26 (°C-days)</td>
<td>31.7</td>
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<tr>
<td>HDD18(°C-days)</td>
<td>1372.2</td>
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</tbody>
</table>
2.3.2 Thermal parameters

Thermal properties of materials used in the wall structure are given in Table 3.

<table>
<thead>
<tr>
<th>Material</th>
<th>( k ) (W/m·K)</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( c ) (J/kg·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>brick</td>
<td>0.810</td>
<td>1800</td>
<td>1050</td>
</tr>
<tr>
<td>cement plaster</td>
<td>0.930</td>
<td>1800</td>
<td>1050</td>
</tr>
<tr>
<td>expanded polystyrene (EPS)</td>
<td>0.042</td>
<td>30</td>
<td>1380</td>
</tr>
</tbody>
</table>

2.3.3 Indoor design temperature

In this study, natural ventilation is taken into account only in cooling season. Hence, the value of indoor design temperature is determined based on the outdoor air temperature. In cooling season, the indoor design temperature is assumed to be 26°C when the outdoor air temperature is higher than 26°C. Otherwise, natural ventilation is employed which makes the indoor design temperature equal to the outdoor air temperature. In this situation, cooling transmission load is not existed. In heating season, the indoor design temperature is fixed at 18°C.

2.3.4 Relevant calculation parameters

The solar absorptivity of opaque wall is selected as 0.56 for cement plaster surfaces. The combined heat-transfer coefficient at the indoor is 8.7 W/m\(^2\)·K and at the outdoor wall surfaces is taken to be 19 W/m\(^2\)·K for cooling season and 23 W/m\(^2\)·K for heating season, respectively (Lu, 2007, GB 50176-93).

2.4 Numerical procedure

Equations (1)-(5), which are used to describe the transient heat conduction problem, are discretized by using an implicit finite-difference method. In the discretization process, a uniform grid is chosen and the special nodes are respectively set on the outside surface, on the inside surface and at the interface of layers \((j)\) and \((j+1)\). Meanwhile, the hourly meteorological data of typical meteorological year (TMY-DATA, 2005) for each of the four cities is input as the boundary condition on the outside surface. Then, the set of finite difference equations is solved by using TDMA method in FORTRAN procedure.

2.5 Independence verification

(1) Grid spacing independence verification

The uninsulated wall shown in Fig.1 (a) is selected as the research object in this section. The yearly cooling transmission load according to different grid spacing is calculated respectively. The time step is fixed at 120s while the grid spacing is set as 0.1mm, 0.2mm, 0.5mm, 1mm, 2mm, 5mm, 10mm and 20mm respectively. Fig. 3 indicates the influence of the grid spacing on the yearly cooling transmission load. It is obvious that yearly cooling transmission load decreases with the increase of the grid spacing. When the grid spacing is less than 1mm, the influence of the grid spacing on the yearly cooling transmission load can be ignored. Therefore, it can be accepted that grid independent solution is acquired as the grid spacing is 1mm.

(2) Time step independence verification

In this section, the research object is the same as the above, the yearly cooling transmission load according to different time step is calculated respectively. In the calculation, the grid spacing is fixed at 1mm while the time step is assumed as 1s, 10s, 60s, 120s, 180s, 300s, 360s, 600s, 900s, 1800s and 3600s respectively. Fig.4 shows the influence of the time step on the yearly cooling transmission load. It is seen that the yearly cooling transmission load decreases as the time step increases. When the time step is less than or equal to 120s, the influence of the time step on the cooling transmission load is relatively small. So, it can be received when the time step is 120s. In other words, time step independence solution is obtained.

From what has been discussed above, considering economy and computing speed, the grid spacing is chosen as 1mm and the time step is set to be 120s respectively in the subsequent calculation.
2.6 Validation of the method and program

In order to verify the method and program, a study is conducted on a single-layer wall which consists of brick of 240mm by comparing the simulation value with the analytical solution of the center temperature of the wall. It is assumed that at first the temperature field of the wall has a uniform value of 18°C. The temperatures of the left and right boundaries are suddenly enhanced respectively to 35°C and 20°C, the task is to determine the variation of temperature for the center temperature of the wall versus time. The 1-D transient heat transfer equation of the above problem can be written as

\[
\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2}
\]  

where, \( a \) is thermal diffusivity (m\(^2\)/s).

