Modification on prediction method of heat transfer coefficient from a vertical-finite-length cylinder in saturated film boiling to discuss local heat transfer performance near the lower corner

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Abstract
Saturated film boiling heat transfer around a vertical-finite-length cylinder has been investigated analytically to predict the local heat transfer coefficients at the bottom and lower vertical surfaces. Correlations of heat transfer for the vertical cylinder with top and bottom horizontal surface has been already analyzed by Momoki et al.(2007) and this correlations equations for heat transfer are in good agreement with the experimental data for the cylinders in saturated water. In the present study, this correlations are applied to estimate the local and average heat transfer rate through each surfaces of the cylinder. To predict the local heat transfer coefficient at the corner of the bottom surface and vertical lateral surface, Shigechi et al.(1999) analysis for film boiling heat transfer on horizontal bottom surface was modified in order to get finite vapor film thickness at the end of the bottom surface to predict finite value of local heat transfer coefficient at this end. In this modification, the vapor film thickness at the end of the bottom surface can be predicted and the prediction of the average heat transfer rate through all surfaces are in good agreement with the results of the Shigechi et al.’s method. Moreover, the local heat transfer coefficient at the end of the bottom surface can be predicted to discuss the local heat transfer characteristic of the bottom surface. The local heat transfer rate through the bottom surface and vertical lateral surface are described in terms of local Nusselt number with degree of superheat. The results on local heat transfer coefficient shows the highest value at the corner of the bottom surface and vertical lateral surface and it can be confirmed the experimental results of the vapor film collapse start from the corner of the bottom surface and vertical lateral surface of the cylinder in saturated film boiling.

Keywords: Film boiling, Film collapse, Heat transfer rate, Local heat transfer coefficient

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>c</td>
<td>coefficient for slip and non-slip condition</td>
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<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
<td>[J/(kg·K)]</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of cylinder</td>
<td>[m]</td>
</tr>
<tr>
<td>g</td>
<td>gravity acceleration</td>
<td>[m/s²]</td>
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<tr>
<td>$Gr$</td>
<td>Grashof number</td>
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<tr>
<td>$h$</td>
<td>local heat transfer coefficient</td>
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<tr>
<td>$k$</td>
<td>thermal conductivity</td>
<td>[W/(m·K)]</td>
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<tr>
<td>$\ell$</td>
<td>latent heat</td>
<td>[J/kg]</td>
</tr>
<tr>
<td>$L$</td>
<td>height of cylinder</td>
<td>[m]</td>
</tr>
<tr>
<td>$L_{B1}$</td>
<td>vertical length for smooth vapor-liquid interface</td>
<td>[m]</td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow rate per unit circumference</td>
<td>[kg/(m·s)]</td>
</tr>
<tr>
<td>$M$</td>
<td>mass flow rate</td>
<td>[kg/s]</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
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</table>
$Pr$ : Prandtl number [-]
$R$ : radius of cylinder [m]
$r$ : radial position [m]
$r^*$ : $r - R$ [m]
$Sp$ : dimensionless degree of superheating [-]
$Sp^*$ : modified dimensionless degree of superheating [-]
$T$ : temperature [$^\circ$C]
$T_W$ : wall temperature [$^\circ$C]
$T_{sat}$ : saturated temperature [$^\circ$C]
$\Delta T_{sat}$ : degree of superheating [K]
$u$ : fluid velocity [m/s]
x : vertical position along the vertical surface [mm]
y : position normal to the bottom surface [mm]
$\tau$ : time [s]
$\delta$ : vapor film thickness [mm]
$\lambda$ : vapor-film-unit length [m]
$\lambda_0$ : capillary length [m]
$\lambda_{cr}$ : critical wave length [m]
$\mu$ : dynamic viscosity [Pa·s]
$\nu$ : kinematic viscosity [m$^2$/s]
$\rho$ : density [kg/m$^3$]
$\sigma$ : surface tension [N/m]

Superscripts

$-$ : average value
$\sim$ : dimensionless

Subscripts

A : downward facing bottom surface
B : vertical lateral surface
B1 : vertical lateral surface with smooth vapor-liquid interface
B2 : vertical lateral surface with wavy vapor-liquid interface
C : upward facing horizontal surface
L : liquid at film temperature
V : vapor at film temperature

1. Introduction

Film boiling occurs in manufacturing process of material such as metal quenching, emergency core cooling of nuclear reactor and during normal operation in boilers tubes, and so on. For the safety effect, it is useful to predict the heat transfer performance and to correlate the occurrence of the vapor film collapse. It is also important to know the heat transfer rate of the specific area that the vapor film collapse start to occur. As the mechanism of vapor film collapse is in complex phenomena and there are so many research on prediction of heat transfer rate at the minimum heat flux condition or vapor film collapsed on three dimensional body. The study on film boiling heat transfer from the single surface such as a vertical surface, a horizontal cylinder, a sphere and upward facing horizontal surface have been carried out and a lot of practical correlation of film boiling heat transfer coefficient have been proposed for the saturated and subcooled film boiling. However, it is still difficult to predict accurately the film boiling heat transfer rate and the condition at the film collapse around the three dimensional bodies owing to insufficient knowledge of the heat transfer process.

