Analysis of thermosolutal natural convection and entropy generation within a three-dimensional inclined cavity with various aspect ratios

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Abstract
Three-dimensional thermosolutal natural convection and entropy generation within an inclined enclosure is investigated in the current study. A numerical method based on the finite volume method and a full multigrid technique is implemented to solve the governing equations. Effects of various parameters, namely, the aspect ratio ($A_z$), buoyancy ratio ($N$) and inclination angle ($\gamma$) on the flow patterns, heat and mass transfer rates as well as entropy generation are predicted and discussed. A comparison of 2D and 3D models at normal situation $\gamma=0^\circ$ is conducted when $N$ varied in the transition range $-2\leq N \leq -0.6$ demonstrating that the 2D assumption can be adopted for the 3D flows when $-0.5 \leq N \leq 0$. The numerical outcome of the present study shows that, the thermal and solutal isosurfaces exhibit a central stratification that significantly strengthens as $A_z$ is augmented. It is also found that decreasing the aspect ratio value $A_z$ leads to weakening the total entropy generation and reducing the 3D effects exhibited within the cavity. Special attention is attributed to analyze the periodic flow pattern that appears for $Ra=10^4$, $A_z=2$ and $\gamma=75^\circ$. In terms of irreversibility criterion at the oscillatory regime, total entropy generation ($S_{tot}$) and Bejan number (Be) are seen to oscillate with the same frequency but in opposing phases and with different amplitudes.

Key words: Double-diffusive natural convection, Heat and mass transfer, Three-dimensional fluid flow, Entropy generation, Tilted enclosure

Nomenclature

\[ A_x, A_y, A_z \quad \text{aspect ratio in } x, y \text{ and } z \text{ directions, respectively} \]
\[ Be \quad \text{Bejan number, } Be = \left( S_{th} + S_{diff} \right) / \left( S_{th} + S_{fr} + S_{diff} \right) \]
\[ c \quad \text{dimensional concentration, (kg/m}^3\text{)} \]
\[ C_C \quad \text{concentration at the cold wall, (kg/m}^3\text{)} \]
\[ C_H \quad \text{concentration at the hot wall, (kg/m}^3\text{)} \]
\[ C \quad \text{dimensionless concentration, } C = (c - c_C) / c_H - c_C \]
\[ D \quad \text{depth of the enclosure, (m)} \]
mass diffusivity, \( \text{m}^2/\text{s} \)

d

dimensionless frequency

\( f \)

Grashof number, \( \text{Gr}=\text{Ra}/\text{Pr} \)

\( \text{Gr} \)

acceleration of gravity, \( \text{m}/\text{s}^2 \)

\( g \)

height of the enclosure, \( \text{m} \)

\( H \)

solutal transfer coefficient, \( \text{m} / \text{s} \)

\( h_c \)

heat transfer coefficient, \( \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1} \)

\( h_T \)

conductivity \( (\text{J} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \cdot \text{K}^{-1}) \)

\( k \)

Lewis number, \( \text{Le} = \alpha / d = \text{Sc} / \text{Pr} \)

\( \text{Le} \)

buoyancy ratio, \( N = \beta_c (c_{H} - c_{C}) / \beta_T (T_H - T_C) \)

\( N \)

average Nusselt number

\( \text{Nu} \)

pressure, \( \text{Pa} \)

\( P \)

dimensionless pressure

\( p \)

Prandtl number, \( \text{Pr} = \nu / \alpha \)

\( \text{Pr} \)

Rayleigh number, \( \text{Ra} = g W^3 \beta_T (T_H - T_C) / \nu \alpha \)

\( \text{Ra} \)

gas constant

\( \text{R} \)

dimensionless entropy generation

\( S \)

Schmidt number, \( \text{Sc} = \nu / d \)

\( \text{Sc} \)

average Sherwood number

\( \text{Sh} \)

cold wall temperature, \( \text{K} \)

\( T_C \)

hot wall temperature, \( \text{K} \)

\( T_H \)

temperature, \( \text{K} \)

\( T \)

dimensionless time

\( t \)

width of the cavity, \( \text{m} \)

\( W \)

dimensionless velocity components in \( x, y \) and \( z \) directions

\( u, v, w \)

dimensionless Cartesian coordinates

\( x, y, z \)

dimensional Cartesian coordinates \( (\text{m}) \)

\( X, Y, Z \)

Greek Symbols

\( \alpha \)

thermal diffusivity, \( (\text{m}^2/\text{s}) \)

\( \beta_T \)

coefficient of thermal expansion, \( (\text{K}^{-1}) \)

\( \beta_c \)

coefficient of solutal expansion, \( (\text{m}^3/\text{kg}) \)

\( \gamma \)

Inclination angle, \( (\degree) \)

\( \Delta \)

difference value

\( \theta \)

dimensionless temperature, \( \theta = (T - T_C) / (T_H - T_C) \)
1. Introduction

Thermosolutal natural convection, which refers to the convection driven by a combination of temperature and concentration gradients, has continued to be a very active area for researchers during the past few decades due to its industrial and geophysical applications. Double diffusive convection occurs in a wide variety of fields such as astrophysics, oceanography, geology, channel type solar energy collectors, biology, chemical processes and, geophysical problems (Kuznetsov & Sitnikov, 2002; Chakraborty & Dutta, 2003; Chamkha, & Al-Mudhaf, 2008; Khadiri et al., 2011; Rahman et al., 2012; Zhao et al. 2012; Al-Rashed et al., 2017). The heat and mass transfer and fluid flow phenomenon has been extensively studied within two-dimensional (2D) confined cavities and detailed reviews on the 2D investigations are easily available in the literature (Morega & Nishimura, 1996; Er-Raki et al., 2008; Soto-Meca et al. 2016, Oueslati et al., 2014). In fact, many situations of practical interest that deal with heat and mass phenomena can be encountered in inclined enclosures such as solar collectors Hadidi et al. (2013) or solar distillers (Chouikh et al. 2006; Ghachem et al. 2012). For instance, the double diffusive natural convection investigation in an inclined enclosure having parallelogrammic shape has been conducted by Costa, (2004). The author reported that augmenting the Rayleigh (Ra) or buoyancy ratio number (N) values always leads to an increase of the heat and mass transfer rates in the 2D cavity. It was also confirmed that selected combinations of inclination angle and aspect ratio can provide an extremum overall heat and mass transfer rates. Chen et al. (2012) examined the 2D thermosolutal convection in an inclined enclosure. The authors showed that when the cavity inclination decreases from \(\gamma = 0^\circ\) to \(90^\circ\) the critical thermal Grashof for the onset of stationary instability increases exponentially while that for the onset of oscillatory instability decreases exponentially. They also stated that below a critical tilt angle, the first onset of instability is oscillatory, rather than stationary. The 2D double diffusive convection was also predicted by Al-Farhany & Turan (2012) for the case of a tilted enclosure filled with porous media. The results showed that, as the aspect ratio increases, the average Nusselt and Sherwood numbers are reduced, whereas they decrease when the angle of inclination is augmented. Concerning the buoyancy ratio effects, it was observed that the Nusselt number increases when the buoyancy ratio is augmented for values over -1 and decreases for N less than this specific value. A similar behavior was also predicted for the Sherwood number profiles.

