Effect of axial conduction on heat transfer in a rectangular microchannel with local heat flux

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Abstract

Based on the continuum assumption and Navier-Stokes equation, three-dimensional fluid flow and conjugate heat transfer in heat sinks with rectangle microchannels have been studied by numerical simulation. The validation of this approach has been demonstrated by comparisons with analytical solutions. Results were obtained for the detailed description of the local and average heat transfer characteristics including wall and fluid temperatures, heat flux, Nusselt number (Nu). Effects of the Peclet number (Pe), the relative conductivity (k_w/k_f) and the relative thickness (W/D) of solid substrate were presented and discussed. At low Pe flow, axial conduction dominates the heat transfer, which results in a large upstream region which is pre-warmed by axial conduction, so axial conduction cannot be neglected at small Pe . The increase of Pe induces an enhancement of convection, the heat exchange mainly happens on heat region by convection under this condition. A more uniform streamwise temperature distribution at fluid-solid interface indicates the enhancement of the pre-warmed effect along with the increase of k_w/k_f . The study also reveals that the Nu_x of low k_w/k_f material is higher than the Nu_x of high k_w/k_f material. The increase of wall thickness has an effect on axial distribution of the Nu_x , the trend is similar to that of an increase of the wall thermal conductivity k_w/k_f . It was concluded that the average Nusselt number is proportional to thermal conductivity of the wall. For low Pe , the fluid is heated by the wall by means of convection on heat region, when $Pe \leq 1$, a slight increase of the $\overline{Nu}$ on heat region is shown as the increase of Pe . When $Pe > 10$, the convection is the dominant factor in heat transfer, an obvious increase of $\overline{Nu}$ is shown with increase of Pe . For high Pe flow, the heat transfer enhancement as increase of Pe is weak.

Keywords : Microchannel, Laminar flow, Local heat flux, Axial conduction, Heat transfer

Nomenclature

a	thermal diffusivity, $m^2.s$
A_{wall}	cross-sectional area of the microchannel, m^2
D	hydraulic diameter, m
h	convective heat transfer coefficient of the liquid flow, $W/(m^2.^\circ C)$
H	height of the microchannel heat sink, m , see Fig. 1
H_{ch}	height of the microchannel, m , see Fig. 1
H_{w1}	substrate thickness on top side of microchannel heat sink, m , see Fig. 1
H_{w2}	substrate thickness on down side of microchannel heat sink, m , see Fig. 1
k	thermal conductivity, $W/(m.^\circ C)$
L	length of the microchannel heat sink, m , see Fig. 1

L_0	length of heat region, m
L_e	thermal entrance length, m
Nu_x	Nusselt number
Pe	Peclet number
q	heat flux, W/m ²
q_0	input heat flux, W/m ²
Re	Reynolds number based on channel hydraulic diameter
T	Temperature, K
$T_{f,r}$	solid temperature on the interface of the solid and the liquid, K
$T_{s,r}$	solid temperature on the interface of the solid and the liquid, K
$T_{w,x}$	wall temperature at the x-coordinate position, K
u, v, w	velocity component in the x, y, z direction respectively, m/s
$\vec{V}$	velocity vector
W	width of the microchannel heat sink, m, see Fig. 1
W_{ch}	width of the microchannel, m, see Fig. 1
W_{w1}	substrate thickness on front side of microchannel heat sink, m, see Fig. 1
W_{w2}	substrate thickness on back side of microchannel heat sink, m, see Fig. 1
x, y, z	Cartesian coordinates, m
x/L	dimensionless length along flow direction
<i>Greek symbols</i>	
μ	liquid dynamic viscosity, Pa·s
ρ	density, kg/m ³
<i>Subscripts</i>	
f	fluid
in	inlet
w	wall

1. Introduction

Microchannel heat sink, first introduced by Tuckerman and Pease (1981), has received much attention in recent years due to the high heat transfer efficiency (Wang et al. (2009), Zhang et al. (2010), Ates et al. (2010), Rahimi and Mehryar (2012), Chen et al. (2009)). Along with high development of electronic systems, the continuous increase in heat dissipation at components, module and system levels (the peak local heat flux of these electronic equipments may reach $10^6 \sim 10^7$ W/m²) leads to an increasing demand for highly efficient electronic cooling technologies (Liu and Chen (2013, 2014), Chen et al. (2012)), so the traditional ways of air cooling technologies cannot meet the requirements of heat dissipation. Among the available heat transfer techniques for electronic cooling including heat pipe (Liu et al. (2013), Deng et al. (2017)), phase change materials (Ashraf et al. (2017)) et al., the microchannel heat sink has been proven to be a high performance cooling method (Mudawar (2001)).