The boundary conditions are

\[
T|_{x=0} (t) = 35°C
\]

\[
T|_{x=L} (t) = 20°C
\]

The initial uniform temperature field is

\[
T|_{x=0} (x) = 18°C
\]

The analytical solution for Eqs. (8)-(11) can be written as below:

\[
T(x,t) = \sum_{n=1}^{\infty} c_n \exp \left(-a^2 \left(\frac{n\pi}{l}\right)^2 t\right) \sin \left(\frac{n\pi x}{l}\right) \left[35 + \frac{x}{l}(20 - 35)\right]
\]

where

\[
c_n = \frac{2}{l} \int_0^l \left[18 - \left(\frac{x}{l}\right)(20 - 35)\right] \sin \left(\frac{n\pi x}{l}\right) \, dx
\]

For the calculated case, \( a^2 \) is \( 4.29 \times 10^{-7} \) m\(^2\)/s and \( l \) is 0.24m.

Then, the discrete thermal conductivity model presented in this paper is used to solve this problem. In the calculation, the number of nodes is 241 and time step is 120s. The thermal properties of brick are the same as that listed in Table.3. As a result, the variation of the center temperature of the wall with a time of 24h is obtained. Fig. 5 presents the comparison of simulated and analytical values for the center temperature of the wall. It is seen that the simulated values agree well with analytical solutions, which validates that the method and program are suitable and correct. So, the numerical calculation method can be used to simulate heat transfer of building envelope accurately.
Fig.5 Comparison of simulated value and analytical value for the center temperature of the single-layer wall.

3. Life-cycle cost analysis and the optimization of insulation thickness

LCC method is regarded as the most complete approach to determine optimum insulation thickness because it has considered all the future cost, which includes the insulation cost and the energy consumption cost over the lifetime of the building. In China, the design working life of ordinary buildings is 50 years (GB 50068-2001), and the life of EPS is usually 20 years (Yu, et al., 2009). In this paper, a cash flow chart is used to calculate the net present value of total cost over the building lifetime.

From the economic point of view, optimum insulation thickness is relevant to many factors such as the cost of insulation material, the cost of energy consumption, the yearly heating and cooling loads, coefficients of performance for the cooling and heating equipment, building lifetime, interest and inflation rates, and so on.

In this paper, both cooling and heating equipment are assumed to be the air-source heat pump air conditioner which consume electricity. Therefore, the yearly energy cost for heating transmission load per unit area \( (C_{A,H}, \text{¥/m}^2) \) of the external wall can be obtained from:

\[
C_{A,H} = \frac{Q_H \cdot C_E}{3.6 \times COP_H}
\]  
(14)

where \( Q_H \) is the yearly heating transmission load (MJ/m\(^2\)), \( C_E \) is cost of electricity (¥/kWh) and \( COP_H \) is the coefficient of performance for heating system.

Similarly, the yearly energy cost for cooling transmission load per unit area \( (C_{A,C}, \text{¥/m}^2) \) of the external wall is given by:

\[
C_{A,C} = \frac{Q_C \cdot C_E}{3.6 \times COP_C}
\]  
(15)

where \( Q_C \) is the yearly cooling transmission load (MJ/m\(^2\)), \( COP_C \) is the coefficient of performance for cooling system.

So, the yearly total energy cost \( (C_A, \text{¥/m}^2) \) of the external wall can be calculated by:

\[
C_A = C_{A,H} + C_{A,C}
\]  
(16)

The cost of insulation material is given by:

\[
C_i = C_{ic} \cdot L_i + C_{iad} + C_{idm}
\]  
(17)

where \( C_{ic} \) is the cost of insulation material per unit volume (¥/m\(^3\)), \( L_i \) is the insulation thickness (m), \( C_{iad} \) is the additional cost of installing the insulation material (¥/m\(^3\)) and \( C_{idm} \) is the additional cost of dismantling the invalid insulation material (¥/m\(^2\)).