Despite the importance of irreversibility phenomenon occurring within confined cavities subject to two temperature and concentration gradients, until now, only a few studies dealt with the entropy generation for the case of double diffusive natural convection. For instance, Magherbi et al. (2006) focused on analyzing the effect of inclination angle on the irreversibility phenomena in thermosolutal convection within a tilt 2D cavity. They found that the angle of inclination had a significant effect on entropy generation and heat and mass transfer rates. In addition, the total entropy...
generation was seen to strengthen with the thermal Grashof number and the buoyancy ratio for moderate Lewis numbers. Besides, structures relative to the local entropy generation were observed to be nearly identical and localized at the bottom and the top of the heated and cooled walls respectively. Chen and Du (2011) studied the effects of multiple parameters namely; thermal Rayleigh number, buoyancy ratio and aspect ratio on entropy generation of turbulent double-diffusive natural convection in a rectangle cavity. It was found that the total entropy generation increases with $Ra$ and augments nearly linearly with aspect ratio values. Furthermore, the relative total entropy generation rates due to diffusive and thermal irreversibilities were observed to decrease monotonously against the aspect ratio while that due to viscous irreversibility undergoes a monotonic increasing versus it. Very recently, a numerical analysis of entropy generation in steady-unsteady double diffusive convection through a 2D inclined porous enclosure was carried out by Mchirgui et al. (2014). The influence of the tilt angle and the enclosure aspect ratio on entropy generation, heat and mass transfer rates, and fluid flow were predicted. The authors approved that entropy generation exhibits an oscillatory behavior for specific fixed parameters and undergoes a minimum entropy generation according to the aspect ratio values of 0.5 and 1.

On the other hand, while some thermosolutal flow phenomena have revealed two-dimensional solutions, many subtleties of third dimensionality are still missing in literature. Almost most of studies on the heat and mass transfer phenomena essentially focused on analyzing the heat and mass phenomena in two-dimensional cavities. Nevertheless, only a few studies investigated the double diffusive convection flow within three-dimensional (3D) enclosures. Among the earliest studies dealing with the thermosolutal phenomena, we can cite the work of Sezai and Mohamad (2000) who numerically analyzed the double diffusive convection within a cubic cavity and demonstrated that the flow remains strictly three dimensional for a certain range of the depending parameters. Besides, Bergeon and Knobloch (2002) studied the bifurcations phenomena in double diffusive convection in three-dimensional enclosures driven by opposing horizontal temperature and concentration gradients, and the results were compared with the two-dimensional case. The authors were able to track the various bifurcations and distinguish clearly between states that are fundamentally two-dimensional. Kuznetsov and Sheremet (2011) studied the transient thermosolutal convection in a cubical enclosure having finite thickness walls filled with air, submitted to temperature and concentration gradients. The authors analyzed the effects of the various parameters such as the buoyancy ratio, the Rayleigh number, the dimensionless time, the thermal conductivity ratio and the size of the contaminant source on the appearance of heat and mass transfer regimes. They also pointed out that the mass transfer rate strongly depends on the variation of the thermal conductivity ratio parameter, especially for Rayleigh number values higher than $Ra=10^5$. The 3D thermosolutal convection in a cuboid enclosure with horizontal temperature and concentration gradients has been investigated by Chen et al. (2013). This study analyzed the effect of governing parameters of the problems, namely the Rayleigh number, buoyancy ratio and the Lewis numbers on the appearance of typical pitchfork bifurcation giving appropriate disturbances of the flow regime. Beaume et al. (2013) investigated the 3D thermosolutal convection to compute branches of spatially localized convection. Such states are referred to as convectons. The authors predicted two branches of three-dimensional convectons with perfect symmetry that bifurcate simultaneously from the conduction state and undergo homoclinic snaking. They also claimed that the presence of the additional symmetry, introduced by the third dimension, is responsible for the presence of complex time-dependence at onset even in small domains. For instance, Lioua et al. (2011) numerically predicted the laminar mixed convection and entropy generation in a lid-driven cubic cavity. The authors found that direction of lid at the 3D enclosure is an effective parameter on both entropy generation and heat and fluid flow for low values of Richardson number but it becomes insignificant at higher values. In addition, higher Bejan numbers are formed for Richardson values greater than 10 due to domination of opposing flow and natural convection. In addition, the 3D double diffusive natural convection and entropy generation within a non inclined solar distiller has been conducted by Ghachem et al. (2012). They stated that a high Bejan number is found for $N=1$, indicating the domination of heat and mass irreversibilities outside of which friction irreversibilities are largely dominant. Moreover, at the equilibrium state ($N=1$), the 3D distributions of the generated entropy were seen to be more pronounced and local Nusselt numbers changes with changing buoyancy ratio and takes a complex structure. It is clear from the above literature survey that effects of the aspect ratio, and tilt angle on the heat and mass transfer rates as well as the entropy generation are not studied in earlier works considering three-dimensional inclined enclosures. In fact, to the best knowledge of the authors, until now, most of the studies that performed the double diffusive natural convection inside enclosed spaces are two-dimensional and no attention has been paid to explore the effect of the aspect ratio and the inclination angle on the double diffusive convection and to analyze the irreversibility...
phenomenon characteristics that occur in the 3D resulting flows. From this point of view, the aim of this study is to explore numerically the three-dimensional unsteady double-diffusive natural convection in an inclined enclosure for different aspect ratio varied in the z-direction $A_z$. Four values of $A_z$ are considered in the current study; namely $A_z = 0.5; 1; 1.5$ and $2$, while holding constant the aspect ratio values $A_x$ and $A_y$ in the x and y-directions, respectively. The effects of aspect ratio, buoyancy ratio, inclination angle, on the heat and mass transfer rates and fluid flow patterns are analyzed and discussed for different buoyancy ratios and both specific inclination angles $\gamma = 30^\circ$ and $\gamma = 75^\circ$ leading to minimum and maximum of heat and mass transfer rates, respectively. The results are presented in terms of thermal and solutal isosurfaces, velocity isocontours, as well as Nusselt and Sherwood profiles. Especial attention is also considered for analyzing the occurrence of periodic behavior with the 3D cavity at specific fixed parameters. Of great interest is the comparison of 2D and 3D models of the cavity at normal situation $\gamma = 0^\circ$ in order to analyze the validity of the 2D assumption in the transition range of buoyancy ratio $N$.