Most significant advancement on film boiling heat transfer start from Bromley’s correlation (1950) for natural con-
convection stable film boiling from a vertical tube or a vertical plane surface, it is very well-known and still useful for the estimation of heat transfer coefficient from a single surface. An analytical expression for the film boiling heat transfer coefficient from a horizontal surface was derived by the Berenson (1961) by applying the Taylor-Helmholtz Instability to film boiling from a horizontal surface. By introducing the vapor-film-unit model, heat transfer correlation for natural-convection film boiling with a wavy interface was focused by Nishio and Ohtake, (1992).

In the previous study on the two-dimensional, steady-state, laminar film boiling heat transfer from a downward-facing horizontal plate of a finite size was analyzed by Shigeichi et al. (1989) and the results are examined in terms of vapor film thickness and heat transfer coefficient on bottom surface and compared with experimental data. However, the analysis is underpredicted the values of heat transfer rate by a factor of half or one-third, compare with the experimental data. In 1998, applying the same method, film boiling heat transfer from a downward facing horizontal plate has been studied theoretically by taking into account the effects of the plate edge and profile shapes of velocity and temperature in the vapor film with the integral method. The magnitude of the heat transfer rate in this method varies about twofold dependent of the profile shape of the velocity and temperature in the vapor film and is comparable to the experimental results. Initial study of film boiling around a three-dimensional body has been analyzed by Shigeichi et al. (1999) by modification of the Bromley’s solution for a vertical surface to accommodate the continuity of vapor mass flow rate around the lower corner of the vertical cylinder and the results showed that the effect of the vapor mass flow generated under the horizontal bottom surface is to decrease the film boiling heat transfer rate of the vertical lateral surface. Saturated and subcooled film boiling heat transfer around a vertical finite-length cylinder has been investigated experimentally and analytically by Momoki et al. (2003) and correlated average heat transfer coefficient through all surfaces of the cylinder. The average heat flux from the surface of the cylinder can be correlated within 20 percent using the correlation proposed in this method. They discussed the film boiling characteristic on each surface, and showed that the predicted average heat transfer rate at the vertical lateral surface with wavy vapor-liquid interface \( \overrightarrow{\text{\(n\)}}_2 \) is highest, and the rate at the bottom surface \( \overrightarrow{\text{\(n\)}}_3 \) and vertical lateral surface with smooth vapor-liquid interface \( \overrightarrow{\text{\(n\)}}_1 \) are also lower than the rate at the upward-facing horizontal surface \( \overrightarrow{\text{\(n\)}}_4 \). In 2007, Momoki et al. investigated pool film boiling experiment by quenching 18 kinds of vertical silver cylinder with top and bottom horizontal surface and proposed the correlations of heat transfer for saturated and subcooled film boiling. These correlation equations for heat transfer and lower limit of film boiling are in good agreement with the experimental data for the cylinders in the range of 8 to 100mm diameters and 8 to 160mm lengths in saturated water. And experimental study on effect of the material thermophysical properties upon quench point in transient film boiling around a vertical cylinder was performed by Momoki et al. (2005) and the results is like that the collapse of the vapor film occurs at the lower corner of the silver cylinders and at the upper corner of the cylinder with low thermal conductivity such as carbon and stainless steel.

Moreover, quenching experiments on transient film boiling heat transfer with a vertical-finite-length cylinder to quiescent saturated water at atmospheric pressure have been carried out consistently and investigated the film boiling heat transfer performance through all surfaces of the cylinder and the phenomena of the film collapse. The experimental apparatus used in film boiling experiment have been already described in details in previous proceeding. The collapse behavior of the vapor film during experiment are recorded with high speed video camera with 10000 frames per second speed. Experimental results on direct solid-liquid contacts to the cylinder wall were counted and analyzed the trend and frequency of these solid-liquid contacts and the phenomena changed in cooling rate, the start of the film collapse is confirmed at the lower corner of the vertical finite length cylinder by the visual and photographic observation results (Momoki et al. 2017). In this regard, the experimental observation of vapor film collapse start from the lower edge of the vertical lateral surface specify that there is the small region where the heat transfer rate is so high at the lower corner of the cylinder and attempts to emphasize the local heat transfer coefficient on each surface of the cylinder. So, it is required to predict the local heat transfer rate to discuss the experiment result on film collapse. In this condition, Shigeichi et al. (1999) analysis on effect of horizontal bottom surface on film boiling heat transfer from a vertical cylinder is modified to predict the local heat transfer rate at the end of the horizontal bottom surface.

Therefore, in this study, applying the same method as that used in prediction of heat transfer coefficient, the boundary condition at the end of the bottom surface is modified by assumption there is extended heated surface at the end of the bottom surface of cylinder and its thickness is equal to the vapor film thickness at the start of the vertical lateral surface. The boundary condition at the end of this virtual surface, the gradient of the vapor film thickness is minus infinity. In this modification, the film thickness at the end of the bottom surface is predictable and local heat transfer rate at the end of the horizontal bottom surface is predicted in finite value and local heat transfer coefficient through all surfaces of the cylinder can be estimated for the four cases with different boundary condition of velocity at the vapor liquid interface. The
effect of this virtual extended thickness can be vanished was confirmed by comparing the predicted average heat transfer coefficients by the present method with the previous method by Shigechi et al. method.