The cash flow over the building lifetime in this paper is shown in Fig.6. In the building lifetime, the cost of energy consumption occurs every year, while the cost of insulation appears at 0\(^{th}\), 20\(^{th}\) and 40\(^{th}\) year.
Fig. 6 Cash flow chart over the building lifetime.

The total cost is written as:

\[ C_i = C_{i0} + C_{i20} (1 + r)^{-20} + C_{i40} (1 + r)^{-40} + C_A \cdot PWF \]  \hspace{1cm} (18)

where, \( PWF \) is the net present value factor and is defined as below:

\[ PWF = \left( \frac{1 + r}{r(1 + r)^N} \right)^N - 1 \]

\[ i > g, r = \frac{i - g}{i + g} \]

\[ i < g, r = \frac{g - i}{1 + i} \]  \hspace{1cm} (19)

\[ PWF = \frac{N}{1 + i} \quad i = g \]  \hspace{1cm} (20)

where \( g \) is the inflation rate, \( i \) is the discount rate and \( N \) is the lifetime of building.

4. Results and discussion

4.1 Thermal performance of building walls

4.1.1 Effect of natural ventilation on yearly cooling transmission load

The research on the effect of natural ventilation on yearly cooling transmission load is carried out for an uninsulated wall shown in Fig. 1 (a). All orientations are taken into account in the calculation. Fig. 7 shows the yearly cooling transmission loads of external walls in Chengdu with and without considering natural ventilation for the south, the north, the east and the west orientations, respectively. It is revealed that natural ventilation plays a significant role in reducing the yearly cooling transmission load for all orientations. Moreover, it is seen that the highest yearly cooling transmission load appears in the west while the lowest appears in the north either with or without natural ventilation. Among all orientations, the highest reduction of yearly cooling transmission load is provided by west facing wall while the lowest is obtained in the north facing wall. Therefore, in Chengdu, natural ventilation in the west is the better choice in summer.

The saving rate of yearly cooling transmission load \( S_{CL} \) can be written as follows

\[ S_{CL} = \left( 1 - \frac{Q_{CL}}{Q_{CL}} \right) \times 100\% \]  \hspace{1cm} (21)

where \( Q_{CL} \) and \( Q_{CL} \) are the yearly cooling transmission loads (MJ/m²) with and without natural ventilation, respectively.

As is seen from Fig.8, the saving rates of yearly cooling transmission load for all orientation walls of the four cities are obviously different. Natural ventilation in Chengdu gives the largest \( S_{CL} \) compared with other three cities. The values of \( S_{CL} \) are 69.2-74.3\% in Chengdu, 26.8-29.7\% in Changsha, 21.5-23\% in Hefei and 18.5-23\% in Shanghai depending on different orientations, respectively. In addition, it is clear that the highest \( S_{CL} \) for the four cities respectively appears in the north of Chengdu, the south of Changsha and the west of Hefei and Shanghai. However, the lowest appears in the east of Chengdu and Changsha but the north of Hefei and Shanghai. From Fig.8, it can be concluded that in Chengdu, natural ventilation should be extensively applied to reduce the yearly cooling transmission load.
4.1.2 Effect of insulation location on yearly transmission load

In this section, the effect of insulation location on yearly transmission load is analyzed for the case of without natural ventilation. Fig. 9 indicates the variation of yearly cooling and heating transmission loads versus insulation thickness for west facing wall in Chengdu. It is found that internal insulation of the wall provides a little more yearly cooling transmission load and almost equal yearly heating load compared with external insulation. Besides, both yearly cooling transmission load and heating transmission load decrease as the insulation thickness increases, and their descent rates are more rapid at the smaller values of insulation thickness and become slow at the larger values. Moreover, it is seen that in the climatic condition of Chengdu, the yearly cooling transmission load is much less than the heating transmission load which is dominant in the whole year. Therefore, insulation location almost has no effect on total yearly transmission load of the external wall. These results are the same as those obtained by Al-Sanea and Zedan (2011) and Ozel (2014) in the different climates.