2. Problem formulation

2.1. Physical model and governing equations

Details of the geometry for the configuration are shown by a schematic diagram in Fig. 1. The physical model considered here is a three-dimensional enclosure tilted with an inclination angle $\gamma$ with respect to the horizontal position. The enclosure is a confined 3D cavity of width $W$, height $H$ and depth $D$ having aspect ratios $A_x = 1$, $A_y = \frac{D}{W} = 2$ and a longitudinal one $A_z = \frac{H}{W}$. Note that we propose to vary the aspect ratio in the vertical direction $z$ meaning approaching the two horizontal walls. For this purpose, four values of $A_z$ will be considered in the current investigation; $A_z = 0.5; 1; 1.5$ and $2$, while keeping $A_x$ and $A_y$ constant. The cavity is filled with humid air where different temperature and concentrations are specified between the left hot $(T_H, c_H)$ and right cold vertical walls $(T_C, c_C)$, with $T_H > T_C$ and $c_H > c_C$. The two remaining horizontal faces are assumed to be adiabatic. Furthermore, the thermophysical properties of the incompressible fluid are taken to be constant except for the density variation in the buoyancy terms, where the Boussinessq approximation is considered as:

![Fig. 1. Schematic of the tilted 3D enclosure](image-url)
where \( \rho_0 \) is the fluid density at the reference temperature \( T_0 = T_c \) and concentration \( c_0 = c_c \). The terms \( \beta_T = -(1/\rho_0)(\partial \rho / \partial T)_{p,c} \) and \( \beta_c = -(1/\rho_0)(\partial \rho / \partial c)_{p,T} \) are the thermal and concentration expansion coefficients, respectively. Note that \( \beta_T > 0 \) and \( \beta_c < 0 \) which means that the molecular weight of the solute is higher than that of the gas. By employing the aforementioned assumptions, the conservation equations of mass, momentum, temperature and species can be expressed in their dimensionless forms as follows:

\[
\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} + \frac{\partial \tilde{w}}{\partial \tilde{z}} = 0
\]  

(2)

\[
\frac{\partial \tilde{u}}{\partial \tilde{t}} + u \frac{\partial \tilde{u}}{\partial \tilde{x}} + v \frac{\partial \tilde{u}}{\partial \tilde{y}} + w \frac{\partial \tilde{u}}{\partial \tilde{z}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \text{Pr} \left( \frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} \right) + Ra \Pr(\theta + NC) \sin \gamma
\]  

(3)

\[
\frac{\partial \tilde{v}}{\partial \tilde{t}} + u \frac{\partial \tilde{v}}{\partial \tilde{x}} + v \frac{\partial \tilde{v}}{\partial \tilde{y}} + w \frac{\partial \tilde{v}}{\partial \tilde{z}} = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \text{Pr} \left( \frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{z}^2} \right)
\]  

(4)

\[
\frac{\partial \tilde{w}}{\partial \tilde{t}} + u \frac{\partial \tilde{w}}{\partial \tilde{x}} + v \frac{\partial \tilde{w}}{\partial \tilde{y}} + w \frac{\partial \tilde{w}}{\partial \tilde{z}} = -\frac{\partial \tilde{p}}{\partial \tilde{z}} + \text{Pr} \left( \frac{\partial^2 \tilde{w}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{w}}{\partial \tilde{y}^2} + \frac{\partial^2 \tilde{w}}{\partial \tilde{z}^2} \right) + Ra \Pr(\theta + NC) \cos \gamma
\]  

(5)

\[
\frac{\partial \tilde{\theta}}{\partial \tilde{t}} + u \frac{\partial \tilde{\theta}}{\partial \tilde{x}} + v \frac{\partial \tilde{\theta}}{\partial \tilde{y}} + w \frac{\partial \tilde{\theta}}{\partial \tilde{z}} = \left( \frac{\partial^2 \tilde{\theta}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\theta}}{\partial \tilde{y}^2} + \frac{\partial^2 \tilde{\theta}}{\partial \tilde{z}^2} \right)
\]  

(6)

\[
\frac{\partial \tilde{C}}{\partial \tilde{t}} + u \frac{\partial \tilde{C}}{\partial \tilde{x}} + v \frac{\partial \tilde{C}}{\partial \tilde{y}} + w \frac{\partial \tilde{C}}{\partial \tilde{z}} = \frac{1}{\text{Le}} \left( \frac{\partial^2 \tilde{C}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{C}}{\partial \tilde{y}^2} + \frac{\partial^2 \tilde{C}}{\partial \tilde{z}^2} \right)
\]  

(7)

The dimensionless quantities \( \tilde{x}, \tilde{y}, \tilde{z} \), \( \tilde{u}, \tilde{v}, \tilde{w} \), \( \tilde{t}, \tilde{p}, \tilde{\theta} \) or \( \tilde{C} \) denote the coordinates space, velocity component in the \( \tilde{x} \) direction, time, hydrodynamic pressure, temperature, and concentration, respectively. The governing equations (2-7) are non-dimensionalized using scales \( W, \alpha / W, \alpha^2 / W^2, \rho_0 \alpha^2 / W^2, \Delta T, \Delta c \) for coordinate space, velocity, time, pressure, temperature, concentration, respectively, where the characteristics of temperature and concentration scales \( \Delta T \) and \( \Delta c \) are defined as: \( \Delta T = T_h - T_c \) and \( \Delta c = c_h - c_c \).

The dimensionless temperature \( \tilde{\theta} \) and concentration \( \tilde{C} \) are defined as \( \tilde{\theta} = (T - T_c) / \Delta T \) and \( \tilde{C} = (c - c_c) / \Delta c \).

The above system of equations introduce the following dimensionless parameters: \( \Pr = \nu / \alpha \), \( N = \beta_c (c_h - c_c) / \beta_T (T_h - T_c) \), \( Ra = g W^3 \beta_T (T_h - T_c) / \nu \alpha \), \( Le = \alpha / d \) denoting the Prandtl, buoyancy ratio, Rayleigh and Lewis number, respectively.

Here \( \nu \) is the kinematic viscosity of the fluid, \( \alpha \) and \( d \) the thermal and mass diffusivities, respectively.
and $g$ is the acceleration due to gravity. Schmidt number can also be introduced as $Sc = Pr \times Le$. It is to mention that in the current investigation, all computations were performed for $Pr=0.71$, $Le=0.85$ and $Sc=0.6035$ which cover water vapor diffusion into air. For appropriate boundary conditions, non-slip boundary conditions are imposed over the walls ($u=v=w=0$). Thermal and solutal boundary conditions are as follows:

At $x = 0$ : $\theta = C = 1$, 

At $x = 1$ : $\theta = C = 0$ , 

Elsewhere: $\frac{\partial \theta}{\partial n} = \frac{\partial C}{\partial n} = 0$, where $n$ is the coordinate of the normal surface.

On another hand, the heat and mass transfer rates in the 3D cavity can be evaluated using the well known Nusselt and Sherwood numbers using the following analysis. In fact, the Nusselt number is the ratio of the total heat transfer to the conductive thermal rate. Hence, equating the heat transfer by convection to the heat transfer by conduction at the hot wall gives $h_t \Delta T = -k \left( \frac{\partial T}{\partial X} \right)_{X=0,H}$. Here, $h_t$ and X denote the heat transfer coefficient and the dimensional coordinate space in the x-direction, respectively.

