Through-flow effects on Nusselt number and torque coefficient in Taylor-Couette-Poiseuille flow investigated by large eddy simulation

Akihiro OHSAWA *, Akira MURATA * and Kaoru IWAMOTO*

*Tokyo University of Agriculture and Technology
2-24-16 Nakacho, Koganei, Tokyo 184-8588, Japan
E-mail: ohsawacchyan@gmail.com

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Abstract

Flow in a concentric annular passage with a rotating inner cylinder, which is called Taylor-Couette flow, is important in industrial applications, such as electric motor which requires not only effective cooling of rotating shaft but also saving power required for the axis rotation. When the through flow is superposed, which is called Taylor-Couette-Poiseuille flow, it affects the cooling efficiency and the torque required for the axis rotation. To the authors' knowledge, previous studies have been focused on either the Nusselt number or the torque coefficient in the Taylor-Couette-Poiseuille flow. Therefore, it is difficult to estimate the through-flow effects on both of them under the same geometry and flow conditions. In this study, the through-flow effects on both the Nusselt number and the torque coefficient in the Taylor-Couette-Poiseuille flow under the same geometry and flow conditions were investigated by performing large eddy simulation. The through-flow Reynolds number, Re , was varied from 500 to 8000 under constant Taylor and Prandtl numbers of $Ta=4000$ and $Pr=0.71$, respectively. The Nusselt number and the torque coefficient had similar trend to each other with the increase of Re . They decreased by 25% for the change of Re from 0 to 1000 and were nearly constant for the change of Re from 4000 to 8000. Contribution of the advection, turbulent transport and diffusion terms to the Nusselt number and the torque coefficient were evaluated by using the equations proposed by the authors. The contribution of the advection term was nearly zero for Re from 500 to 8000, which was contrary to the case without through-flow ($Re=0$). As Re increased, the contribution of the turbulent transport term decreased but that of the diffusion term did not change so much. The friction factor in the axial direction varied as $Re^{-0.75}$ of which power was between laminar (Re^{-1}) and turbulent ($Re^{-0.25}$) correlations in a smooth stationary pipe flow.

Key words : Taylor-Couette-Poiseuille flow, Through flow, Nusselt number, Torque coefficient, Friction factor

1. Introduction

Flow in a concentric annular passage with a rotating inner cylinder is called the Taylor-Couette flow and the similar situations are widely seen in industrial applications, such as journal bearings, turbo machinery and rotor-stator gap of electric motor. Beyond a critical rotation speed of the inner cylinder, the Taylor vortex appears due to centrifugal force (Taylor, 1923) and the flow regime changes from laminar vortex to turbulent vortex via wavy vortex with the increase of rotation speed (Coles, 1965). The dimensionless parameter of rotation speed is the Taylor number, $Ta=r_i\omega H/\nu$ (sometimes called the rotation Reynolds number and r_i , ω , H , ν are radius of the inner cylinder, rotation speed of the inner cylinder, flow passage height ($=r_o-r_i$) and kinematic viscosity, respectively).

When the through-flow is superposed on the Taylor-Couette flow, it is called the Taylor-Couette-Poiseuille flow, and the critical Taylor number is increased with the through-flow Reynolds number, $Re=2Hu_{zm}/\nu$ (u_{zm} is mean velocity in the axial direction). This phenomenon had been studied by the stability analysis at very low through-flow Reynolds number, $Re\sim 1-100$ (DiPrima and Pridor, 1979, Takeuchi and Jankowski, 1981). Recently, Cotrell and Pearlstein (2004) performed stability analysis to obtain the curve of critical Taylor number up to $Re=9900$, as shown by the broken dotted

line (a) in Fig. 1. Lueptow et al. (1992) performed experiments and categorized the flow patterns of which experimental range is shown by (b) in Fig.1. The critical Taylor number without through-flow ($Re=0$) was about $Ta_c=100$ at which the Taylor vortex appeared. With the superposition of the through-flow, the onset of the Taylor vortex was delayed until $Ta_c=200$ for $Re=40$, and at the highest limit of their experimental condition of $Ta=3000$ and $Re=40$, the flow pattern became turbulent vortex. From this experimental study, it is thought that the flow is turbulent in the range of several thousand of $Ta \sim 10^3$ and $Re \sim 10^3$.