2. Prediction of film boiling heat transfer coefficient from the vertical-finite-length cylinder

Figure 1 shows the physical model of a finite-length-vertical cylinder of diameter \(D\) and length \(L\), in the saturated film boiling condition, which is used in the present study. Generally, the prediction of the heat transfer performance on bottom, vertical lateral surface and upward facing horizontal surface of the cylinder based upon the following model and correlations of the previous author.

In this estimation, the heat transfer performance of the vertical cylinder was taken into account each convective heat transfer on the bottom (A) surface, vertical lateral (B) surface and top (C) surfaces of the cylinder. Furthermore, the vertical lateral surface is divided as lower vertical surface (B1) where the vapor-liquid interface is smooth and considered as laminar and upper vertical surface (B2) where the vapor-liquid interface is wavy and considered as turbulent by the length \(L_B1\). The value of the \(L_B1\) is obtained by the Momoki et al. (2003) based on the empirical equation for the critical wave length of water at the atmospheric pressure in saturated condition. The film boiling heat transfer from the downward-facing bottom surface \((h_A)\) and the vertical surface with smooth vapor liquid interface \((h_{B1})\), can be calculated by the present method which is modified the correlation of Shigechi et al. (1999). The vapor-film-unit model presented by Nishio and Ohtake (1992) was applied to predict the heat transfer rate through the vertical surface with wavy vapor-liquid interface of the cylinder \((h_{B2})\). Heat transfer coefficient on the upward-facing surface \((h_C)\) is estimated by applying the Berenson (1961)’s correlation. In briefly, local heat transfer coefficient on horizontal bottom surface and vertical surface with smooth vapor-liquid interface are simultaneously predicted by the present modification on Shigechi et al. (1999) based on Bromely’s (1956) method and heat transfer coefficient on vertical surface with wavy vapor-liquid interface and the upward-facing horizontal surface are predicted by applying the Nishio and Ohtake (1992) model and the Berenson’s correlation (1961) respectively in this study. Prediction of local heat transfer coefficient on bottom surface and vertical lateral surface with smooth vapor-liquid interface, \((h_A)\) and \((h_{B1})\) in present study are described in the following section.

2.1. Downward facing horizontal surface (A) and vertical lateral surface with smooth vapor-liquid interface (B1)

The correlation for the heat transfer coefficient on each surfaces of the vertical cylinder are already presented by Momoki et al. (2007) and predicted average heat transfer coefficient on each surface of the cylinder are in good agreement with the experimental results on cooling rate of the vertical cylinder in the transient film boiling condition. On the other hand, the estimate highest temperature point on the surface of the cylinder by these predicted average heat transfer rate did not agree the observed starting point of the film collapse (Momoki et al., 2007). In this regard, we want to discuss the local heat transfer coefficient on each surface of the vertical cylinder with the experimental result on film collapse. The prediction method for local and average film boiling heat transfer under the downward facing horizontal surface is theoretically proposed by Shigechi et al. (1999) and their method is described briefly here. The boundary layer equations for the vapor at the bottom surface were solved by an integral method and the resulting second order boundary value problem was solved analytically by giving boundary condition at \(r = R\) (which is at the end of the bottom surface), the gradient of the vapor film thickness is \(-\infty\). In this analytical solution, the amount of heat transfer rate near the end of the bottom surface become infinity as the film thickness approach to zero at this point. In order to eliminate such inconvenience to apply in prediction of local heat transfer coefficient, we modified the position of the boundary condition.
as $\frac{d\delta_A}{dr} = 0$ at $r = 0$ and $\frac{d\delta_A}{dr} = -\infty$ at $r = r_E$. It is noted that $\frac{d\delta_A}{dr} \bigg|_{r=r_E} = -\infty$ means the tangent line of the vapor-liquid interface at $r = r_E$ is a vertical line and that also $\delta_A = 0$ satisfies at $r = r_E$ in the present model. It was also assumed that the bottom surface of the cylinder is extended from $R (= D/2)$ to $r_E (= R + \delta_{b,0})$. The physical model and coordinate system ($r, y$) for this model are shown in Fig. 2 to clarify the present modification.

For the bottom surface, the momentum and energy equations in y-direction by neglecting the inertia and convective terms, are obtained as,

$$-(\rho_L - \rho_V) \frac{d\delta_A}{dr} + \mu_V \frac{d^2\delta_A}{dy^2} = 0$$

(1)

$$\frac{1}{2} \frac{d^2T}{dy^2} = -\lambda_V \frac{\partial T}{\partial y}$$

(2)

The boundary conditions are given at the cylinder wall, $y = 0$, and at the vapor-liquid interface, $y = \delta_A$ : $T = T_w$ at $y=0$, and $T = T_{sat}$ at $y = \delta_A$. The velocity profiles at the vapor-liquid interface are considered as the non-slip condition corresponding to Case (A-ns) and the slip condition corresponding to Case (A-s). For the Case (A-ns), $u = 0$ and for the Case (A-s), $\frac{d\delta_A}{dy} = 0$. There is one more boundary condition at the vapor-liquid interface as the rate of heat conduction through the vapor film to the interface is balanced to the rate of latent heat of vaporization and it can be expressed as

$$-\lambda_V \frac{\partial T}{\partial y} \bigg|_{\delta_A} = \ell \int_{\delta_A}^{r_E} \frac{d}{dy} \left[ \frac{1}{2} y \rho_V u dy \right].$$