In conventional channels, the axial heat conduction of the solid wall can be neglected in the convection heat transfer due to the fact that the thickness of the solid is usually very small as compared with the hydraulic diameter. On the contrary, the thickness of the solid wall is usually equal in size or larger than the hydraulic diameter of microchannel. Cole and Cetin (2011) studied the wall effects on heat transfer using a layer description of the fluid flow in microchannel, although Cole obtained detailed results for both temperature and Nusselt number for a variety of channel geometries and over a wide range of Pe using the infinite-domain Fourier transform, the study just considered the conditions of 2D microchannel, and the heat transfer along circumferential was ignored. Kosar (2010) studied the effects of thermal conductivity and substrate thickness on convective heat transfer in laminar internal flows. The effects of axial heat conduction are more notable under conditions of short channel, thick wall or low Reynolds number

(Maranzana et al (2004), Hetsroni et al. (2005), Li et al. (2007)). From the heat transport viewpoint, the characteristic time for convection and conduction become comparable for the microscale flow, therefore the convection term no longer dominates the heat transfer in longitudinal direction. In general, the ratio of convection term and diffusion term is characterized by the dimensionless number, Peclet number (Pe). The effect of axial conduction in fluid on the heat transfer has been studied for both parallel-plate microchannel (Jeong and Jeong (2006)) and microtube (Cetin et al. (2009)) with boundary conditions defined by constant wall temperature and constant wall heat-flux. In these studies, it can be concluded that the effect of the axial conduction in fluid cannot be neglected when the Pe is small.

The local temperature and the local wall heat flux at the interface between fluid and solid which needed in estimating the convection heat transfer coefficients (h) are very difficult to implement during the experiment. Shown by Herwig and Hausner (2003), uniform wall heat flux density along the whole channel and the fluid temperature keep constant during the flow in channel are not always valid for mini-micro-channels. Uneven heating or local heating often appears in microscale heat transfer research. Based on field coordination theory (Guo et al. (1998), WANG et al. (2004)), the intensity of convective heat transfer depends on the coordination relation between the velocity field and the temperature gradient field. Axial heat transfer effect caused by local heating is different from the case which heat source applied to the whole channel. All in all, the interest on the effect of axial conduction on laminar flow and heat transfer in microchannel has received much attention for many years, but the mechanisms of laminar flow and heat transfer in three dimensional microchannel with asymmetric heated by local heat source have not been fully revealed. In our study, the axial conduction in the wall is being taken into account.

In this paper, the study focus on the effect of axial conduction of the wall and the fluid on heat transfer in a three dimensional rectangular microchannel with up wall heated by a local heat source, which has not been studied before. The effect of the axial and circumferential conduction in the wall, the axial conduction in fluid, and the convection between the wall and the fluid are considered conjugately. Results were generated for the detailed description of the local and average heat transfer characteristics including temperatures of wall and fluid, heat flux and Nu . Influences of Pe , the relative conductivity (k_w/k_f) and the relative thickness (W/D) of solid substrate were presented and discussed.

2. Laminar flow and heat transfer in microchannel

2.1 Mathematical Model

For microchannel liquid flow with the equivalent hydraulic diameter of $1\mu\text{m} \sim 1\text{mm}$, the velocity slip condition can be neglected, and the continuum assumption and the Navier-Stokes equations are applicable (Guo and Li (2003)). Based on the continuum assumption and Navier-Stokes equation, three-dimensional fluid flow and conjugate heat transfer in heat sinks with rectangle microchannels have been analyzed numerically in this paper.

Fig. 1 Schematic of rectangular microchannel heat sink: (a) structure of parallel microchannel, (b) unit cell of microchannel.

Table 1 Dimensions of microchannel heat sink used in validation (Shah and London (1978))

$W(\mu\text{m})$	$W_{ch}(\mu\text{m})$	$H_{w1}(\mu\text{m})$	$H_{ch}(\mu\text{m})$	$H_{w2}(\mu\text{m})$
21.5	57	270	180	450

The rectangular microchannel heat sink is shown in Fig. 1, where a unit cell consisting of one channel is selected because of the symmetry of the structure. The unit cell can be represented by an asymmetric rectangular channel with the cross-section dimensions as shown in Table 1.