Introducing the dimensionless variables used in the governing equation gives the local Nusselt defined as

$$Nu = - \left( \frac{\partial \theta}{\partial X} \right)_{x=0,1}$$

Similarly, the Sherwood number represents the ratio of the convective mass transfer to the rate of diffusive mass transport. Hence, by equating the extracted mass transfer by convection to the added mass transfer to the enclosure leads to $h_c \Delta c = -d \left( \frac{\partial c}{\partial X} \right)_{X=0,W}$, where $h_c$ is solutal heat transfer coefficient.

Likewise, using the dimensionless variables previously introduced gives the local Sherwood number

$$Sh = - \left( \frac{\partial C}{\partial X} \right)_{x=0,1}$$

Finally, the average Nusselt and Sherwood numbers are obtained by integrating the above local Nusselt and Sherwood number over the hot left wall:

$$\bar{Nu} = - \frac{1}{A_yA_z} \int_0^{A_y} \int_0^{A_z} \left( \frac{\partial \theta}{\partial X} \right)_{x=0,1} dydz$$

$$\bar{Sh} = - \frac{1}{A_yA_z} \int_0^{A_y} \int_0^{A_z} \left( \frac{\partial C}{\partial X} \right)_{x=0,1} dydz$$

Note that the solutal and heat transfer coefficient $s$ as well as the dimensional Cartesian coordinates ($X$, $Y$, $Z$) were added and defined in the nomenclature.

2.2. Equations for entropy generation

In thermosolutal natural convection problem, the associated irreversibilities are due to heat transfer, fluid friction and diffusion.

In literature, a multiple studies proposed the dimensionless form of the local entropy generation in convective heat transfer. In this context one may cite those of Shohel & Roydon (2002) and Tasnim & Shohel (2002) who predicted the ratio between the local entropy generation rate and a characteristic entropy transfer rate $So$. According to Bejan (1996),
the characteristic entropy transfer rate \( S_{0}^{loc} \) is expressed by \( S_{0}^{loc} = k \left( \frac{\Delta T}{WT_{0}} \right)^{2} \). Here, \( k \), \( W \), \( T_{0} \) and \( \Delta T \) are respectively, the thermal conductivity, the characteristic length of the enclosure, a reference temperature and a reference temperature difference.

According to local thermodynamic equilibrium of linear transport theory (Lioua et al., 2011; Ghachem et al., 2012; Zahmatkesh, 2008; Mukhopadhyay, 2010; Kaluri & Basak, 2011; Oueslati et al., 2013), the dimensionless form of local entropy generation \( S_{loc}^{th} \) due to heat transfer \( S_{th}^{loc} \), fluid friction \( S_{fr}^{loc} \) and to diffusion \( S_{dif}^{loc} \) for a 3D heat and mass and a fluid flow is given by:

\[
S_{loc} = S_{th}^{loc} + S_{fr}^{loc} + S_{dif}^{loc}
\]  

(10)

Where \( S_{th}^{loc} \), \( S_{fr}^{loc} \) and \( S_{dif}^{loc} \) are expressed by:

\[
S_{th}^{loc} = \left( \frac{\partial \theta}{\partial x} \right)^{2} + \left( \frac{\partial \theta}{\partial y} \right)^{2} + \left( \frac{\partial \theta}{\partial z} \right)^{2}
\]  

(11)

\[
S_{fr}^{loc} = \varphi_{1} \left[ 2 \left( \frac{\partial u}{\partial x} \right)^{2} + \left( \frac{\partial v}{\partial y} \right)^{2} + \left( \frac{\partial w}{\partial z} \right)^{2} \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^{2} + \left( \frac{\partial w}{\partial z} + \frac{\partial v}{\partial y} \right)^{2} \right] \right]
\]  

(12)

\[
S_{dif}^{loc} = \varphi_{2} \left[ \left( \frac{\partial C}{\partial x} \right)^{2} + \left( \frac{\partial C}{\partial y} \right)^{2} + \left( \frac{\partial C}{\partial z} \right)^{2} \right] + \varphi_{3} \left[ \left( \frac{\partial \theta}{\partial x} \right) \left( \frac{\partial C}{\partial x} \right) + \left( \frac{\partial \theta}{\partial y} \right) \left( \frac{\partial C}{\partial y} \right) + \left( \frac{\partial \theta}{\partial z} \right) \left( \frac{\partial C}{\partial z} \right) \right]
\]  

(13)

Where \( \varphi_{i} (1 \leq i \leq 3) \) denotes the irreversibility coefficient ratios defined by:

\[
\varphi_{1} = \frac{\mu T_{0}}{k} \left( \frac{\nu}{W\Delta T} \right)^{2}
\]  

(14)

\[
\varphi_{2} = \frac{R d T_{0}}{k c_{0}} \left( \frac{\Delta c}{\Delta T} \right)^{2}
\]  

(15)

\[
\varphi_{3} = \frac{R d}{k} \left( \frac{\Delta c}{\Delta T} \right)
\]  

(16)

which are taken constant at \( \varphi_{1} = 10^{-4} \), \( \varphi_{2} = 0.5 \) and \( \varphi_{3} = 0.01 \) in the present work and denote irreversibility distribution ratios related to velocity gradients, concentration gradients, and mixed product of concentration and thermal gradients, respectively (Magherbi et al. 2006; Bouabid et al. 2011; Ghachem et al. 2012).

Here, \( c_{0} \) and \( T_{0} \) are respectively the reference concentration and temperature. Besides, the parameters \( R \), \( d \), \( \mu \), \( \nu \), denote the gas constant, the mass diffusivity, the dynamic viscosity and kinematics viscosity; respectively.

The dimensionless total entropy generation \( S_{tot}^{loc} \) in the 3D cavity is given by the summation of the total entropy generation due to heat transfer \( S_{th}^{loc} \), fluid friction \( S_{fr}^{loc} \), and to diffusion \( S_{dif}^{loc} \) which in turn are obtained via
integrating the local entropy generation rates $S_{th}^{loc}$, $S_{fr}^{loc}$ and $S_{dif}^{loc}$ over the domain $\Omega$:

$$S_{tot} = \int_{\Omega} S_{th}^{loc} d\Omega + \int_{\Omega} S_{fr}^{loc} d\Omega + \int_{\Omega} S_{dif}^{loc} d\Omega = S_{th} + S_{fr} + S_{dif}$$  \hspace{1cm} (17)

Another dimensionless parameter is considered which is the Bejan number $Be$ representing the ratio of heat and mass transfer irreversibility to the total irreversibility due to heat and mass transfer and fluid friction and defined as:

$$Be = \frac{S_{th} + S_{dif}}{S_{th} + S_{fr} + S_{dif}}$$  \hspace{1cm} (18)