(3)

Solving the momentum equation, Eq. (1), together with the velocity boundary conditions, the equation for the velocity profile can be obtained as

$$u = \frac{1}{2} \left( \frac{\mu_L - \mu_V}{\mu_V} \right) \frac{d\delta_A}{dr} \left[ \frac{\nu}{\delta_A} \left( \frac{y}{\delta_A} \right) - \left( \frac{y}{\delta_A} \right)^2 \right],$$

(4)

where the numerical values of coefficient $c_{A1}$ depends upon the boundary condition at the vapor liquid interface and for Case (A-ns), the value $c_{A1}$ is 1 and for Case (A-s), the value of $c_{A1}$ is 2. Also, solving energy equation of Eq. (2), together with temperature boundary condition, the equation for temperature profile can be obtained as

$$T = T_w - \Delta T_{sat} (y/\delta_A),$$

(5)

where $\Delta T_{sat} \equiv T_w - T_{sat}$. By applying the velocity and temperature profile equations, Eqs. (4) and (5), to heat balance equation at the vapor-liquid interface, Eq. (3), the differential equation for the vapor film thickness $\delta_A$ is obtained as

$$\frac{1}{r} \frac{1}{r} \frac{d}{dr} \left[ \frac{\rho_L - \rho_V}{\mu_V} \frac{d\delta_A}{dr} \right] = -c_{A2} \frac{\nu}{\delta_A} \left( \frac{y}{\delta_A} \right) - \left( \frac{y}{\delta_A} \right)^2 \frac{\Delta T_{sat}}{\ell} \frac{1}{\delta_A},$$

(6)

Introducing the following dimensionless variables; $\tau \equiv r/D$, $\delta_A \equiv (\delta_A/D) \left( Gr_A/\delta_V \right)^{1/3}$ and dimensionless parameters $Gr_A$ and $\delta_V$, where $Gr_A$ is the Grashof number that is equivalent to the $(\mu_D/\nu^2) \left( \rho_L/\rho_V \right) - 1]$, and $\delta_V$ is the dimensionless degree of superheating that is equivalent to $c_{PV} \Delta T_{sat}/(Pv \ell)$, the differential equation of the vapor film thickness is transformed to

$$\frac{1}{\tau^3} \frac{d}{d\tau} \left[ \frac{\rho_L - \rho_V}{\mu_V} \frac{d\delta_A}{d\tau} \right] = -c_{A2} \frac{1}{\delta_A},$$

(7)
Defining the function $\phi \equiv \frac{d\phi}{d\tau}$ in Eq. (7), and the form of the differential equations is transformed to following ordinary differential equation.

$$\frac{d\phi}{d\tau} = -\left[ \frac{3}{\delta_A} \right] \phi^2 + \left( \frac{1}{\tau} + \frac{c_{A2}}{\delta_A^3} \right).$$  \hspace{1cm} (8)

The values of numerical coefficient, $c_{A2}$ used in Eqs. (6), (7) and (8) corresponding to the slip and non-slip condition at the vapor liquid interface at surface (A) are described as 12 for Case (A-ns) and 3 for Case (A-s) respectively.

Applying the Runge-Kutta method to the above Eq. (8) with the assumption of $\delta_A$ at $\dot{\tau} = 0$, that is $\delta_A = 0$, and the profile of vapor film thickness, $\delta_A(\dot{\tau})$ and $\delta_B(\dot{\tau})$ are calculated using the differential equation of vapor film thickness, $\frac{d\delta_A(\dot{\tau})}{d\dot{\tau}}$ and the value of $\delta_B(\dot{\tau})$ can be determined. The vapor film thickness at the lower end of the vertical surface, $\delta_B(0)$ is calculated by the following method, which is based upon the continuity of the mass of the vapor at the corner between the vertical and downward facing surface. Applying the continuity condition of the vapor mass flow rate around the corner of the cylinder i.e at $\dot{\tau} = 1/2$ and at the lower end of the vertical surface, $\delta_B(0)$ can be determined.

$$M_A|_{\dot{\tau}=1/2} = M_B|_{\delta_B=0} = \rho \nu u \int_0^{\delta_B} 2\pi (R + r^*) dr^*.$$  \hspace{1cm} (9)

It should be noted that the value of $r^*$ is much smaller than the radius of the cylinder, $R$. Therefore the vapor on the vertical lateral surface of the cylinder is treated as the flow on the plain surface in the present study, and we can transform the vapor mass flow rate at the lower corner, $M_B|_{\delta_B=0}$ to be $\rho \nu u \int_0^{\delta_B} 2\pi R = \delta_B|_{\delta_B=0} = \delta_B|_{\dot{\tau}=1/2}$.