Some assumptions are applied to simplify the model of conventional Navier-Stokes and energy equations. The fluid employed in this study is incompressible, the solid and fluid have constant properties. And the flow is steady laminar flow, and the inner wall is no-slip. In this study, the radiation heat transfer is neglected. Based on the simplified assumptions above, the continuity, momentum and energy equations for a fully developed 3D flow heat transfer are,

$$\nabla \cdot \vec{V} = 0 \quad (1)$$

$$\rho(\vec{V} \cdot \nabla \vec{V}) = -\nabla P + \mu \nabla^2 \vec{V} \quad (2)$$

$$\vec{V} \cdot \nabla T = a \nabla^2 T \quad (3)$$

The energy equation of the solid thermal conduction is,

$$\nabla^2 \cdot T = 0 \quad (4)$$

Where P is the pressure of liquid, ρ is the density of liquid, T is the temperature of liquid, and μ is the viscosity, $\vec{V}$ is the velocity vector and a is the thermal diffusion coefficient.

The boundary conditions on the interface between the solid and liquid can be written as

$$T_{s,r} = T_{f,r} \quad (5)$$

$$-k_s \frac{\partial T_s}{\partial n} \bigg|_r = -k_f \frac{\partial T_f}{\partial n} \bigg|_r \quad (6)$$

$$u|_r = v|_r = w|_r = 0 \quad (7)$$

The boundary conditions for velocity and temperature at channel inlet are,

$$\text{for } x = 0, \quad u = u_{in}, \quad v = w = 0, \quad T = T_{in} \quad (8)$$

The flow is fully developed at channel outlet,

$$\text{for } x = L, \quad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial w}{\partial x} = 0, \quad \frac{\partial T}{\partial x} = 0 \quad (9)$$

For outer surfaces of the channel, the adiabatic boundary conditions are applied except the heat sink top wall.

$$\text{for } 1 \leq x/L \leq 2, \quad 0 \leq y \leq W, \quad z = H, \quad -k_s \frac{\partial T}{\partial z} = q_0 \quad (10)$$

The local Nusselt number, Nu_x , is used to describe the internal laminar flow and heat transfer characteristics of microflow. Nu_x along the channel at the x -coordinate is,

$$Nu_x = \frac{h_x L_0}{k_f} \quad (11)$$

$$\text{Where } \overline{T_w} = \frac{1}{A_{wall}} \int_{wall} T_{w,x}(x, y, z) dS, \quad \overline{T_f} = \frac{\int T \rho \vec{v} \cdot d\vec{A}}{\int \rho \vec{v} \cdot d\vec{A}}, \quad h_x = \frac{\overline{q_x}}{(\overline{T_w} - \overline{T_f})}.$$

The average Nusselt number downstream the thermal entrance section is,

$$\overline{Nu} = \frac{\int_{L_e}^L Nu_x dx}{L - L_e} \quad (13)$$

Where L_e is the length of thermal entrance section.

2.2. Numerical Method

The finite-control-volume method and the SIMPLE algorithm together with a uniform structured mesh based on rectangle grid elements were used to obtain the numerical solution of the above model along with the boundary conditions described above. The resulting system of algebraic equations is solved using the Gauss–Seidel iterative technique, with successive over-relaxation to improve the convergence time. Several different mesh sizes are used to test the grid independence. The simulation results obtained using the final mesh system is proved independent of the mesh size. Double precision was used in the computations, and solutions were considered converged when the residual levels of velocity and temperature below 10^{-6} and the heat and mass reach balances.

In order to verify the validity of the numerical analysis using the numerical code, the flow and heat transfer characteristics of rectangular microchannel heat sink (the geometric parameters are given in Table 1) are studied. The dimensionless velocity distribution for fully developed flow in microchannel obtained in this section are plotted in Fig. 2. The dimensionless velocity here is defined as $u/u_{\max}$ (in which $u_{\max}$ is the maximum velocity located at the center of rectangular channel). Excellent agreement is found between the present numerical prediction and the analytical solution (Qu and Mudawar (2002), Shah and London (1978)). As can be seen from the comparison between the analytical solutions and numerical velocity profiles at plane $z = 0$ and plan $y = 0$ which were presented in Fig. 2, the difference is so small that we have reason to believe that the two results are virtually identical.

Fig. 2 Comparison between the analytical solutions by Shah and London (1978) and numerical predictions:
(a) dimensionless velocity distribution of $z = 0$ section, (b) dimensionless velocity distribution of $y = 0$ section,
(c) Nu_x distribution along flow direction in a rectangular microchannel.

In order to verify the solution for the energy equation, the surrounding solid region is removed and a pure convective heat transfer problem in the rectangular channel is considered. The constant heat flux applied to the top wall of the channel is uniform. Based on numerical results, the Nusselt number Nu_x along the channel is evaluated and plotted in Fig. 2(c). Dates in Fig. 2 suggests that the flow characteristics and heat transfer predictions using this mathematical model is accurate for micro-scale flow.