Previous studies on the heat transfer in the Taylor-Couette-Poiseuille flow are as follows. Jeng et al. (2007) performed heat transfer measurement over the inner cylinder for $Re=60-2400$ and $Ta=0-2922$ with and without ribs by using thermocouples. Becker and Kaye (1962) experimentally showed that for $Re=0-5900$ and $Ta=65-1175$ the Nusselt number declined along with the through-flow Reynolds number and reached minimum (at $Re=1592$) before it increased again. Simmers and Cony (1979) experimentally showed that the Nusselt number decreased with the increase of the through-flow Reynolds number and it was nearly constant beyond a certain value of the through-flow Reynolds number for $Re=0-3000$ and $Ta=200-6500$. Tachibana and Fukui (1964) proposed empirical correlation with and without slots on the inner cylinder surface for $Re=380-4200$ and $Ta=120-5900$. Aubert et al. (2015) performed heat transfer experiment for $Re=7500-11200$ and $Ta=3350-9430$ and proposed empirical correlation in which the Nusselt number was expressed as $Nu \propto Re^\alpha Ta^\beta Pr^\gamma$. Fénot et al. (2011) reviewed previous studies on the heat transfer characteristics in the Taylor-Couette flow with and without through-flow in detail. They reported that the heat transfer characteristics in the Taylor-Couette-Poiseuille flow depended not only on the global parameters (the Taylor number, the through-flow Reynolds number and the geometry of the system) but also on the entrance conditions for the through flow. Aubert et al. (2015) reported that the Nusselt number near the entrance was much higher than that near the center in the axial direction, which was due to the developing nature of the flow. The variation of the Nusselt number in the axial direction was confirmed by large eddy simulation of Murata and Iwamoto (2011) for $Re=1000$ and $Ta=4000$.

Previous studies on the torque in the Taylor-Couette flow with and without the through flow are as follows. In some studies, torque required to rotate inner cylinder in the Taylor-Couette flow (without through-flow) was used to identify the critical Taylor number at which the torque increased suddenly (Chandrasechar, 1962). The relationship between torque, G , and Taylor number has been studied and was found experimentally as $G=K(Ta^*)^\gamma$ (K and γ are constants) up to high Taylor number, $Ta=10^6$ (Lathrop et al., 1992). Yamada (1962b) performed experiments of the through-flow effect on the torque coefficient with and without circumferential grooves on the inner cylinder surface for $Re=800-30000$ and $Ta=5000-30000$. Without the grooves at constant Taylor number, the torque coefficient decreased with the through-flow Reynolds number and it reached a minimum (at about $Re=5000$) before it increased again, which is a similar behavior to the through-flow effect on the Nusselt number. Manna and Vacca (2009) performed direct numerical simulation to study the torque reduction by the through-flow for $Re=0-400$ and $Ta=1000-1500$.

The through-flow effect on both the Nusselt number and the torque coefficient is important for rotating machineries, such as electric motors which require not only cooling the rotating shaft effectively but also saving power required for rotating the axis. To the authors' knowledge, as most studies are focused on either Nusselt number or torque coefficient, it is difficult to estimate through-flow effect on them under the same geometry and flow conditions. In this study, through-flow effect on both the Nusselt number and the torque coefficient under the same geometry and flow condition was investigated by large eddy simulation. The through-flow effect on the contribution of the advection, turbulent transport and diffusion terms to the Nusselt number and the torque coefficient was investigated with the equations obtained by the authors (Ohsawa et al., 2016). Furthermore, rotation effect of the inner cylinder on the friction factor of the through flow was investigated.