Neglecting the effect of curvature of a cylinder, the vertical lateral surface of the cylinder is treated as a plane surface and by applying momentum and energy balance upon the vertical surface and specified boundary condition at the vapor liquid interface as in the analysis of the bottom surface, the velocity profile on vertical surface can be obtained as

$$u = \frac{1}{2} \left[ \frac{\rho_L - \rho_V}{\mu_V} \frac{g}{\delta_B^2} \right] \left[ \frac{\rho_L - \rho_V}{\mu_V} \frac{g}{\delta_B^2} \right] \left[ \frac{\rho_L - \rho_V}{\mu_V} \frac{g}{\delta_B^2} \right].$$  \hspace{1cm} (11)

The coefficient of the Eq. (11), $c_{B1}$ depends upon the boundary condition of slip (B-s) and non-slip (B-ns) at the vapor-liquid interface of the vertical surface and the values are 1 and 2 for Case (B-ns) and Case (B-s) respectively. By substitution equation of velocity profile at the vertical surface, Eq. (11), into Eq. (10), in their respective cases, the value of $\delta_B$ can be determined by the following equations

$$m_B \equiv \frac{1}{c_{B1}} \left[ \frac{\rho_L - \rho_V}{\mu_V} \frac{g}{\delta_B^2} \right] \delta_B^3.$$  \hspace{1cm} (12)

where the coefficient $c_{B1}=12$ for case (B-ns) and $c_{B1}=3$ for case (B-s). The numerical result of $(\delta_B)$ calculated from the Eq. (12), is the film thickness at the bottom surface of the vertical surface and it can be designated as $\delta_B(0)$. The above procedure is looped from the assumption of the $\delta_A$ to satisfy the specified condition of Eq. (8), to the prediction of $\delta_B$ in Eq. (12), till the value of the $\delta_B$ addition to the radius of the cylinder is equivalent to the radius $R_E$, where the gradient of the vapor film thickness approach to $-\infty$. Table 1 shows example of the film thickness of the $\delta_B$ at degree of superheat $\Delta T_{sat} = 300$ K predicted by above method for four cases. These numerical results are obtained by repeated assuming of the $\delta_A$ at $\dot{\tau} = 0$ to satisfy the Eq. (8), and the resultant value of $\delta_B(0)$ was rechecked by substitution in the predetermined value of $\dot{\tau}$. When the value of $\dot{\tau}$ is equal to the radius of the cylinder plus the value of $\delta_B(0)$ predicted by the continuity condition of the vapor mass flow rate around the corner, the film thickness at the bottom of the vertical surface ( $\delta_B$) at respective degree of superheat is obtained.

Profile of the vapor film thickness upon the vertical surface, can be calculated by the following method. For the vertical lateral surface with smooth vapor-liquid interface, the assumption are the same as the bottom surface and the boundary equation were solved by integral method. And also the differential equation of the film thickness on vertical lateral surface can be written as

$$\frac{d\delta_B^4}{dx} = c_B3 \left[ \frac{v}{(\rho_L - \rho_V) g} \right] \frac{k_V \Delta T_{sat}}{\ell^4}.$$  \hspace{1cm} (13)
2.2. Vertical surface with wavy vapor-liquid interface (B2) and upward facing horizontal surface (C)

As mentioned in section 2, the vapor-liquid interface at the upper vertical surface (B2) and upward-facing horizontal surface (C) is wavy and considered as turbulent. It was assumed that, in these surfaces (B2) and (C), local heat transfer rate is changed slightly with the position compared to the other surfaces and the local heat transfer coefficient is assumed as uniform. Heat transfer rate on the upper vertical surface is already analyzed by Nishio-Ohtake (1992) and upward-facing horizontal surface is analyzed by Berenson (1961)'s correlation. In this study, heat transfer rate on the upper vertical surface and upward-facing horizontal surface are estimated by the prediction method which is proposed in that analysis (Momoki et al. 2007). The heat transfer coefficient upon the vertical surface with wavy vapor-liquid interface, (B2) can be estimated as

\[ h_{B2} = 0.740(k_v/\lambda)(Gr_{B2}/Sp)^{1/4}, \]  

Introducing the following dimensionless variables \( \tilde{x}_B \equiv x_B/L, \tilde{\delta}_B \equiv (\delta_B/L) (Gr_B/Sp)^{1/4} \) and parameters for the vertical surface, \( Gr_B \), the differential equation for the dimensionless film thickness (\( \tilde{\delta}_B \)) can be defined as

\[ \frac{d\tilde{\delta}_B}{d\tilde{x}_B} = \psi_3. \tag{14} \]

In this Eq. (13), and (14), \( \psi_3 = 16 \) for Case (B-ns) and 4 for Case (B-s). Solving Eq. (14), together with the initial condition of \( \tilde{\delta}_B = \tilde{\delta}_{B,0} \) at \( \tilde{x}_B = 0 \), the exact solution for \( \tilde{\delta}_B \) is obtained with the coefficient \( \psi_3 = 2 \) for Case (B-ns) and \( \sqrt{2} \) for Case (B-s) as

\[ \tilde{\delta}_B = \psi_{3B} \tilde{x}_B + \left( \tilde{\delta}_{B,0}/\psi_{3B} \right)^{1/4}. \tag{15} \]

The representative length for the \( L \) is equal to \( L_{B1} \) for the vertical surface with smooth vapor-liquid interface. The value of \( L_{B1} \) is calculated by the following empirical equation based on the critical wave length, \( \lambda_{cr} \), and the relation, \( L_{B1} = S \lambda_{cr} = 2\pi \lambda_{cr} \), in which \( \lambda_{cr} \) is the capillary length defined by \( \lambda_{cr} \equiv \sigma/\left[ g (\rho_{sat} - \rho_v) \right]^{1/2} \). The value of empirical parameter \( S \) was obtained as 0.5 using observation results for saturated film boiling around the vertical cylinder of different aspect ratio with water at atmospheric pressure (Momoki et al., 2007).