3. Results and discussion

Based on the continuum assumption and Navier-Stokes equation, three-dimensional fluid flow and conjugate heat transfer in heat sinks with rectangle microchannels heated over region $1 < x/L_0 < 2$ have been analyzed numerically. The liquid in microchannel is deionized water. Dimensions of microchannel heat sink are summarized in Table 2. At first, the effects of the Pe number on heat transfer are discussed. Influences of the relative conductivity (k_w/k_f) on heat transfer are analysed following. Finally, the variations of the heat transfer characteristics caused by the relative thickness of solid substrate (W/D) are presented and discussed.

Table 2 Dimensions of microchannel heat sink in numerical analysis.

W (μm)	W_{ch} (μm)	H_{w1} (μm)	H_{ch} (μm)	H_{w2} (μm)
100	100	100	100	100

Nu_x is one way to view heat transfer behavior in microchannel. The normalized temperature difference between the wall and the fluid is another perspective to analyze the heat transfer details. The normalized temperature difference

between the wall and the fluid is,

$$(\overline{T_w} - \overline{T_f}) \cdot k_f / (q_0 L_0) \quad (14)$$

$$\text{Where } \overline{T_w} = \frac{1}{A_{wall}} \oint T_{wx}(x, y, z) dS, \quad \overline{T_f} = \frac{\int T \rho \bar{v} \cdot d\bar{A}}{\int \rho \bar{v} \cdot d\bar{A}}.$$

3.1 Heat Transfer Behaviours of the Microchannel

3.1.1 Effect of Axial Conduction in Fluid

Peclet number (Pe) describes the ratio of convection and conduction in fluid. The results no considering the wall thermal conduction ($W/D = 0$) for several Pe are presented in Fig. 3. The proportion of the convection in heat transfer is proportional to Pe . But when $W/D = 0$, the convective heat transfer only occur in heating region ($x/L_0 = 1 \sim 2$, see Fig. 3(a)), so the upstream fluid is heated by heat source through conductivity of the fluid.

With the decrease of Pe , both the conduction of the fluid and the preheating effect of heat source for upstream fluid increase, so dimensionless average temperature difference between the wall and the fluid $(T_w - T_f) \cdot k / (q_0 \cdot L)$ increases. Under high Pe condition, the weak conductivity of the fluid leads to the most heat comes from heat source added to the fluid of heat region, so the maximum of Nu_x appears at starting of the heat region. And with development of the flow, Nu_x of heat region decreases (see Fig. 3(c)), this trend is similar to the thermal entrance effect of microchannel. Differently, owing to the strong preheating effect of heat source for upstream fluid for small Pe , the average temperature difference between the wall and the fluid is large, so the thermal entrance effect of Nu_x is less obvious at starting of the heat region. And under heat source with constant heat density, the Nu_x of heat region increases monotonically for small Pe condition.

The upstream fluid is heated by the heat comes from the fluid of heat region. The smaller the Pe is, the stronger the conduction of the fluid is, and the bigger the temperature difference between the wall and the fluid in upstream fluid is. Therefore, the axial conductivity of the fluid can not be neglected under low Pe condition.

Fig. 3 Effects of Pe on (a) q_x/q_0 , (b) $(T_w - T_f) \cdot k / (q_0 \cdot L_0)$ and (c) Nu_x for $W/D = 0$.

3.1.2 Effect of Axial Conduction in Wall

Axial conduction of the wall can not be ignored when the hydraulic diameter of the microchannel is close to the thickness of the wall in microscale flow. To analyze the proportion of heat conduction in wall during heat transfer process, Fig. 4 illustrates the flow field distribution under conditions of $Pe = 0.1$, $k_w/k_f = 3$, $W/D = 0, 1$.

Different from the case of $W/D = 0$, considering the wall heat conduction, a portion heat of heat source transfers to upstream by conduction of the wall, so the thermal entrance effect forms near entrance of the channel, meanwhile the maximum of q_x/q_0 , $(T_w - T_f) \cdot k / (q_0 \cdot L)$ and the Nu_x appear at this location. Under influences of convective heating of the wall and conduction of the fluid, $(T_w - T_f) \cdot k / (q_0 \cdot L)$ decreases along flow direction from channel entrance to the end of heat region.