Nomenclature

C_f	friction factor	L	axial length, m
C_M	torque coefficient	Nu	Nusselt number
C_s	Smagorinsky constant	Pr	Prandtl number ($=0.71$)
G	torque, Nm	Pr_t	turbulent Prandtl number ($=0.9$)
h	heat transfer coefficient, W/(m ² K)	q_i	heat flux on the inner cylinder, W/m ²
H	flow passage height ($=r_o-r_i$), m	q_o	heat flux on the outer cylinder ($=-q_i r_i/r_o$), W/m ²
J	calculated value at inner cylinder, m ⁴ /s ²	Q	second invariant of deformation tensor

r, θ, z	radial, circumferential, and axial coordinates, m	κ	thermal diffusivity, m^2/s
r_i, r_o	radii of inner and outer cylinders, m	κ_{sgs}	subgrid-scale thermal diffusivity, m^2/s
Re	through-flow Reynolds number ($=2Hu_{zm}/\nu$)	ν	kinematic viscosity, m^2/s
S_{ij}	rate of strain tensor, 1/s	ν_{sgs}	subgrid-scale eddy viscosity, m^2/s
T	Temperature, K	λ	thermal conductivity, $\text{W}/(\text{mK})$
Ta	Taylor number ($=r_i\omega H/\nu$)	Subscripts and superscripts	
u_r, u_θ, u_z	velocity components in r, θ, z directions, m/s	$\langle \rangle$	surface average in r - θ plane
u_{zm}	mean through-flow velocity, m/s	$'$	fluctuating component
u_τ	friction velocity, m/s	$-$	time average
η	radius ratio ($=r_i/r_o=0.87$)	$+$	dimensionless value based on inner scale
Δ	filter width, m		
ω	inner cylinder angular speed, rad/s

Fig. 1 Parameter range of previous and present studies. From c to j, the maximum of Re and Ta were plotted. Rectangle, triangle and circle are the Nusselt number, the torque coefficient and both the Nusselt number and the torque coefficient, respectively. a: Ta_c curve by Cottrell and Pearlstein, 2004. b: Lueptow et al., 1992. c: Jeng et al., 2007. d: Becker and Kaye, 1962. e: Simmers and Cony, 1979. f: Tachibana and Fukui, 1964. g: Aubert et al., 2015. h: Murata and Iwamoto, 2011. i: Yamada, 1962a. j: Manna and Vacca, 2009.

2. Computational method

Figure 2 shows the schematic view of the Taylor-Couette-Poiseuille flow which is the superposition of through-flow on the system studied in the authors' previous paper (Ohsawa et al., 2016). Numerical method for this study was the same as the authors' previous paper. The cylinder axes are aligned with the z axis of the cylindrical coordinate system in which r and θ denote radial and azimuthal coordinates, respectively. The inner cylinder with radius of r_i rotates at a constant angular velocity, ω , while the outer cylinder with radius of r_o is at rest. The through-flow is in the axial (z) direction. The flow passage has height of H , and it is heated by a constant and uniform heat flux of q_i on the inner cylinder and cooled by a constant and uniform heat flux of $q_o = -q_i r_i/r_o$ on the outer cylinder. The coefficient of r_i/r_o appears in order to keep mean fluid temperature constant in the axial direction. Because entrance conditions have some effects on the heat transfer as reported in Fénot et al. (2011) and Aubert et al. (2015), periodic boundary condition in the z direction was adopted to exclude the entrance effect and to achieve fully developed condition. Computational domain size was adopted to be 90° in the circumferential direction and axial length, $L=18H$, was twice as that of the authors' previous paper (Ohsawa et al., 2016). This computational domain size was the same as that of Ohsawa et al. (2015).

Fig. 2 Schematic view of the Taylor-Couette-Poiseuille flow. Dimensions for present simulation are as follows: $H=2$, $L=18H$, $r_i=6.75H$, $r_o=7.75H$ and $\eta=r_i/r_o=0.87$.