3. Numerical technique and code validation

3.1. Numerical methodology

The dimensionless governing equations (2-7) were numerically predicted using an in-house code named «NASIM» (Oueslati et al., 2013; Oueslati et al., 2011; Oueslati et al., 2014; Ben-Beya and Lili, 2009; Naffouti et al. 2014) based on finite volume method and the following numerical technique. The time derivative in the momentum equations is performed by an Euler backward second-order implicit scheme. Linear terms are evaluated implicitly, while the non-linear terms are explicitly evaluated by means of an Adams–Bashforth extrapolation. The strong velocity–pressure coupling present in the continuity and the momentum equations is handled by implementing the projection method (Brown et al. 2001). A Poisson equation with homogeneous boundary conditions is then solved and leads to update pressure and free divergence velocity fields. The finite volume method has been also implemented on a staggered grid system in order to discretize the system of equations to be solved. Besides, the QUICK scheme of Hayase et al. (Hayase et al. 1992) is used to minimize the numerical diffusion for the advective terms. The Poisson equation which is solved using an accelerated full multigrid method (Ben-Cheikh et al. 2007), while the discretized equations are computed using the red and black point successive over-relaxation method (Hadjidimos, 2000) with the choice of optimum relaxation factors. In order to secure the steady state conditions, the following criterion has to be satisfied:

$$\max \left( \left| \frac{\Phi_{ijk}^{m+1} - \Phi_{ijk}^m}{\Phi_{ijk}^m} \right| \right) \leq 10^{-8}$$  \hspace{1cm} (19)

The generic variable $\Phi$ stands for $u$, $v$, $w$, $p$, $\theta$ or $C$ and, $m$ indicates the iteration time levels. In the above inequality, the subscript sequence $(i, j, k)$ represents the space coordinates $x$, $y$, and $z$ respectively.

It is worth noting that all simulations of the present study were conducted with non-uniform grids of $32 \times 64 \times 16$, $32 \times 64 \times 32$, $32 \times 64 \times 48$ and $32 \times 64 \times 64$, with respect to aspect ratio values investigated $A_z = 0.5$; $1$; $1.5$ and $2$, respectively. Moreover, because of the presence of large gradients near the walls, we generate a centro-symmetric grid with clustering near the walls using the following $n$ grid point distribution:

$$x_i = \frac{1}{2} \left( 1 + \frac{\tanh[\Gamma(2i/n - 1)]}{\tanh(\Gamma)} \right)$$  \hspace{1cm} (20)

Where $\Gamma = 1.25$ and $1 \leq i \leq n$. Similar grid point distributions were used in the three directions of the 3D tilted enclosure 3D as described in Fig. 2.
3.2. Code validation

Firstly, the in-house «NASIM» code is validated by considering the 3D double diffusive natural convection flow within a non tilted cubic cavity ($\gamma = 0^\circ$) using a grid of $64^3$. For this purpose, two test cases are examined corresponding to the investigation of Sezai and Mohamad (2000) and also Chakraborty and Dutta (2003), with respect to different Lewis number $Le=0.1$, 1 and 10. It is to state that the predicted results were extracted from Figure 2 presented in the study of Chakraborty and Dutta (2003). The Prandtl, buoyancy ratio and Rayleigh number values were kept fixed at $Pr=10$, $N = -0.5$ and $Ra=10^5$, respectively. The results computed of the average Nusselt and Sherwood numbers at the left hot wall together with values from both references are depicted in Table 1. As observed, good agreements between the present results and those of Sezai and Mohamad (2000) and also Chakraborty and Dutta (2003) are obtained showing a maximum relative error less than 1%.

| Table 1 |
|-----------------------|-----------------------|-----------------------|
| Average Nusselt and Sherwood numbers compared with those of Sezai and Mohamad (2000), and Chakraborty and Dutta (2003) for different Lewis number values $Le=0.1$, 1 and 10 ($Pr=10$, $N = -0.5$ and $Ra=10^5$) |
| $Le=0.1$ | $Le=1$ | $Le=10$ |
| $\bar{Nu}$ | $\bar{Sh}$ | $\bar{Nu}$ | $\bar{Sh}$ | $\bar{Nu}$ | $\bar{Sh}$ |
| Present study | 3.02 | 1.04 | 3.48 | 3.48 | 3.83 | 6.52 |
| Sezai and Mohamad (2000) | 3.02 ($0\%$) | 1.03 ($0.97\%$) | 3.48 ($0\%$) | 3.48 ($0\%$) | 3.83 ($0\%$) | 6.52 ($0\%$) |
| Chakraborty and Dutta (2003) | 3.01 ($0.33\%$) | 1.03 ($0.97\%$) | 3.49 ($0.28\%$) | 3.49 ($0.28\%$) | 3.82 ($0.26\%$) | 6.51 ($0.15\%$) |

On another hand, in order to demonstrate the performance of the present code in its 2D version in terms of irreversibility phenomena, total entropy generation $S_{tot}$ values obtained at different Grashof number ($Gr=Ra/Pr$) and buoyancy ratio values together with those of Magherbi et al. (2006) are reported in Table 2. It is worth noting that due to the lack of benchmark solutions available in literature relative to entropy generation for 3D tilted enclosures, the considered validation is performed for the 2D case relative to an inclined square cavity tilted with an angle equal to $60^\circ$.
with respect to the vertical position and with a rotation in the clockwise direction. All computations are obtained for the fixed parameters (Sc=1.5, Le=2, Pr=0.75) and with irreversibility coefficient ratios values $\varphi_1 = 10^{-4}$, $\varphi_2 = 0.5$ and $\varphi_3 = 0.01$. It is worth noting that the results were extracted from Figure 3 relative to the paper of Magherbi et al. (2006) for the corresponding parameters and illustrated in Table 2. As it is observed, the maximum relative error is within 1%. This fair agreement between the present results and those obtained by Magherbi et al. (2006) demonstrates the capability of the present code again in terms of irreversibility rates, meaning that the two-dimensional assumption remains valid. In general, both 2D and 3D flows patterns exhibit a large similarity, especially for integral quantities.

### Table 2
Comparison of the computed results of total entropy generation $S_{\text{tot}}$ with those of Magherbi et al. (2006) at various Grashof number and buoyancy ratio values, for a tilted angle 60° and at fixed parameters (Sc=1.5, Le=2 and Pr=0.75 ) and corresponding to irreversibility ratios ($\varphi_1 = 10^{-4}$, $\varphi_2 = 0.5$ and $\varphi_3 = 0.01$).

<table>
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<td>13.6472</td>
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</table>

#### 3.3. Comparison of 2D and 3D models at normal position

On another hand, to further discuss the validity of the 2D and 3D models at the transition range, we carried out several numerical simulations by varying N in range [-2, 0] in the (x-z) mid-plane.

Fig. 3 displays the corresponding comparison in terms of variation of $\overline{Nu}$ and $\overline{Sh}$. In this figure, the flow pattern is described by the streamlines in 2D and 3D cases as well as the isocontours corresponding to the v-velocity component. The solid lines and dashed ones respectively denote the counterclockwise and clockwise rotation vortices. It is seen that both average Nusselt and Sherwood numbers plots exhibited by the 2D model undergoes totally different pattern than that of the 3D one.