The fluid of the heat region is heated by convection of heat source, Fig. 4(c) also shows that the Nu_x of heat region increases monotonously. And affected by the convection of the wall, $(T_w - T_f) \cdot k / (q_0 \cdot L)$ becomes smaller and another peak of Nu_x appears near the end of heat region. After the entrance, the values mentioned above (q_x/q_0 , $(T_w - T_f) \cdot k / (q_0 \cdot L)$ and Nu_x) decrease rapidly. Take no account of the wall conduction ($W/D = 0$), the heat of heat region transfers to upstream merely by conduction of the fluid, so the effect of thermal entrance section does not appears. From the analysis above, we can conclude that the conduction in the wall can not be neglected under low Pe conditions.

 Fig. 4 Effects of W/D on (a) q_x/q_0 , (b) $(T_w - T_f)k_f / (q_0 L_0)$ and (c) Nu_x for $Pe = 0.1$, $k_w/k_f = 3$.

3.1.3 Effect of the Heat Source Location

To have an insight into the effects of heat source location, Fig. 5 demonstrates the flow field distributions of $Pe = 0.1$, $k_w/k_f = 3$, $W/D = 1$ under condition of local heating and overall heating. The conduction is the dominant mechanism for heat transfer in the fluid for small Pe . When heat source added to the whole channel, there is an obvious thermal entrance section near the channel entrance. And the maximum of q_x/q_0 , $(T_w - T_f)k_f / (q_0 L_0)$ and Nu_x appear at the channel entrance. After the thermal entrance, q_x/q_0 decreases firstly and then keeps constant almost. Meanwhile the Nu_x of downstream after thermal entrance decreases rapidly. The fluid is heated by a portion heat of the heat region when the channel heated locally, so the thermal entrance effect forms near channel entrance, and the maximum of q_x/q_0 and Nu_x appear at channel entrance. At local heating boundary conditions, q_x/q_0 increases again affected by local heat source, in addition q_x/q_0 decrease rapidly in downstream of the heat region (Fig. 5(a)).

The Nu_x for heat region increases monotonically when the channel heated by whole heat source. Differently, at local heating condition, the Nu_x increase monotonously near start of the heat region, and the Nu_x keep increasing until end of the heat region. Affected weakly by heat source, in downstream of the heat region, the Nu_x decreases again (Fig. 5(c)).

 Fig. 5 Effects of the heat source location on (a) q_x/q_0 , (b) $(T_w - T_f)k_f / (q_0 L_0)$ and (c) Nu_x for $Pe = 0.1$, $k_w/k_f = 3$.

3.2 Influences of Pe

Dimensionless wall heat flux q_x/q_0 is an important design feature in thermal design. Fig. 6(a) shows q_x/q_0 of different Pe as a function of dimensionless longitudinal distance x/L_0 (L_0 is the length of heat region). Thanks to entrance effect of flow, thin thickness of the boundary layer near channel inlet makes the convective heat transfer resistance smaller, so the coefficient of convective heat transfer enhanced. Attributed to the strong convective heat transfer in developing region of the thermal boundary layer near channel inlet, a high ratio of q_x/q_0 is found near channel inlet. With fast growth of the thermal boundary layer, the convective heat transfer resistance increases, so the convection between the fluid and the wall becomes weaker gradually, and that leads to the decrease of q_x/q_0 . As Pe increases, the entrance effect on q_x/q_0 decreases, and the length of thermal entrance becomes shorter. The wall conduction makes heat to transfer axially upstream to pre-heat the fluid which has lower temperature than wall. And the effect of axial conduction in fluid becomes more pronounced as Pe decreases (Cole and Cetin (2011)). For low Pe flow ($Pe = 0.1$), the proportion of the convection in heat exchange is small, so the axial conduction along the channel is the main way of heat exchange. At

$Pe = 0.1$, the peak of q_x/q_0 appears near channel entrance, it can be observed that the axial conduction in both the fluid and the wall cannot be neglected for small Pe .

Distributions of q_x/q_0 for different Pe which presented in Fig. 6(a) show that q_x/q_0 for high Pe is larger than that for low Pe in heat region. The increases of q_x/q_0 in heat region also imply the enhancement of convection as Pe rises. As can be seen from Fig. 6(a) the heat exchange mainly happens in heat region by convection for high Pe . Data in Fig. 6(b) suggests that the normalized mean temperature difference between the mean temperature of fluid and fluid temperature of inlet $(T-T_{inlet})^*k / (q_0^*L)$ for each Pe tends to rises monotonically along flow direction before end of heat region. The reason is that all the outer boundaries of the channel in addition to the heated region are insulated, so the heat added to fluid remains in fluid. For same fluid properties and channels with fixed geometry, mean velocity of the fluid is proportional to Pe . With increase of Pe , the convection between the wall and the fluid becomes more pronounced, and the pre-warmed effect of heat region for upstream becomes weaker, so $(T-T_{inlet})^*k / (q_0^*L)$ between the wall and the fluid becomes obvious. In Fig. 6(b), at $Pe = 0.1$, $(T-T_{inlet})^*k / (q_0^*L)$ between the wall and the fluid is too small to be visible. Furthermore, $(T-T_{inlet})^*k / (q_0^*L)$ is fairly close at entrance of heated region ($x/L_0 = 1$).