Finally, heat transfer coefficient and Nusselt number for the bottom surface \( (A) \) can be calculated by the following equations

\[ h_A \equiv k_v \frac{1}{\Delta T_{sat}} \left[ \frac{\partial T}{\partial y} \right]_{y=0} = \frac{k_v}{\tilde{\delta}_A} \tag{16} \]

\[ Nu_A = \left(1/\tilde{\delta}_A \right) (Gr_A/Sp)^{1/5} \tag{17} \]

\[ Nu_{B1} = \sqrt{8} \left[ \int_{0}^{1/2} \left( \tilde{r}/\tilde{\delta}_A \right) d\tilde{r} \right] (Gr_{B1}/Sp)^{1/5}. \tag{18} \]

The Nusselt number (\( Nu_{B1} \) and \( Nu_{B2} \)) and heat transfer coefficient (\( h_{B1} \) and \( h_{B2} \)) upon vertical lateral surface with smooth vapor-liquid interface (B1) can be calculated by the following equations

\[ h_{B1} \equiv k_v \frac{1}{\Delta T_{sat}} \left[ \frac{\partial T}{\partial y} \right]_{y=0} = \frac{k_v}{\tilde{\delta}_B} \tag{19} \]

\[ Nu_{B1} = \left(1/\tilde{\delta}_B \right) (Gr_{B1}/Sp)^{1/4} \tag{20} \]

\[ Nu_{B2} \equiv \frac{h_{B2} \cdot L_{B1}}{k_v} = \left[ \int_{0}^{1} \left(1/\tilde{\delta}_B \right) d\tilde{x}_B \right] (Gr_{B2}/Sp)^{1/4}. \tag{21} \]

Table 1 Example of predicted vapor film thickness of water at the bottom and vertical surface of the cylinder with \( D = L = 32 \) mm at \( \Delta T_{sat} = 300 \) K

<table>
<thead>
<tr>
<th>Case</th>
<th>( \tilde{\delta}_{A,0} )</th>
<th>( \tilde{r}_e )</th>
<th>( \int_{0}^{1/2} \left( \tilde{r}/\tilde{\delta}_A \right) d\tilde{r} )</th>
<th>( \tilde{\delta}_{B,0} )</th>
<th>( \tilde{\delta}_{B,0} ) [mm]</th>
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<td>Ans-ns</td>
<td>1.27315</td>
<td>0.50524</td>
<td>0.12424</td>
<td>1.01321</td>
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<td>0.67446</td>
<td>0.116193</td>
</tr>
</tbody>
</table>
where the Grashof number for this surface is considered as \( \text{Gr}_{B2} \equiv \left( g \lambda^3 \nu^2 \right) \left[ \left( \rho_1 / \rho_v \right) - 1 \right] \), the modified dimensionless degree of superheating as \( \text{Sp}^+ \equiv c_p \Delta T_{sat} / \left[ Pr (T + 0.5c_p \Delta T_{sat}) \right] \), and the vapor-film-unit length as 
\[
\lambda = 16.2 \left[ \left( \text{Sp}^+ \text{Gr}_{B2} \right) \right]^{1/11} \lambda_0.
\]

For the upward-facing horizontal surface (C),
\[
h_C = 0.425 \left( k_C / \lambda_0 \right) \left( \text{Gr}_C / \text{Sp} \right)^{1/4},
\]
where \( \text{Gr}_C \equiv \left( g \lambda^3 / \nu^2 \right) \left[ \left( \rho_1 / \rho_v \right) - 1 \right] \) is coincides with the calculation method of the \( \text{Gr}_{B2} \) used in calculation of the vapor-film-unit length.

3. Results and discussion on estimated average and local heat transfer coefficients

To predict the heat transfer coefficient upon all surfaces of the vertical-finite-length cylinder, first we modified the prediction method of the Shigechi et al. and can predict the local heat transfer coefficient at the end of the bottom surface. In this modification, the value of average heat transfer rate on each surface of the cylinder is the same as predicted by Shigechi et al.’s one for Case (A-ns+B-ns) and local heat transfer rate by the present method is only about 1 % smaller than predicted by the Shigechi et al.’s method except near the end of bottom surface. Furthermore, during film boiling, the calculated vapor film thickness decreased and heat transfer coefficient increased as the degree of superheat decreased as usual. Here, local heat transfer coefficient is predicted for the degree of superheat \( \Delta T_{sat}= 200, 300, 400 \), and 500K but only described the results on \( \Delta T_{sat} = 300K \) with possible two cases. The highest local heat transfer rate is at the corner of the bottom and vertical surface where there is the film collapse start by the experimental observation.