For incompressible flow, continuity, Navier-Stoke and energy equation are Eq. (2-1), Eq. (2-2) and Eq. (2-3), respectively. The last term in Eq. (2-2) is a mean pressure gradient term which drives the through-flow in the axial direction.

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2-1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} \left((\nu + \nu_{sgs}) \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial \bar{p}}{\partial x_j} \delta_{jz} \quad (2-2)$$

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left((\kappa + \kappa_{sgs}) \frac{\partial T}{\partial x_j} \right) \quad (2-3)$$

Where,

$$\nu_{sgs} = C_s^2 \bar{\Delta}^2 \sqrt{2S_{ij}S_{ij}}, \quad (2-4)$$

$$S_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right), \quad (2-5)$$

$$\kappa_{sgs} = \nu_{sgs} / Pr_t. \quad (2-6)$$

Prandtl number, Pr , was set to 0.71 and turbulent Prandtl number, Pr_t , was set to 0.9 (Kawamura and Iwamoto, 2010). Although the value of the turbulent Prandtl number of SGS (sub-grid scale) model in the present computation was higher than the usual value, the volume in which the SGS viscosity exceeded 10% of the molecular viscosity was less than 2% of the total computational domain volume for all cases of Re in this study. Therefore, the effect of the turbulent Prandtl number of SGS model was minor and the results were hardly changed by this difference. The Lagrangian dynamic subgrid-scale model was adopted in which C_s was averaged along path line for a certain distance (Meneveau et al., 1996), and subgrid-scale viscosity, ν_{sgs} was calculated. The dimensionless coefficient for calculating the time scale was set to 1.5. Finite volume method was adopted and OpenFOAM (ver.2.2.1) was used for the computation. Crank-Nicolson method was adopted for time advancement of all variables. The structured grid was adopted and the

grids were concentrated to the inner and outer cylinders in the radial direction, while those were equally spaced in the circumferential and axial directions. The computation was performed on FX10 (Information Technology Center, the University of Tokyo).

The Taylor number was set to $Ta=4000$ and the through-flow Reynolds number was varied in $Re=500, 1000, 4000$ and 8000 . Table 1 shows the numerical conditions and the results. The same grid number was adopted for $Re=500-4000$. This grid number gave similar grid resolutions normalized by inner scale (friction velocity, u_τ , was calculated from circumferentially and axially averaged wall shear stresses) and it was the same as that of Ohsawa et al. (2015) in which the independency of the grid resolution and circumferential domain size were verified under the same geometry and the Taylor number as present study but uniform heat flux on both the inner and outer cylinder. For supplemental data, the results of circumferential domain size of 90° and 180° for $Re=4000$ are listed in Table 1, which shows the independency of the circumferential domain size (for $Re=4000$, as the angle between the velocity vector and the z axis is 46.7° as shown in the caption of Fig. 6, nearest to 45° , the dependency of the circumferential domain size must be largest). For $Re=8000$, higher grid number was adopted to achieve similar grid resolution to other cases. Statistical values were calculated for more than 20 revolutions of the inner cylinder and 10 sweeps of main flow through the passage in the axial direction, after statistically steady condition was established. The definitions of the Nusselt number, Nu , and the torque coefficient, C_M , are Eq. (2-7) and Eq. (2-8), respectively. Eq. (2-10) is the definition of the torque, G .

$$Nu = \frac{2Hq_i}{\lambda \Delta T} \quad (2-7)$$

$$C_M = \frac{G}{\rho(r_i \omega)^2 2\pi r_i^2 L} \quad (2-8)$$

Where,

$$\Delta T = \langle \bar{T} \rangle \Big|_{r_i} - \langle \bar{T} \rangle \Big|_{r_o}, \quad (2-9)$$

$$G = 2\pi L \rho \omega_i^2 \left(\frac{\partial \langle \bar{u}_\theta \rangle}{\partial r} - \frac{\langle \bar{u}_\theta \rangle}{r} \right) \Big|_{r_i} = 2\pi L \rho \omega_i^3 \left(\frac{\partial}{\partial r} (\langle \bar{u}_\theta \rangle / r) \right) \Big|_{r_i} \quad (2-10)$$

Table 1 Numerical conditions and the results.