For seek the detailed heat transfer behaviors in microflow, Nu_x along flow direction is plotted in Fig. 6(c). When Pe is small, axial conduction dominates the heat transfer, so a long upstream region is pre-warmed. The evidences are high dimensionless wall heat flux q_x/q_0 near channel entrance as detailed in Fig. 6(a) and small dimensionless temperature difference between the wall and the fluid which displayed in Fig. 6(b) for low Pe flow. At $Pe = 100$, Nu_x is large at entrance of heat region ($x/L_0 = 1$) and falls monotonously through whole heat region. The distribution of Nu_x at $Pe = 10$ shows reasonable behavior as expected. The shape of Nu_x when $Pe = 10$ relatively flat than $Pe = 100$ over the heat region ($1 < x/L_0 < 2$). Differently, at low Pe conditions (such as $Pe = 0.1, 1$), the heat transfer between the wall and the fluid is dominated by the axial conduction, the Nu_x increases monotonically over entire heat region. And it is notable that a local peak of Nu_x appears in downstream region (about $x/L_0 = 2.25$) when $Pe = 0.1$ and 1. The local peak of Nu_x is leaded mainly by two reasons. Under the continuous heating effect of the wall, closely following the end of the heat region (about $x/L_0 = 2.25$), the temperature difference between the wall and the fluid is small. And at this location, the ratio of q_x/q_0 is relative high, according to equation (11), a local peak of Nu_x appears in downstream of the heat region (about $x/L_0 = 2.25$).

Fig. 6 Effects of Pe on (a) q_x/q_0 , (b) $(T-T_{inlet})^*k_f / (q_0^*L_0)$ and (c) Nu_x for $W/D = 1$, $k_w/k_f = 3$.

3.3 Influences of the Wall Thermal Conductivity (k_w/k_f)

The effects of axial heat conduction are more relevant for short ducts, thick walls and low Pe . Axial distributions of Nu_x are plotted for different k_w/k_f when $W/D = 1$, $Pe = 100$ (Fig. 7(a)). As can be seen from Fig. 7(a), q_x/q_0 along flow direction becomes more uniform as k_w/k_f rises. With increase of k_w/k_f , thermal resistance of conduction in wall decreases, so the pre-warmed effects of the heat region in wall is enhanced along with the increase of k_w/k_f . For this reason, a more uniform streamwise temperature distribution at fluid-solid interface can be found for high k_w/k_f . There is reason to believe that the temperature distribution of fluid-solid interface is more uniform in materials with high conductivity than that in ones which have low conductivity.

Moreover, the conductive resistance decreases as k_w/k_f increases, so q_x/q_0 near channel entrance becomes larger. At $Pe = 1000$, convection is the dominant mechanism for heat transfer. Judging from the trends of q_x/q_0 , it can be obtained that the larger the heat conductivity of the wall is, the wider the area affected by the heat source is. For all cases, the k_w/k_f of this section is larger than 1, owing to the lower temperature in fluid than in wall, the fluid is heated by the wall.

Therefore, with increase of k_w/k_f the heat transfer by conduction of the wall increases, so the distribution of the heat source becomes more uniform, and this contributes to the smaller temperature difference between wall and fluid, that is the local effect of the heat source is less obvious. On the contrary, as k_w/k_f decreases, the local influence of the heat source becomes more obvious, this trend leads to the temperature difference between wall and fluid increases monotonously. From Fig. 7(b), it can be seen that the temperature difference for $k_w/k_f = 3$ is close to the value which no considering the conduction of the wall.

Affected by the thin boundary layer near channel entrance, the coefficient of convective heat transfer enhanced, a peak of Nu_x can be found near channel entrance for different k_w/k_f . Owing to the fast growing of thermal boundary layer, the thermal resistance of convective heat transfer increases, Nu_x decreases quickly closely following the channel inlet because of the weakened convection between fluid and wall. It is obvious in Fig. 7(c), a local peak of Nu_x can be found near the entrance of heat region ($x/L_0 = 1$) for materials with relative low conductivity ($k_w/k_f = 3$). The reason is that the enhancement of conduction between wall and fluid near the entrance of heat region. For same Pe , with increase of k_w/k_f , the influence region of heat source is enlarged by proportion increase of the wall conduction. From Fig. 7(c), one can conclude that the trend of Nu_x is similar to the trend which heat source added to the whole channel for materials with $k_w/k_f = 300$. Much more, the influence for convection caused by heat source is more pronounced when the wall materials is less conductive relative to the fluid. Namely the Nu_x of low conductivity material is higher than that for materials with high conductivity. And the conclusion is consistent with results of Nonino et al. (2009).