The saving rate of yearly cooling transmission load for west facing wall with different insulation thickness in Chengdu is shown in Fig.10. It is clear the saving rate of yearly cooling transmission load for the external insulation is slightly higher than that for the internal insulation. Based on the previous analysis, natural ventilation for west facing wall ( uninsulated) in Chengdu can lead to a saving rate of yearly cooling transmission load of 73%. From Fig.10, one will notice that natural ventilation has the same saving rate as the insulation thickness of 50mm for external insulation or 52mm for internal insulation.
4.1.3 Effect of combining natural ventilation and insulation location on yearly cooling transmission load

Figure 11 (a) illustrates the change of yearly cooling transmission load with insulation thickness for west facing wall in Chengdu, both natural ventilation and insulation location are taken into consideration. As is seen from the figure, compared with internal insulation, external insulation can provide lower yearly cooling transmission load under the condition of natural ventilation. Furthermore, the variation pattern of yearly cooling transmission load for external insulation is different from that for internal insulation. Combining natural ventilation with external insulation, yearly cooling transmission load sharply decreases with the increase of the insulation thickness to about 60mm. After that, the variation of yearly cooling transmission load is not obvious. However, under the action of internal insulation and natural ventilation, yearly cooling transmission load continuously decrease with the increase of the insulation thickness. Fig.11 (b) shows the variation of total yearly transmission load versus insulation thickness for west facing wall in Chengdu with the consideration of both natural ventilation and insulation location. It can be seen from the figure that the total yearly transmission loads for the external and internal insulated walls are nearly the same. This is due to the fact that their approximately equal yearly heating transmission loads are much larger than their yearly cooling transmission loads.

Figure 12 shows the variation of saving rate for yearly cooling transmission load caused by natural ventilation versus external insulation thickness in Chengdu, Changsha, Hefei and Shanghai. It is clear that the variation trend in Chengdu is completely different from that in the other three cities, which is maybe due to the special basin climate. As seen from Fig.12 (a),
the saving rates for all orientation walls increase rapidly up to the highest values (95.84% in the east, 96.55% in the north, 96.41% in the south and 96.99% in the west) with the increase of external insulation thickness, and the corresponding values of insulation thickness are 86mm, 71mm, 86mm and 131mm. After these points, the saving rates decrease slightly. In addition, it is seen that when the external insulation thickness increases to about 100mm, the saving rates for all orientation walls are nearly equal. Before this point, the saving rates ranked from high to low are in the north, the south, the east and the west, however, after this point, the highest saving rate is provided by the west and the saving rates in other three orientations are much the same. So, in general, natural ventilation can significantly improve the saving rate of yearly cooling transmission load, and it should be emphasized in the design of building.

Fig. 12. Variation of saving rate for yearly cooling transmission load caused by natural ventilation versus external insulation thickness in four cities
(a) Chengdu  (b)  Changsha  (c)  Hefei  (d)  Shanghai

Figures 12 (b) and (c) show that the variation pattern for saving rate of yearly cooling transmission load caused by natural ventilation versus external insulation thickness in Changsha is similar to that in Hefei. What is more, the variation trends of all orientation walls are quite similar. It is seen that the saving rates of all facing walls increase as the external insulation thickness increase. Furthermore, the saving rates ranked from high to low are the north, the south, the west and the east in Changsha. However, the order in Hefei is the west, the east, the south and the north.

As seen in Fig.12 (d), in Shanghai, the saving rates of all orientation walls will rise with the increase of external insulation thickness, and their values become closer except for the south. Moreover, it is obvious that the highest saving rate appears in the south facing wall, while the lowest appears in the north facing wall.

4.2 The optimum insulation thickness

Optimum insulation thickness is the value which leads to the minimum total cost. In this study, the optimum thicknesses of insulation material EPS is calculated based on LCC method in which the total yearly transmission load obtained from dynamic
heat-transfer model is used as the input. In addition, because the total yearly transmission loads for external and internal insulation are almost equal at the same insulation thickness, the same optimum thicknesses can be obtained regardless of insulation location. Therefore, in this paper, all the optimizations are implemented for external insulation. The related parameters and their corresponding values are given in Table 4.