It is interesting to note that in the 2D case flow reversal occurs at a negative buoyancy ratio N of -0.5 as reflected by the end of the conductive regime, whereas in the 3D model the reversal of the main flow rotation occurs at N=-1 corresponding to the minimum of both $\overline{Nu}$ and $\overline{Sh}$. In the range $-2 \leq N \leq -0.6$, the difference between the 2D and the 3D model results of both $\overline{Nu}$ and $\overline{Sh}$ values are very high as a result of secondary flow formation in the transverse planes. For lower negative buoyancy ratio N three secondary flow vortices form about the diagonal cavity as observed from the projection of flow patterns on the mid (x-z) plane. Increasing N in steps of 0.1 accordingly to the 3D case, the system undergoes a bifurcation to another mode at N=-1 which consists of two counterrotating cells. At the limit flow situation (N=0), the flow patterns regains a three cellular mode with respect to the diagonal axis in the reversal direction.

By increasing N from its lower value, the v- isocontours corresponding to the mid- plane (y=1), demonstrate that the flow is strictly three-dimensional in the whole range corresponding to the bicellular pattern until N=-0.6 excepting the value N=-1 for which the flow becomes slightly bi-dimensional. It worth noting, that the 2D model assumption can be adopted for N in range (-0.5, 0). It is important to note that Sezai and Mohamad (2000) as well as Chen et al. (2013) carried out similar comparisons of both 2D and 3D models in the same range $-2 \leq N \leq 0$ for cubic and cuboid enclosures, respectively. Both studies confirmed also that 2D model approaches the 3D one for specific ranges.
Fig. 3 Comparison of 2D and 3D model in terms of average Nusselt and Sherwood numbers variations with respect to $N$ in range $(-2 \leq N \leq 0)$ when $\gamma = 0^\circ$ for $Ra = 10^4$.

4. **Results and discussion**

4.1. **Influence of aspect ratio on the flow pattern**
Fig. 4. Flow pattern for different $A_z$ at $\gamma = 30^\circ$, $Ra = 10^4$ and $N=5$: (a) isosurfaces of the temperature and (b) concentration isocontours.

In order to understand the effect of aspect ratio on the flow characteristic, we illustrate in Fig. 4 the thermal and concentration fields for $A_z = 0.5$, 1, 1.5 and 2. It is worth mentioning that $\gamma = 30^\circ$ corresponds to the angle where the heat and mass transfer rates are maximum at the equilibrium state as will be demonstrated later at the last section. For the greater value of aspect ratio investigated ($A_z = 2$), both isosurfaces and isocontours of temperature and concentration present a slight distortion at the enclosure core with excessive gradients at the vicinity of the upper part of the hot wall and the lower part of the cold wall. As the 3D cavity is made shorter, the central stratification is seen to be less important meaning that the double diffusion natural convection is preponderant within cavities that are more extended in vertical direction.

4.2. Effect of $A_z$ on the entropy generation

In terms the effect of the aspect ratio $A_z$ on the irreversibility phenomena manifested in the flow, total entropy generation profiles are illustrated in Fig. 5. The predictions are presented for fixed parameters $\gamma = 30^\circ$, $Ra = 10^4$ and $N=5$. By further increasing the value of $A_z$, an augmenting trend is clearly exhibited according to all evolutions of total entropy generations, total entropy generation $S_{tot}$, due to heat transfer $S_{th}$, fluid friction $S_{fr}$, and diffusion $S_{dif}$. This monotonous growing behavior of the profiles demonstrates that the rate of irreversibility criterion is enhanced in enclosures with greater shape ratio.

Fig. 5. Effect of $A_z$ on the evolutions of the total entropy generations for $\gamma = 30^\circ$, $Ra = 10^4$ and $N=5$.

This observation was clearly confirmed in Fig. 6 which plots the local entropy generation fields at the plane $(x-z)$ for
different aspect ratio $A_z$ considering the same chosen parameters ($\gamma = 30^\circ$, $Ra = 10^4$ and $N=5$). In fact, the isosurfaces of thermal local entropy generation $S_{th}^{loc}$ was seen to diminish by reducing the aspect ratio value. Moreover, significant local thermal entropy is observed near the hot wall and at the upper adiabatic one which means that augmentation of temperature gradient near the walls contributes to an enhancing of entropy generation near these zones. In addition, by making the horizontal walls closer, thermal entropy is less transmitted from the top walls to the cavity center. Besides, isosurfaces of local entropy generation due to fluid friction $S_{fr}^{loc}$ shown in Fig. 6 (b) are seen to only occur at the vicinity of the active vertical walls while the whole cavity core remains in an equilibrium state. The corresponding structures are symmetrically distributed with respect to the cavity diagonal. Nevertheless, despite the perfect symmetry that remains for all $A_z$ values, the entropy generation due to fluid friction is also reduced when the confined cavity loses volume. This phenomenon may be attributed to the fact that weakening the aspect ratio has led to a decrease of the surface contact. As a result, the heated fluid particles weaken and hence the irreversibility rate becomes fewer and causes a net decrease of entropy generation.

Fig. 6. Characteristics of the local entropy generation fields for different aspect ratio $A_z$ at the fixed parameters $\gamma = 30^\circ$, $Ra = 10^4$ and $N=5$: (a) $S_{th}^{loc}$, and (b) $S_{fr}^{loc}$ versus $A_z$.
4.3. **Combined effects of buoyancy ratio and aspect ratio on the heat and mass transfer rates**

In this sub-section we attempt to reveal the combined influence of both parameters; the buoyancy ratio $N$ and aspect ratio $A_z$ on the heat and mass rates as well as the entropy generation in the 3D enclosure. The computations were carried out at fixed parameters $Ra = 10^4$, $\gamma = 30^\circ$ and with buoyancy values varied in range $-5 \leq N \leq 5$ and aspect ratio values $A_z$ (2, 1.5, 1, and 0.5).

![Fig. 7. Combined effects of buoyancy ratio values N (-5, 5) and aspect ratio A_z (2, 1.5, 1, and 0.5) on heat and mass transfer rates for (Ra = 10^4 and \(\gamma = 30^\circ\)): (a) average Nusselt number, and (b) average Sherwood number versus N.](image)

