Numerical investigation on a travelling wave thermoacoustic heat pump

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Abstract
A thermoacoustic heat pump heater is a device having the ability to produce heat from acoustic power. The heat pump investigated was composed of an acoustic driver, a branched tube, and a looped tube. A porous component composed of many stacked steel screen meshes, called regenerator, was installed inside the looped tube. When an acoustic wave is supplied to the looped tube, filled with nitrogen gas at 0.5 MPa, heat pumping spontaneously occurs inside the regenerator. A numerical code was developed and tested. Then, the coefficient of performance was calculated for different temperatures by optimizing the regenerator flow channel radius and position inside the looped tube. It was found that the temperature affects considerably the regenerator optimum parameters.

Keywords: Thermoacoustic, Heat pumping, Performance, Numerical optimization

1. Introduction

When an acoustic wave propagates into a narrow tube, the pressure and density of the working medium molecules change and a temperature gradient is then created. Oppositely, when a critical temperature gradient is imposed to a tube wall, gas molecules inside the tube begin to oscillate spontaneously and an acoustic wave is then generated. These two opposite phenomena are the basics of thermoacoustic heat pumps and thermoacoustic engines, respectively. They are considered as the two most known applications of thermoacoustics. Thermoacoustic devices have simple structure with no moving parts and use harmless working gases which make them not only efficient but also low cost and environmentally friendly devices (Zink et al., 2010). In 1999, based on Yazaki et al. (1998) works, Backhaus and Swift constructed a thermoacoustic Stirling heat engine having efficiency reaching 41% of Carnot efficiency which is comparable to conventional engine efficiency (Backhaus and Swift, 1999). Recently, Tijani and Spoelstra (2011) designed and built a thermoacoustic engine which reached a record performance of 49% of Carnot efficiency. Many thermoacoustic heat pumps working as a cooler were also constructed and investigated (Yazaki et al., 2002; Dai et al., 2006; Bassem et al., 2011a; Sharify et al., 2017). In 2009, an efficient travelling wave thermoacoustic refrigerator which theoretically reached 60% of Carnot coefficient of performance was proposed by Ueda et al. (2010). However, few researches were interested on thermoacoustic heat pumps working as a heater. A first attempt was done by Tijani and Spoelstra (2012) when they constructed a thermoacoustic device working in the temperature range of 10 °C to 80 °C. In 2011, a thermoacoustic heater reaching a temperature above 350 °C was constructed (Bassem et al., 2011b). Based on the experimental analysis, it was shown that the thermal power was generated through a thermodynamic cycle similar to the inherently reversible Stirling cycle (Ceperley, 1979) allowing an efficient energy conversion from acoustic power into thermal power. Later in 2015, Kikuchi et al. (2015) constructed a thermoacoustic heat pump and measured its performance. Both studies (Bassem et al., 2011b; Kikuchi et al., 2015) investigated the thermoacoustic device only experimentally. However, no numerical simulation has been performed yet. In this work, numerical simulations were done to estimate the performance of the travelling wave thermoacoustic heater.
2. Calculation

2.1 Calculation model

The travelling wave thermoacoustic heater is schematically illustrated in Fig. 1. This heat pump is composed of an acoustic driver (linear motor), a branched tube, and a looped tube containing a regenerator sandwiched between two heat exchangers. The looped and branched tubes are connected by a T-shape tube. The regenerator was positioned at \( \frac{L_H}{L_{loop}} = 0.75 \), where \( L_H \) is the distance from the center of the T-shape tube to the hot end of the regenerator [see Fig. 1] and \( L_{loop} \) is the looped tube length. In order to validate our calculation with experimental results, we chose to use the same parameters as the experiments done by Kikuchi et al. (2015).

![Fig. 1 Schematic illustration of the thermoacoustic heat pump.](image)

The lengths of the branched and looped tubes were set to 2.2 m and 1.5 m, respectively. The inner diameters of the branched and looped tubes were 56 mm and 41 mm, respectively. The regenerator was modelled as an array of circular narrow flow channels having the characteristic radius of 33 µm and its length was equal to 50 mm. The characteristic radius was calculated by using the empirical equation defined by Ueda et al. (2009) as: \( \sqrt{D_h \times d} / 2 \), where \( D_h \) is the hydraulic diameter inside the regenerator and \( d \) is a diameter of wires comprising the regenerator. Two heat exchangers were installed at both sides of the regenerator. The heat exchangers installed at the hot and ambient sides of the regenerator are called hereafter hot and ambient heat exchangers, respectively. Their lengths were set to 8 mm and 5 mm, respectively. The two heat exchangers were modelled as 1-mm-thick parallel plates with 1 mm spacing, so that their porosity was equal to 0.5. The thermal buffer tube is a part of the looped tube from the hot end of the regenerator which insures a thermal buffer between the hot heat exchanger temperature and the ambient temperature. Its length was 0.2 m. The thermal buffer tube and the regenerator are assumed to be thermally insulated. The temperatures of the cold heat exchanger and the looped tube were fixed at 25 °C and the gas properties of nitrogen at 0.5 MPa were used in the calculation.

2.2 Calculation method

In the numerical calculation, we used the transfer matrix method based on the following first order differential equation:
Here, $P$ and $U$ are the oscillatory gas pressure and velocity, respectively. $A$ is the cross-sectional area of the tube, $i$ is the imaginary number, $\omega$ is the angular frequency of the acoustic wave, $\gamma$ is the specific heats ratio and $Pr$ is the Prandtl number. $\rho_m$, $P_m$ and $T_m$ are the mean density, mean pressure and mean temperature, respectively. $X_v$ and $X_a$ are thermoacoustic functions depending on the geometry of the flow channel (Tominaga, 1995; Ueda, 2008).