Ta	Re	Grid number ($r \times \theta \times z$)	Grid resolution ($\Delta r^+ \times \Delta \theta^+ \times \Delta z^+$)	Time step, Δt^+ (normalized by $\nu/(u_\tau)^2$)	Nu	$C_M \times 10^3$
4000	500	100×256×384	(0.26-6.4)×8.1×9.1	0.37	16.3	2.22
	1000	100×256×384	(0.3-6.8)×8.6×9.7	0.31	13.4	1.86
	4000	100×256×384	(0.3-7.1)×9.0×10.1	0.36	11.6	1.67
	4000(180°)	100×512×384	↑	↑	11.3	1.67
	8000	150×384×384	(0.38-6.1)×8.1×13.6	0.33	11.4	1.60

3. Results and discussions

Figure 3 shows the through-flow effect on the Nusselt number and the torque coefficient at constant Taylor number of $Ta=4000$. Both the Nusselt number and the torque coefficient are normalized by the values without through-flow ($Re=0$) which was obtained by the authors (Ohsawa et al., 2016). The through-flow effects on the Nusselt number and the torque coefficient have very similar trend to each other. Both the Nusselt number and the torque coefficient decrease sharply from $Re=0$ to 1000 , more than 25% reduction, and they are nearly constant for the change of Re from 4000 to 8000 . The ratio, Nu/C_M , stays at about unity. This simulation results demonstrate that the through-flow reduces the Nusselt number and the torque coefficient similarly for turbulent region under the same geometry and fully developed flow condition.

The contribution of the advection, turbulent transport and diffusion terms to the Nusselt number and the torque coefficient was evaluated by using the following Eq. (2-11) and (2-12) which were derived by the authors (Ohsawa et al., 2016). The equations show the Nusselt number and torque coefficient are decomposed into the three terms which are advection, turbulent transport and diffusion terms, respectively.

$$Nu = \frac{2}{r_i \kappa \Delta T} \int_{r_i}^{r_o} (r \langle \bar{u}_r \bar{T} \rangle + r \langle \bar{u}_r' T' \rangle - r \langle \kappa \frac{\partial \bar{T}}{\partial r} \rangle) dr \quad (2-11)$$

$$C_M = \frac{1}{(r_i \omega)^2 r_i^2 H} \int_{r_i}^{r_o} (r^2 \langle \bar{u}_r \bar{\theta} \rangle + r^2 \langle \bar{u}_r' \theta' \rangle - r^3 \nu \partial_r (\langle \bar{u}_r \theta \rangle / r)) dr \quad (2-12)$$

Fig. 3 Through-flow effects on the Nusselt number and the torque coefficient for $Ta=4000$. Rectangle and circle are, respectively, the Nusselt number and the torque coefficient normalized by the values of $Re=0$ which was obtained by the authors (Ohsawa et al., 2016). Broken line is the ratio of Nu/C_M .

Figure 4 shows the contribution of each term to the Nusselt number, which is obtained by the numerical integration of Eq. (2-11). The striking feature is that the through-flow decreased contribution of the advection term from 78.4 % for $Re=0$ to nearly zero (less than 0.18%) and it makes the turbulent transport term main contributor to the Nusselt number. Both the contribution of the turbulent transport and the Nusselt number (Fig. 3) decrease as Re increases from 500 to 4000 and they are nearly constant for the change of Re from 4000 to 8000.