![Figure 3](image-url)  
**Fig. 3** Vapor film thickness at the end of bottom surface (water at atmospheric pressure, \( \Delta T_{sat} = 300 \text{ K} \))

Figure 3 shows vapor film thickness distribution on downward-facing bottom surface predicted by two types of prediction methods. Vapor film thickness predicted by the boundary condition at the end of the bottom surface by Shigechi et al. are represented by dotted lines and still undetermined the vapor film thickness at that end since the gradient of the vapor film thickness is minus infinity. Present predicted results are represented by the solid lines with the boundary condition at the virtual radius, \( r_{v} \), the gradient of the vapor film thickness is minus infinity and the vapor film thickness at the end of the bottom surface can be determined. While the Shigechi et al. method for the downward-facing bottom surface (A), is not affected by the vertical surface (B1), the present method is slightly affected by slip or non-slip condition at vapor-liquid interface at the surface (B1). Therefore, to focus on the difference between two methods, we also presents the results at dimensionless radius at \( f = 0.5 \). In this Fig. 3, the obtained film thickness for all cases, A-ns+B-ns, A-ns+B-s, A-s+B-ns and A-s+B-s by the present method and those for cases A-ns and A-s for the Shigechi et al. method are shown for the water at degree of superheat 300K. However, the vapor film thickness at the center of the bottom surface (\( \delta_0 \) at \( r = 0 \)) is slightly thicker than the previous one by Shigechi et al.'s but this small value is not effective in prediction the heat transfer rate through the entire bottom surface. Furthermore, we can predicted the thickness of the vapor film at the end of the bottom surface with their respective cases as \( \delta_0 \) for Case (A-ns+B-ns) = 0.239 mm, \( \delta_0 \) for Case (A-ns+B-s) = 0.214 mm, \( \delta_0 \) for Case (A-s+B-ns) = 0.185 mm and \( \delta_0 \) for Case (A-s+B-s) = 0.165 mm in order of decreasing.
The results on local and average Nusselt number changed with degree of superheat upon the horizontal bottom surface were shown in Fig. 4 for only two Cases (A-ns + B-s) and (A-s+B-s) as the effect of the case in the region (B1) on the heat transfer coefficient in the region (A) was small. It is clear that the value of slip condition at the vapor-liquid interface is much higher than the condition at non-slip. The local Nusselt number is described with the position of the dimensionless radius \( \tilde{r} = 0 \) to \( \tilde{r} = 0.5 \) and the results on present study are shown at degree of superheat at \( \Delta T_{sat} = 200, 300, 400 \) and 500K respectively and which is defined by the ratio of dimensionless number on \( x \)-axis of the Fig. 4, as \( Sp/Gr_B \).

Here, \( Sp/Gr_B \) value of \( 3.3 \times 10^{-10} \) represents to \( \Delta T_{sat} = 200K \), \( 7.1 \times 10^{-10} \) represents to \( \Delta T_{sat} = 300K \), \( 1.3 \times 10^{-9} \) represents to \( \Delta T_{sat} = 400K \) and \( 2.2 \times 10^{-9} \) represents to \( \Delta T_{sat} = 500K \), respectively. As the dimensionless radius approach to 0.5, that is at the end of bottom surface, local Nusselt number is the highest on both cases and for both studies. As the different of the result on local Nusselt number predicted by Shigechi et al. method and present study start from \( \tilde{r} = 0.48 \) and at the end of the bottom surface, local Nusselt number predicted by the previous method approach to infinity. At that time, local Nusselt number at the end of the bottom surface can be predicted by the present modification method. Furthermore, the average Nusselt number predicted by the present study are almost the same those of the average Nusselt number predicted by the Shigechi et al. method. At the end of the bottom surface at low degree of superheat (which is near the lower limit of film boiling condition) the local Nusselt number is highest and as the degree of superheat higher, increasing rate of local Nusselt number is not reasonable amount. For all cases, the local Nusselt number predicted by the present method in the area \( \tilde{r} \leq 0.48 \) are almost the same to the Shigechi et al.’s method and as increasing \( \tilde{r} > 0.48 \), the results on local Nusselt number by the present methods are smaller than the Shigechi et al. one’s.

The results of local and average Nusselt number upon the vertical surface were shown in Fig. 5, although considered for the combination of four cases, but only for two combination Cases, (A-ns + B-ns) and (A-ns + B-s) are described as the limited space is available and the effect of slip and non-slip condition in the bottom surface upon vertical surface is not so large. Local and average Nusselt numbers on vertical surface are predicted by Shigechi et al.’s method and also by the present method and compared the results. As the prediction of \( \delta_{9,0} \) at the lower end of the vertical surface predicted by the present study is smaller than the value of film thickness predicted by Shigechi et al., all values predicted at the vertical lateral surface by the present study are just only about 1% greater than those by the Shigechi et al.’s method. Local Nusselt number upon vertical surface with smooth vapor-liquid interface are described with the value at vertical position, at the lower end of the vertical lateral surface (\( x_B = 0 \)) the value of local Nusselt number is the highest and as the distance \( x_B \) from the lower end of the vertical surface increased, the value of local Nusselt number decreased and finally approaches to...
Fig. 5 Nusselt number on the vertical surface and degree of superheat

constant value. The difference between the highest Nusselt number at \( x_B = 0 \) mm and lowest Nusselt number at \( x_B = 9.6 \) mm becomes smaller as increasing degree of superheat, that is the value of \( Sp/Gr_B \) increase at \( x \)-axis for both cases. From the results on local Nusselt number upon the vertical surface, combination of non-slip condition at the bottom surface and slip condition at the vertical surface gives the highest heat transfer rate at the lower end of the vertical surface.