In order to investigate the effect of natural ventilation and LCC model on optimum insulation thickness, the optimization is carried out for three cases. The lifetimes in LCC model and the situation of natural ventilation for the three cases are listed in Table 5. The total cost versus insulation thickness for the west facing wall in the four climatic conditions for the cases without and with natural ventilation is shown in Fig.13. Furthermore, the optimization results for Cases 1, 2 and 3 are listed in Table 6.

Table 4 The parameters used in calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_C$</td>
<td>0.6 ¥/kWh</td>
</tr>
<tr>
<td>$COP_{EC}$</td>
<td>2.3 (JGJ 134-2010)</td>
</tr>
<tr>
<td>$COP_{HCOP}$</td>
<td>1.9 (JGJ 134-2010)</td>
</tr>
<tr>
<td>$i$</td>
<td>8% (CPEEMP, 2006)</td>
</tr>
<tr>
<td>$g$</td>
<td>4% (China Statistical Yearbook, 2012)</td>
</tr>
<tr>
<td>$N$</td>
<td>50 years (GB 50068-2001)</td>
</tr>
<tr>
<td>$C_{f}$</td>
<td>850 ¥/m³ (Quota, 2009)</td>
</tr>
<tr>
<td>$C_{iad}$</td>
<td>49 ¥/m² (Quota, 2009)</td>
</tr>
<tr>
<td>$C_{idm}$</td>
<td>10 ¥/m² (Quota, 2009)</td>
</tr>
</tbody>
</table>

1 ¥=0.16$.

Table 5 The lifetime in LCC model and the situation of natural ventilation for three cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Lifetime in LCC model(year) Building/EPS</th>
<th>natural ventilation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>50/20</td>
<td>No</td>
</tr>
<tr>
<td>Case 2</td>
<td>50/20</td>
<td>Yes</td>
</tr>
<tr>
<td>Case 3</td>
<td>20/20</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Figure 13 shows variation of total cost to insulation thickness for the west facing wall in the four climatic conditions. It is seen that the insulation cost increases linearly with insulation thickness, however, the energy cost decreases with increasing insulation thickness. Total cost is the sum of insulation and energy cost, which firstly decreases and then increases with the increase of insulation thickness. The insulation thickness at which the total cost is the minimum is taken as the optimum insulation thickness. Moreover, from Fig.13, it is obvious that at the same insulation thickness, the energy cost without natural ventilation is higher than that with natural ventilation, which leads to the total cost without natural ventilation opossessing the higher value. Furthermore, compared with the case without natural ventilation, the total cost with natural ventilation achieves the minimum at smaller insulation thickness. That is to say, natural ventilation can not only reduce the total cost but also result in an obvious decrease in optimum insulation thickness. In the four cities, the effect of natural ventilation on optimum insulation thickness in Changsha is the most obvious.

From Table 6, it can be seen that although the four cities belong to the same climatic zone of China, their optimum insulation thicknesses are different from each other. The thicker optimum insulation thickness occurs in Hefei while the thinner appears in Chengdu. Moreover, the effect of orientation on the optimum insulation thickness cannot be ignored, especially in Changsha, Hefei and Shanghai. Therefore, in the design of external insulation of the wall, inhomogeneous distribution should be considered in different orientations and cities.

As seen in Table 6, optimum insulation thicknesses of Case 2 are lower than those of Case 1. That is to say, no matter in which city of the four cities or which orientation, natural ventilation can result in an obvious decrease in optimum insulation thickness. Besides, in the four cities, the highest decrease of 7mm appears in west facing wall in Changsha, however, the lowest decrease of 1mm is provided by the north orientation walls in Hefei and Shanghai.

Moreover, it is obvious in Table 6 that the different LCC models lead to significant distinction in optimum insulation thickness. Compared with the LCC model in which the lifetimes of building and EPS are 20 years, LCC model with the
lifetimes of 50 years for building and 20 years for EPS provides thinner optimum insulation thickness. So, it is important to choose an appropriate economic model for the optimization of insulation thickness.