Figure 5 shows the contribution of each term to the torque coefficient, which is obtained by the numerical integration of Eq. (2-12). Similar trend to the Nusselt number (Fig. 4) is observed. The through-flow decreases contribution of the advection term from 76.4 % for $Re=0$ to nearly zero (less than 0.4%) and it makes the turbulent transport term main contributor to the torque coefficient. Both the contribution of the turbulent transport term and the torque coefficient (Fig. 3) decrease as Re increases from $Re=500$ to 4000 and is nearly constant for the change of Re from 4000 to 8000.

The reason why the advection term disappeared in Figs. 4 and 5 is as follows. The advection term in case of $Re=0$ is due to the Taylor Vortices which circulate between inner and outer cylinders in the r - z plane. When the through-flow is superposed, it pushes the Taylor Vortices down to the axial direction. Therefore, under the time average and space average in θ - z plane, the advection term disappeared.

Figure 6 shows contour surface of the second invariant of deformation tensor, $Q=1000$, for the instantaneous velocity field. The angles between the flow vectors and the z axis amid of the flow passage are referred in the caption. For $Re=500$, circumferentially almost-uniform ring structure and slightly inclined streaky structure on it are observed, and they are advected in the axial direction by the through-flow. For $Re=1000$, similar structures to those of $Re=500$ are observed (not shown). For $Re=4000$ and 8000, streaky structures are inclined to the axial direction. The outer structures (view from the outer cylinder) are coarser than the inner ones, which suggest that turbulence is depressed in the outer cylinder side. In other word, the through-flow decreases turbulent transport in the outer cylinder side. As the turbulent transport term is the main contributor of the Nusselt number and the torque coefficient for $Re=500$ and 8000, the

reduction of the turbulent transport leads to the reduction of the Nusselt number and the torque coefficient.

Fig. 4 Contribution of each term to the Nusselt number.

Fig. 5 Contribution of each term to the torque coefficient.

Fig. 6 Contour surface of the second invariant of deformation tensor, $Q=1000$, for the instantaneous velocity fields. The through-flow directs from left to right. The angle between velocity vectors and the z axis amid of the flow passage are 80.3° for $Re=500$, 46.7° for $Re=4000$ and 28.7° for $Re=8000$.

Fig. 7 shows the radial profiles of the integrand of the turbulent transport term in Eq. (2-11), which corresponds to the contribution of the turbulent transport term to the Nusselt number at r . The turbulent transport term decreases for the change of Re from 500 to 4000 across the radial direction and has similar profiles for $Re=4000$ and 8000. The reduction of the turbulent transport term from $Re=500$ to 8000 for $(r-r_i)/H=0.1$, 0.5 and 0.9 are respectively 0.3×10^{-3} , 0.35×10^{-3} and 0.57×10^{-3} , which shows that the through-flow reduces turbulent transport term near the outer cylinder more than near the inner cylinder. This result backs up the result that the through-flow depresses turbulence and reduces the turbulent transport in the outer cylinder side as discussed on Q (Fig. 6).

Fig. 7 Profiles of the turbulent transport term. Solid line without symbol, open, filled and broken line without symbol are for $Re=500$, 1000, 4000 and 8000, respectively.

Fig. 8(a) shows circumferential mean velocity. Slopes near the inner cylinder, which correspond to the torque coefficient, decrease with the increase of Re . Fig. 8(b) shows the axial mean velocity. For $Re=4000$ and 8000 , peaks are located in the outer cylinder side, because the strength of the Reynolds stress $\langle -u_r u_z \rangle$, which was normalized by not the friction velocity but $(r_i \omega)^2$ to estimate the effects of Re , is lower in the outer cylinder side than in the inner cylinder side as shown in Fig. 8(c). This phenomenon is similar to the case of the rotating shear flow in which the Reynolds stress is lower in the suction side than in the pressure side and the peak of the mean velocity moves to the suction side (Hattori and Nagano, 2002). Fig. 8(d) shows the Reynolds stress $\langle -u_r u_\theta \rangle$ which is closely related to the turbulent transport as shown in Eq. (2-12). The reduction in absolute value from $Re=500$ to 8000 at $(r-r_i)/H=0.9$ is about 1.5 times as large as that at $(r-r_i)/H=0.1$. This means that the through-flow depress the turbulent transport in the outer cylinder side more than in the inner cylinder side.