Fig. 7 Effects of k_w/k_f on (a) q_x/q_0 , (b) $(T_w - T_f)k/(q_0 * L_0)$ and (c) Nu_x for $W/D = 1$, $Pe = 100$.

Figure 8 clearly demonstrates the profiles of q_x/q_0 , $(T_w - T_f)k/(q_0 * L_0)$ and Nu_x for several conductivities when $W/D = 1$, $Pe = 0.1$. With decreases of k_w/k_f , the heat of wall conduction during heat transfer process decreases, increase of the heat which transfers from heat source to fluid by convection leads to the increase of q_x/q_0 . The proportion of the wall conduction is proportional to k_w/k_f , so with increase of k_w/k_f , the pre-heating effect for fluid by the wall conduction enhances, and this leads to the decreases of $(T_w - T_f)k/(q_0 * L_0)$.

For different k_w/k_f , the trends of Nu_x are similar. Both in heat region and upstream, the differences of Nu_x for different k_w/k_f are small. The reason is that the small proportion of the wall conduction for small k_w/k_f makes more heat transfers to fluid by convection, by this reason the peak of Nu_x in downstream region for low k_w/k_f is bigger than that for high k_w/k_f .

Fig. 8 Effects of k_w/k_f on (a) q_x/q_0 , (b) $(T_w - T_f)k/(q_0 * L_0)$ and (c) Nu_x for $W/D = 1$, $Pe = 0.1$.

3.4 Influences of the Wall Thickness (W/D)

The distribution of q_x/q_0 , $(T_w - T_f) * k / (q_0 * L)$ and Nu_x for different W/D when $k_w / k_f = 75$, $Pe = 1000$ are plotted in Fig. 9. At $k_w / k_f = 75$, $Pe = 1000$, which is convection dominated, the q_x/q_0 determined by the heat which the wall stored. The increase of W/D enhances the heat transfer rate of the wall, and the enhancement in heat transfer rate of the wall caused by increase of W/D enhances the convection between wall and fluid. And this makes the heat transfer by convection more uniform axially, not only the increase of q_x/q_0 in upstream and downstream, but also the decrease of q_x/q_0 in heat region are the corresponding performances of this trend (see Fig. 9(a)). The local heating effect is weakened by enhancement of the convection owing to the increase of W/D . Therefore, with increase of W/D the pre-heating effect of the wall for fluid increases, so $(T_w - T_f) * k / (q_0 * L)$ decreases. Compared with smaller W/D case, the $(T_w - T_f) * k / (q_0 * L)$ curve for larger W/D becomes more gentle (Fig. 9(b)).

Fig. 9 Effects of W/D on (a) q_x/q_0 , (b) $(T_w - T_f) * k / (q_0 * L_0)$ and (c) Nu_x for $k_w / k_f = 75$, $Pe = 100$.

Fig. 10 Effects of W/D on (a) q_x/q_0 , (b) $(T_w - T_f) * k / (q_0 * L_0)$ and (c) Nu_x for $k_w / k_f = 3$, $Pe = 0.1$.

At $k_w / k_f = 75$, affected by the wall conduction, a peak of Nu_x appears near channel entrance (Fig. 9(c)). The small difference of Nu_x near channel entrance ($x/L_0 = 0$) indicates that W/D has little influence on convection near channel entrance for high Pe . Comparison of the Nu_x in Fig. 7 and Fig. 9 indicates that the increase of W/D has an effect on Nu_x equivalents to increasing the conductivity of the wall. Thermal resistance of the conduction is inversely proportional to the wall thickness, so the thermal resistance of conduction decreases with increase of W/D , this cause more heat to move axially, and that leads the decrease of Nu_x , and this trend is in conform with the conclusions of Kosar (2010)..

Different from the results shown in Fig. 9, Fig. 10 demonstrates the curves for $Pe = 0.1$ when $k_w / k_f = 3$. Compared the curves in Fig. 8 with those in Fig. 10, one can conclude that the effects which caused by the increase of W/D is similar to the effects owing to the increase of k_w/k_f .