3.1. Local heat transfer coefficient on each surface of the vertical cylinder

In order to discuss the effect of local heat transfer coefficient on film collapse, in this study, we consider two velocity profiles of non-slip and slip condition at the vapor-liquid interface for the bottom surface and vertical lateral surface with smooth vapor-liquid interface. This physically possible combination of two velocity profile are referred as Case (A-ns + B-ns), Case (A-ns + Bns), Case (A-s + B-ns) and Case (A-s + B-s). So we predict for these four cases and discuss here for the applicable cases because one of the objectives are discussion about the highest heat transfer point and the film collapse. The predicted local heat transfer coefficients on all surfaces of the vertical cylinder are described in Fig. 6, for the combination of Cases (A-ns + B-ns) and (A-ns + B-s) and the average heat transfer coefficients for Case (A-ns + B-ns) are only described for the comparison. Local heat transfer rate through all surfaces of the vertical cylinder are predicted for the degree of superheat at \( \Delta T_{sat} = 200, 300, 400 \) and 500K. Hence, the value of local heat transfer coefficient at degree of superheat 300K is described in Fig. 6, because all the results show the similar trend. It will be noted that the trend of local heat transfer coefficient upon vertical lateral surface with smooth vapor-liquid interface \( (h_{B1}) \) is two fold depend upon the slip condition (B-s) and non-slip condition (B-ns) at the vertical surface.

We discuss that the distribution of the predicted local heat transfer coefficients on all surface of the cylinder based on the present method. In the author’s previous study, the estimated heat transfer rate based on the average heat transfer coefficients for the Case (A-ns + B-ns) agrees the experimental results, and the non-slip condition at the bottom surface (A) may be reasonable because the velocity of the vapor beneath the downward-facing horizontal surface is not so large and also there is no significant configuration at the bottom surface to be considered as slip condition. Therefore, the estimated result for the non-slip vapor-liquid interface condition for the downward-facing surface (A-ns) is described in Fig. 6. For the vertical surface with smooth vapor-liquid interface (B1), the estimated results for both slip and non-slip condition at the vapor liquid interface are discussed because the velocity of the vapor might become so large at some region near the bottom due to the supplied vapor from the bottom surface passed a corner or vantage point and slip condition might be occurred. Furthermore the combination of non-slip condition (A-ns) at the bottom surface and slip
condition at the vertical surface (B-s), that is Case (A-ns + B-s) gives higher heat transfer rate than that of Case (A-ns + B-ns). The reason that \( h_{B1} \) for Case (A-ns + B-s) is the highest possible in actual application, is liked that as consider for the cylinder configuration, the velocity at the corner of the bottom surface and vertical surface might be high enough to be considered as non-slip condition at the vapor-liquid interface as the film thickness at that edge is invisible small. But for the bottom surface, we have already studied that the effect of the vapor mass flow generated under the horizontal bottom surface is to decrease the heat transfer rate. However, in actual application, overestimation of the heat transfer rate should be avoided and it can be conducted that even in non-slip condition at the vertical surface, local heat transfer coefficient at the corner of the vertical surface is highest compare to other surfaces. In prediction of the lowest temperature point on the cylinder surface with the average heat transfer coefficients, it seems to be the upper corner of the cylinder due to the much higher heat transfer coefficient at the upward-facing horizontal surface \( \bar{h}_C \) and at the upper region of the vertical lateral surface \( \bar{h}_{B2} \), than that of the downward-facing horizontal surface \( \bar{h}_A \) and lower vertical surface \( \bar{h}_{B1} \). As the results on predicted local heat transfer coefficient on vertical cylinder, the highest heat transfer coefficient are found at the end of bottom surface (at \( x_A = L \)) and at the lower corner of the vertical surface (at \( x_B = 0 \)) of the cylinder as in Fig. 6, and it suggests that the lowest temperature point may become the lower corner of the cylinder. That the highest heat transfer coefficient at the lower corner of the cylinder agrees the author’s previous experimental results on the film collapse start from the lower corner of the vertical cylinder (Momoki et al. 2017).

4. Conclusion

Local heat transfer rate through each surface of the vertical finite-length cylinder are predicted at saturated film boiling condition of water at atmospheric pressure and discuss the highest local heat transfer point with the film collapse. The heat transfer coefficient at the lower corner of the bottom surface and vertical lateral surface of the vertical cylinder are predicted at saturated condition by modification of the Shigechi et al. (1999) analysis. The present modification shows the result of average heat transfer rate through each surface of the vertical cylinder coincide with the predicted one’s by Shigechi et al. Moreover, it was found that the local heat transfer coefficients at the corner of downward facing bottom...
surface and the vertical surface give finite value for the entire area of the cylinder surface and in good agreement with the experimental results of the vapor film collapse start at the corner of the vertical cylinder.

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References