Figure 7 reflects the effects of the buoyancy ratio $N$ parameter in range of $-5 \leq N \leq 5$ on variations of average Nusselt and Sherwood numbers for various aspect ratio values $A_z$ (2, 1.5, 1, and 0.5). It is clearly noticed from this figure that the heat and mass transfer rates are less in the opposing flow situation ($N < 0$) than for the corresponding $N$ in the aiding flow one ($N > 0$). In fact, both $\overline{Nu}$ and $\overline{Sh}$ values augment progressively by enhancing the absolute value of $N$. This can be attributed to the fact that the magnitude of solutal buoyancy strengthens and overpowers the thermal buoyancy as the magnitude of $N$ (absolute value) increases in opposing and aiding flow situations. Besides, for the same value of $N$, values of the average Nusselt number are seen to be higher for both situations, which demonstrates that the double diffusive flow is dominated by thermal gradients. Furthermore, as the aspect ratio value is increased, the effect of both thermal and solute buoyancy forces are enhanced, which is to say that the double diffusive convection phenomena dominates within 3D confined enclosures with higher aspect ratio. In addition, close scrutiny of the average Nusselt and Sherwood trends reveals that the flow rates are minimum at the buoyancy ratio value $N=1$ over which it increase monotonously with respect to all $A_z$ values. It is due to the fact that, at this specific value of buoyancy ratio, the effects of thermal and solutal buoyancy are of equal magnitude as revealed by the 2D numerical work of Morega and Nishimura (1996). Hence, it is to conclude that at this specific buoyancy ratio value, the heat and mass transfer are minimized regardless the aspect ratio value. The thermal and solutal rates seem to be independent of the shape of the enclosure when the corresponding gradients are of equal magnitudes. It is also to mention that a similar trend of the flow rates presenting a minimum at $N=-1$ was also observed by Sezai and Mohamad (2000) for the case of the double-diffusive natural convection within a cubic enclosure.
4.4. Influence of N and $A_z$ on the irreversibility phenomena

On another hand, in terms of the influence of buoyancy ratio on development of irreversibility phenomena within the 3D cavity, we illustrate in Fig. 8 the variations of the total entropy generation ($S_{tot}$) and Bejan number (Be) versus N for different values of $A_z$ for $Ra = 10^4$ and $\gamma = 30^\circ$. Contrarily to the heat and mass transfer rates analyzed previously, the distribution of total entropy generation is found to lower with the increase of N for all aspect ratios. The decreasing trend of $S_{tot}$ profiles shows that the irreversibility phenomenon is affected by the augmenting of the buoyancy ratio value especially at the aiding flow situation ($N>0$) as well as the cavity shape. Enhancing the N value leads to an important reduction of the entropy generation for all aspect ratios of the 3D cavity as reflected by Fig. 8 (a). Furthermore, acccording to all $A_z$ values, $S_{tot}$ profiles exhibit a decreasing trend at the opposing trend ($N<0$) and continues to diminish reaching their minimum values at the cooperating situation ($N>0$) where the values remains nearly stagnant.

Contrarily to the entropy generation profiles, the Bejan number profiles exhibit a decreasing trend at the opposing situation ($N<0$), and reach their minimum at at the limit situation ($N=0$) corresponding to a purely thermal domination and where the solutal gradient are the weakest. In fact at the limit situation ($N=0$), the double-diffusive flow is driven solely by the buoyancy effect associated with temperature gradients, while the solutal contribution becomes negligible. This may be explained by the fact that, in contrast to temperature field; for ($N=0$) the solute field becomes a passive scalar which is decoupled from the momentum equation. At the cooperating situation ($N>0$), an augmenting behavior of Bejan values is clearly observed with higher values that becomes significant for the weakest aspect ratio $A_z=0.5$. In fact, generally, the values of (Be) indicate the importance of entropy generation due to heat and mass transfer or fluid friction irreversibilities. Therefore, smaller obtained values of Be indicate dominance of fluid flow irreversibilities and hence some available energy is lost leading to less average thermal and solutal rates. The change in the behavior of the total entropy generation and Bejan number profiles when moving from opposing to cooperating situation may be due to the drastic change at the transitional situation in range [-2, 0] previously analyzed.

![Fig. 8. Variation of the total entropy generation and Bejan number with the buoyancy ratio N at different values of $A_z$ ($Ra = 10^4$ and $\gamma = 30^\circ$): (a) variation of $S_{tot}$ and (b) Be with respect to N.](image-url)
4.5. Thermal and solutal rates in the periodic behavior of the flow

![Figure 9](image)

Fig. 9. Periodic pattern of the flow for different aspect ratio values for $Ra = 10^4$, $N=5$ and $\gamma = 75^\circ$ (a) time histories of the $u$-velocity component at the central monitoring point $M_1$ ($A_x / 2$, $A_y / 2$, $A_z / 2$) and (b) frequency spectra the $u$-signal amplitude with the corresponding ($u$, $w$) phase diagram for $A_z = 2$.

As previously mentioned, an interesting feature exhibited by the double diffusive flow is the occurrence of oscillatory solutions for the case of the 3D enclosure having the higher aspect ratio value investigated $A_z = 2$ and tilted with an inclination $\gamma = 75^\circ$. Fig. 9 (a) describes the time signals of the $u$-velocity component predicted at the central monitoring point $M_1$ ($A_x / 2$, $A_y / 2$, $A_z / 2$) for different aspect ratio values at $N=5$ and $Ra = 10^4$. As revealed by the figure, by further enhancing the aspect ratio of the cavity, an established sinusoidal periodic solution is obtained at $A_z = 2$, while the flow remains steady for the remaining weaker $A_z$ values. To understand the phenomenon further, the phase portrait and power spectra of the periodic solution at $N=5$ is illustrated in Fig. 9 (b). Power spectra have been
computed from the time series of $u$-velocity component using the well-known, discrete Fast Fourier Transform frequency techniques. The time history of various quantities has been generated at the same central point $M_1$ ($A_y/2, A_z/2$). The phase portrait $(u, w)$ describing a limit cycle behavior, as well as the existence of a single dominant frequency of 12.736 with its multiples in the power spectrum, clearly show that the solution is of perfect period one. Note that this periodic trend may be caused by a physical mechanism producing time-dependent pattern in thermostal convection induced by the presence of one or more blobs of fluid colder or hotter than their surroundings circulating within the 3D enclosure.

4.6. Irreversibility phenomena in oscillatory flow trend:

For clear visualization of the periodic flow, we presents in Fig. 10 the evolution of entropy generation and Bejan number with the corresponding instant stream lines maps over one period of time $\tau$ at the same parameters $N = 5, \gamma = 75^\circ$ and $A_z = 2$. As can be reflected by this figure, at the four temporal instants of a period, total entropy generation $S_{tot}$ and Bejan number $Be$ seem to oscillate with the same frequency but in opposing phases and with different amplitudes. Concerning the instant streamline structures visualized at the mid-plane ($y = 1$), at $\tau/4$ the flow pattern is described by two counter-rotating convective cells; a primary vortex and a smaller secondary one developed in the ceiling. In fact, at the top wall the $u$-velocity component is responsible for moving hot fluid particles from left toward the right wall, whereas at the bottom one; it is the cooler fluid particles which are moved backward to reach the hot wall (anti clockwise vortex). By time advancing, precisely for the last two instants of the period of time, both counter-rotating undergo a permuted pattern.