3.5. The Analysis of the Mean Nusselt Number on Heat Region

Peclet number describes the ratio of convection and conduction in the fluid. For $Pe = 0.1$, the convection in heat transfer is relatively weak. With the increase of k_w/k_f , the proportion of the heat transferred by conduction of the wall

increases, and the heating effect of the wall for fluid is strengthened, which leads to the increase of the $\overline{Nu}$. From Fig. 11(a), at $Pe = 0.1$, we can conclude that $\overline{Nu}$ is proportional to the wall thermal conductivity for $k_w/k_f \leq 30$, while $\overline{Nu}$ approximately keep constant when $k_w/k_f \geq 30$.

In order to study the effect of the wall thermal conductivity on $\overline{Nu}$ at high Pe conditions, the distribution of the $\overline{Nu}$ along with k_w/k_f when $Pe = 100$ is also plotted in Fig. 11(a). For the higher convective heat transfer capability at high Pe condition ($Pe = 100$), the $\overline{Nu}$ of the heat region shows a quicker increase than the $\overline{Nu}$ at low Pe condition ($Pe = 0.1$).

To analyze the effect of Pe on the distribution of $\overline{Nu}$ on heat region, Fig. 11(b) gave the distribution of $\overline{Nu}$ at different Pe conditions. As shown in Fig. 11(b), $\overline{Nu}$ on heat region rises with the increase of Pe . A slight increase of the $\overline{Nu}$ on heat region is observed as the increase of Pe when $Pe \leq 1$ since the heat is mainly transferred with diffusion. When $Pe > 10$, the convection dominates the heat transfer between the heated wall and fluid. Therefore, an obvious increase of $\overline{Nu}$ is shown in Fig. 11(b) with the increase of Pe . For high Pe flow, the effect of the wall conduction is not significant, and the heat transfer enhancement becomes weak along with the increase of Pe . As shown in Fig. 11(b), when $Pe > 50$, the increase trend of Pe is not obvious, and Pe almost keeps constant.

Fig. 11 Effects of k_w/k_f and Pe on $\overline{Nu}$ of heat region ($1 < x/L_0 < 2$).

4. Conclusions

Three-dimensional fluid flow and conjugate heat transfer in heat sinks with rectangle microchannels have been analyzed numerically. The continuum model, which includes the conventional Navier-Stokes equations and energy equation are used. The validation of this approach has been demonstrated by comparisons with analytical solutions. Results were obtained for the detailed description of the local and average heat transfer characteristics including wall and fluid temperatures, heat flux, Nusselt number. Effects of the Peclet number (Pe), the relative conductivity (k_w/k_f) and the relative thickness (W/D) of solid substrate were presented and discussed. The following conclusions can be drawn from the present results:

1) At low Pe flow, axial conduction dominates the heat transfer, which results in a large upstream region which is pre-warmed by axial conduction, so axial conduction cannot be neglected at small Pe . The increase of Pe induces an enhancement of convection, the heat exchange mainly happens on heat region by convection under this condition. The increases of q_x/q_0 on heat region also imply the enhancement of convection when Pe rises. Although the temperatures of wall and fluid fall, the temperature difference between wall and fluid becomes more pronounced as Pe rises.

2) A more uniform streamwise temperature distribution at fluid-solid interface indicates the enhancement of the pre-warmed effect along with the increase of k_w/k_f . Affected by the entrance region, a peak of Nu_x has been found near channel inlet for different Pe . Also, a local peak of Nu_x appears near entrance of heat region for low conductivity material. The study also reveals that the Nu_x of low k_w/k_f material is higher than the Nu_x of high k_w/k_f material.

3) The increase of wall thickness has an effect on axial distribution of Nu_x , the trend is similar to that of an increase of k_w/k_f .

4) Compared with the average Nusselt number of difference Pe and different W/D , it was concluded that at condition of $Pe = 0.1$, $\overline{Nu}$ is proportional to the wall thermal conductivity for $k_w/k_f \leq 30$, while $\overline{Nu}$ approximately keep constant when $k_w/k_f \geq 30$. At high Pe condition ($Pe = 100$), the convection between the wall and fluid is the dominate

factor in heat transfer, so $\overline{Nu}$ of the heat region shows a more quickly increase than the $\overline{Nu}$ at low Pe condition ($Pe = 0.1$). $\overline{Nu}$ on heat region rises with the increase of Pe . The fluid is heated by the wall by means of convection on heat region, when $Pe \leq 1$, a slight increase of the $\overline{Nu}$ on heat region is shown as the increase of Pe . When $Pe > 10$, the convection is the dominant factor in heat transfer, an obvious increase of $\overline{Nu}$ is shown with increase of Pe . For high Pe flow, the heat transfer enhancement as increase of Pe is weak.

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