Fig. 8 Profiles of time-averaged velocities and Reynolds stresses. Solid line without symbol, open, filled and broken line without symbol are for $Re=500$, 1000 , 4000 and 8000 , respectively. Broken dotted line is for $Re=0$.

Figure 9 shows friction factor in the axial direction. The friction factor, C_f , is defined in Eq. (2-13), in which hydraulic diameter of $2H$ is used. Present result ($Ta=4000$) is in good agreement with the experimental results of Yamada (1962a) who measured the friction factor of the Taylor-Couette-Poiseuille flow with six radius ratios and various combinations of the Taylor number and the through-flow Reynolds number. The result of the numerical simulation by Manna and Vacca (2009) for $Ta=1500$, where the flow regime is laminar vortex, is plotted. Friction factors for both $Ta=1500$ and 4000 are higher than that of a smooth stationary pipe flow, which means that the inner cylinder rotation acts as a resistance on the through-flow. The friction factor varies as $\lambda \propto Re^{-0.75}$ for $Ta=4000$, but in the

turbulent region of the higher through-flow Reynolds number, beyond $Re=8000-12000$ for $Ta=4000$, the friction factor approaches to the value of the Blasius equation, which means that the effect of the inner cylinder rotation can be neglected with respect to the friction in the axial direction. For further Re , more than 12000, the results of Yamada (1962a) show that the friction factor corresponds well to the Blasius relation.

$$C_f = \frac{\Delta p}{L} \frac{4H}{u_{zm}^2} = \frac{\partial p}{\partial z} \frac{4H}{u_{zm}^2} \quad (2-13)$$

Fig. 9 Friction factor in the axial direction. Triangle: Experimental results of Yamada (1962a), $Ta=4000$, linearly-interpolated by using the data of $Ta=3000$ and $Ta=5000$, $\eta=0.885$. Rectangle: Direct numerical simulation of Manna and Vacca (2009), $Ta=1500$ and $\eta=0.98$. Circle: present study. Broken line: slope=-0.75, for comparison with present study.

4. Conclusions

The through-flow effects on the Nusselt number and torque coefficient in the Taylor-Couette-Poiseuille flow was investigated by using large eddy simulation under the same geometry and fully developed flow condition. The Taylor number was set to a constant value of $Ta=4000$ (turbulent vortex region) and the through-flow Reynolds number was varied from $Re=500$ to 8000 (from laminar region to turbulent region). Prandtl number was set to $Pr=0.71$.

The through-flow effects on the Nusselt number and the torque coefficient had very similar trends under the same geometry and flow conditions. They decreased sharply from $Re=0$ to 1000 , more than 25% reduction and they were nearly constant for the change of Re from 4000 to 8000 .

The through-flow effects on the contribution of the advection, turbulent transport and diffusion terms to the Nusselt number and the torque coefficient were evaluated by using equations obtained by the authors (Ohsawa et al., 2016). The through-flow excluded the contribution of the advection term and made the turbulent transport term main contributor, which was contrary to the case without through-flow ($Re=0$). The contribution of the turbulent transport term to the Nusselt number and the torque coefficient decreased as Re increased from 500 to 4000 and was nearly constant for the change of Re from 4000 to 8000 . The contour surface of second invariant of deformation tensor demonstrated that the through-flow depressed turbulence in the outer cylinder side and reduced the Nusselt number and the torque coefficient.

The friction factor in the axial direction was higher than that of the smooth stationary pipe flow, which meant that the inner cylinder rotation acted as a resistance. The friction factor was proportional to $Re^{0.75}$ for the present Taylor number of 4000 and Re of $500-8000$. For $Re=8000$, the friction factor approached to the value of the Blasius equation, which meant that the effect of the inner cylinder rotation could be neglected with respect to the friction in the axial direction.

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