Fig.13 Variation of cost with insulation thickness for the west facing wall in the four cities. (NA: natural ventilation)

<table>
<thead>
<tr>
<th>Table 6</th>
<th>The optimum insulation thicknesses for three cases.</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>Orientation</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Chengdu</td>
<td>East</td>
</tr>
<tr>
<td></td>
<td>South</td>
</tr>
<tr>
<td></td>
<td>West</td>
</tr>
<tr>
<td></td>
<td>North</td>
</tr>
<tr>
<td>Changsha</td>
<td>East</td>
</tr>
<tr>
<td></td>
<td>South</td>
</tr>
<tr>
<td></td>
<td>West</td>
</tr>
<tr>
<td></td>
<td>North</td>
</tr>
<tr>
<td>Hefei</td>
<td>East</td>
</tr>
<tr>
<td></td>
<td>South</td>
</tr>
<tr>
<td></td>
<td>West</td>
</tr>
<tr>
<td></td>
<td>North</td>
</tr>
<tr>
<td>Shanghai</td>
<td>East</td>
</tr>
<tr>
<td></td>
<td>South</td>
</tr>
<tr>
<td></td>
<td>West</td>
</tr>
<tr>
<td></td>
<td>North</td>
</tr>
</tbody>
</table>
4.3 Sensitivity analysis for optimum insulation thickness

In this section, a sensitivity analysis for optimum insulation thickness is carried out for west facing wall in Chengdu. In this study, electricity price ($E_C$), EPS price ($C_{\text{ic}}$), discount rate ($i$) and inflation rate ($g$) are chosen as the uncertain factors. Meanwhile, natural ventilation is taken into consideration in the calculation of total yearly transmission load, and LCC model with the lifetime of 50 years for building and 20 years for EPS is used to determine optimum insulation thickness.

Figure 14 illustrates the sensitivity of optimum insulation thickness following the variation of the four uncertain factors. The abscissa stands for the change rate of uncertain factors relative to the given values listed in Table 4, and the ordinate stands for the change rate of optimum thickness relative to the basic optimum thickness which calculated by using the values listed in Table 4. The sensitivity is denoted by the slope of curve, namely, the greater the slope, the higher the sensitivity.

From Fig.14, it is clear that the sensitivities of optimum thickness on the four uncertain factors, from high to low in turn, are insulation price, electricity price, discount rate and inflation rate. Optimum insulation thickness increases as electricity price and inflation rate increase, while it decreases as the increase of insulation price and discount rate, which agrees well with the results obtained by Al-Sanea and Zedan (2002) in the different climates. Results indicate that an increase of 51% for optimum insulation thickness can be achieved when electricity price rises 90%, while the increase of 90% in the inflation rate leads to about 17.6% increase in optimum insulation thickness. On the contrary, an increase of 90% in insulation price and discount rate result in a decrease of 39% and 29% for optimum insulation thickness, respectively.

5. Conclusions

This study mainly investigates the effect of natural ventilation on yearly transmission loads and optimum insulation thicknesses for external walls. Besides, the effects of insulation location and LCC model on optimum insulation thickness are also studied. The research is carried out for four typical cities such as Chengdu, Changsha, Hefei and Shanghai in hot summer and cold winter zone of China. In this paper, yearly cooling and heating transmission loads of the external wall are calculated using an implicit finite difference method and a computer program developed in FORTRAN is utilized. Meanwhile, a life-cycle cost analysis (LCC) model is applied to determine optimum insulation thicknesses for all orientation external walls. Finally, a sensitivity analysis on optimum insulation thickness for four uncertain factors is performed. The major conclusions derived from this study can be summarized as follows:

(1) Natural ventilation plays a significant role in reducing the yearly cooling transmission load for all orientations walls in the four cities, especially in Chengdu. In addition, natural ventilation can result in an obvious decrease in optimum insulation thickness.

(2) Insulation location almost has no effect on total yearly transmission load. However, with the collaboration of natural ventilation, external insulation provides lower yearly cooling transmission load compared with internal insulation.

(3) The effect of orientation on the optimum insulation thickness cannot be ignored, especially in Changsha, Hefei and Shanghai. Results show that different LCC models lead to significant distinction in optimum insulation thickness. So, it is
important to choose an appropriate economic model for the optimization of insulation thickness.

(4) The sensitivities of the optimum insulation thickness on the four uncertain factors from high to low in turn are insulation price, electricity price, discount rate and inflation rate.

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