On another hand, in order to predict the influence of the Rayleigh number on the appearance of the unsteady behavior, computations were carried out by varying the Ra values in range $5 \times 10^3 < Ra < 2.5 \times 10^4$. Several oscillatory solutions were obtained in this range and the corresponding plots are displayed in Fig. 11 reporting the variation of the frequency of $u$-velocity signal with respect to the Rayleigh number value at $N = 5, \gamma = 75^\circ$ and $A_z = 2$ predicted at
the monitoring point $M_2$ ($A_x / 4, A_y / 2, A_z / 4$). As can be seen, the periodic behavior is enhanced again by augmenting the Ra value and a corresponding established correlation of the frequency as function of the Ra parameter is depicted at the same figure. Besides, further period doublings with a fundamental frequency and its sub-harmonics were predicted according to all Ra values greater than $Ra = 10^4$.

![Graph showing the effect of Rayleigh number on the variation of frequency of the $u$-velocity signal at $N=5$, $\gamma = 75^\circ$ and $A_z = 2$ predicted at the monitoring point $M_2$ ($A_x / 4, A_y / 2, A_z / 4$).](image)

**Fig. 11.** Effect of the Rayleigh number on the variation of the frequency of the $u$-velocity signal at $N=5$, $\gamma = 75^\circ$ and $A_z = 2$ predicted at the monitoring point $M_2$ ($A_x / 4, A_y / 2, A_z / 4$).

### 4.7. Influence of tilt angle

In this section, we attempt to discuss the effect of tilt angle on the heat and mass transfer rates within the 3D cavity for the higher aspect ratio value $A_z = 2$. Figure 12 describes the influence of the inclination angle $\gamma$ about the horizontal plane on the variations of both average Nusselt and Sherwood numbers at the equilibrium state ($N = 1$) where the magnitude of solutal and thermal gradients are equal. The Rayleigh number value is held fixed at $Ra = 10^4$ and the tilt angle is varied in the range $0^\circ \leq \gamma \leq 90^\circ$ with a step of $\Delta \gamma = 15^\circ$. As revealed in Figures 12 (a) and (b), similar profile evolutions versus the inclination angle is exhibited for $\bar{Nu}$ and $\bar{Sh}$ presenting two extrema (a maximum followed by a minimum) precisely at the specific inclination values $\gamma = 30^\circ$ and $\gamma = 75^\circ$. It is worth noting that, in the current investigation, the steady and the periodic behavior of the flow were predicted according to these two particular angles $\gamma = 30^\circ$ and $\gamma = 75^\circ$, respectively. It is also interesting to note that the inclination angle $\gamma = 75^\circ$ presenting a minimum of heat and mass transfer rate is the only angle which gives an oscillatory regime according to the the higher value of aspect ratio $A_z = 2$ and the Ra considered. Hence, this configuration having the greater volume presents a particular interest in the stability point of view. To further analyze the heat and mass fields, we display at the same figure the thermal and solutal isosurfaces at both specific inclination angles $\gamma = 30^\circ$ and $\gamma = 75^\circ$ corresponding to the maximum and minimum of heat and mass transfer rates, respectively. Temperature and concentration isosurfaces reported are seen
to be similar due to the equilibrium flow case. At $\gamma = 30^\circ$ presenting the maximum of $\overline{Nu}$ and $\overline{Sh}$, both thermal and solutal isosurfaces present slight distortion at the core of the enclosure with higher gradients at the vicinity of the vertical active walls. As the enclosure is further inclined until the value $\gamma = 75^\circ$ relative to the weaker heat and mass transfer rates, a central stratification clearly appears while the lower and upper thermal and solute gradients seem to be strengthened significantly.

Fig. 12. Heat and mass transfer rate variations with respect to the tilt angle for $A_z = 2$ and $Ra = 10^4$ at the equilibrium state ($N = 1$) with thermal and solutal isosurfaces at the specific inclination values $\gamma = 30^\circ$ and $\gamma = 75^\circ$: (a) $\overline{Nu}$, and (b) $\overline{Sh}$.

5. Conclusions
A numerical analysis of thermosolutal natural convection and entropy generation in an inclined three-dimensional enclosure is performed in the current investigation. Two opposing walls of the cavity are maintained at fixed but different temperatures and concentrations; while the other two walls are adiabatic. Different aspect ratios and inclination angles where considered to analyze their effects on heat and mass transfer rates, irreversibility characteristics and fluid flow patterns. The main results are drawn as follows.

Firstly, it is observed that the flow continues to be 3D at a normal position of the cavity when $-2 \leq N \leq -0.6$. In particular, it was demonstrated that a 2D model is valid for 3D fluid flows for a buoyancy ratio parameter $N$ varying in the range $(-0.5-0)$.

Considering the aspect ratio effects for $\gamma = 30^\circ$ at which a maximum of heat and mass transfer rate is obtained at the equilibrium state, it was observed that both isosurfaces and isocontours of temperature and concentration present a slight distortion at the core of the 3D enclosure while the lower and upper gradients seem to be significantly strengthened. These central stratifications are seen to be affected by the weakening of the $A_z$ value meaning that the thermosolutal natural convection is more pronounced within cavities which are more extended in vertical direction. In terms of the entropy generation, it is found that increasing the value of $A_z$ leads to strengthening the entropy generation exhibited in the cavity. In addition, for all $A_z$ values, isosurfaces of local entropy generation due to fluid friction are seen to exhibit symmetrical distributions with respect to the cavity diagonal but only occur at the vicinity of the active vertical walls while the whole cavity core remains in an equilibrium state.

For the combined effects of buoyancy ratio and aspect ratio, profiles of $\overline{Nu}$ and $\overline{Sh}$ versus $N$ for various aspect ratio
values $A_z$ revealed that heat and mass transfer rates are less in the opposing flow situation ($N < 0$) than for the corresponding $N$ in the aiding flow one ($N > 0$).

It was also shown that average Nusselt and Sherwood profiles present minimum at the buoyancy ratio value $N=-1$ over which it increase monotonously with respect to all $A_z$ values. However, contrarily to the heat and mass transfer rates, the distribution of total entropy generation is found to decrease with increase of the buoyancy value for all aspect ratios.

Of particular interest is the transition from steady regimes of double-diffusive natural convection to the oscillatory regimes at $Ra = 10^4$, $A_z = 2$ and the inclination angle $\gamma = 75^\circ$ where the heat and mass transfer rates are minimized.

The numerical experiments reveal that the flow monotonously strengthens his periodic trend as the buoyancy ratio or the Rayleigh number values are made higher. According to irreversibility phenomena at the periodic state, total entropy generation $S_{tot}$ and Bejan number $Be$ seem to oscillate with the same frequency but in opposing phases and with different amplitudes during a period of time.

Finally, for the inclination effects, similar profile evolutions versus the tilt angle is exhibited for $Nu$ and $Sh$ presenting two extrema (a maximum followed by a minimum) precisely at the specific inclination values $\gamma = 30^\circ$ and $\gamma = 75^\circ$. Moreover, thermal and solutal isosurfaces seem to undergo central stratifications as the enclosure is further inclined until the value $\gamma = 75^\circ$ corresponding to the weaker heat and mass transfer rates, while the lower and upper thermal and solute gradients seem to be strengthened significantly.

References