This Eq. (1) is the matrix form obtained from the following two equations:

\[
\begin{align*}
\frac{dP}{dx} &= -\frac{1}{A} \frac{\text{i} \omega \rho_m}{1 - \chi_v} U \\
\frac{dU}{dx} &= -\frac{\text{i} \omega \rho_m}{\gamma P_m} U + \frac{X_a - X_v}{(1 - \chi_v)(1 - \text{Pr}T_m)} dT_m U
\end{align*}
\]

Equations (2) and (3) are the momentum and the continuity equations established by Swift (2002) based on Rott’s acoustic approximation (Rott, 1969). Rott proposed the basic equations written with the assumption that the tube radius is much smaller than the length, which implies three simplifications: (1) The radial gradient of the acoustic pressure is neglected throughout the tube, (2) The radial variations of the average temperature and the viscosity are neglected, and (3) The axial heat conduction in the acoustic wave and the friction due to axial temperature gradient are ignored (Rott, 1969). By abandoning the boundary layer approximation adopted previously and proposing a new assumption, Rott succeeded to found theory that is compatible with experiments. Hence, Rott was the first who derived correct expressions for motion and pressure which present the basis in the development of thermoacoustics. Equations (2) and (3) include three important parameters; pressure, velocity, and temperature. In fact, Rott’s acoustic approximation (Rott, 1969) applied to the momentum equation is the origin of pressure gradient in thermoacoustics. In addition, the two terms in Eq. (3) show that a gradient in $U$ can be caused either by pressure or by velocity along the temperature gradient. Hence, Eq. (2) and Eq. (3) are considered two of the principal tools of thermoacoustic analysis, especially in numerical simulations.

By solving Eq. (1), the transfer matrix of the looped tube from $x=0$ to $x=L_{\text{loop}}$ is calculated. Here, $x$ is the axial coordinate along the looped tube whose its origin is set at the center of the T-shape tube [see Fig. 1]. In this step, the transfer matrix of the thermal buffer tube and that of the regenerator are calculated by assuming that the mean temperature variation inside them is linear.

In addition to the hot and cold heat exchangers temperatures, the driving frequency and the pressure at $x=0$ are given as input data. Furthermore, there are two conditions that must be considered (Ueda, 2008); the pressure at $x=0$ is equal to the pressure at $x=L_{\text{loop}}$ and the temperature at $x=0$ is equal to the temperature at $x=L_{\text{loop}}$. Therefore, by using the transfer matrix of the looped tube previously calculated and the given input pressure, the velocity and the pressure amplitudes at any position $x$ inside the looped tube can be obtained. Here, we point out that the advantage of this calculation method is that only pressure at $x=0$ and the geometry of the looped tube are needed to determine the heat pump performance. Other calculation methods, such as DeltaE and THERMOACOUSTICA, need both pressure and velocity to calculate the performance.

According to Rott (1975), the enthalpy flow $H$ can be written as:

\[
H = \frac{1}{2} \text{Re} \left[ P U \left( 1 - \frac{X_a - \bar{X}_v}{(1 + \text{Pr})(1 - \chi_v)} \right) + \frac{\rho_m C_p |U|^2}{2 \text{Re} \left( 1 - \chi_v \right)} \text{Im} \left( X_a - \text{Pr} \bar{X}_v \right) \frac{dT_m}{dx} \right] - \left( A_{\text{solid}} K_{\text{solid}} + A_{\text{solida}} K_{\text{solida}} \right) \frac{dT_m}{dx}
\]
Where the tilde indicates a conjugate, \( \Re \) is the real part, \( \Im \) is the imaginary part, and \( C_p \) is the constant pressure specific heat of the working gas. \( A_{\text{solid}1} \) and \( \kappa_{\text{solid}1} \) are the cross-sectional area and the thermal conductivity of regenerator holder, whereas \( A_{\text{solid}2} \) and \( \kappa_{\text{solid}2} \) are the cross-sectional area and the thermal conductivity of the solid metal which the regenerator is composed of.

Here, the linear theory, which is the only analytical tool taking into account all the thermoacoustic phenomena, is used. In fact, right hand side of Eq. (4) is composed of three quantities; the acoustic power, the heat flow, and the heat conduction. However, this linear theory assumes that the device geometry and the energy flow are one-dimensional. In addition, the effect of the acoustic streaming (Biwa et al., 2007) on \( H \) is neglected. These two points constitute the main limitation of this modelling.

As mentioned before, the regenerator and the thermal buffer tube are assumed to be thermally insulated. Hence, the enthalpy flow along them must be constant. By using Eq. (4), the velocity previously calculated and the solution of Eq. (1), the temperature distribution along the thermal buffer tube and the regenerator are recalculated. Iterative calculations are performed until the values of enthalpy flow of the regenerator and the thermal buffer tube become almost constant.

The transfer matrix of the thermal buffer tube and that of the regenerator are redefined. Therefore, the transfer matrix of the looped tube is recalculated and the acoustic field inside the looped tube can be determined. Then, we evaluate the \( \text{COP} \).

The input acoustic power is expressed as: \( W_{in} = W(x = 0) - W(x = L_{\text{loop}}) \), where \( W \) is the acoustic power and it is defined as: \( W = \frac{A}{2} \Re \{p \bar{U} \} \).

The heating power \( Q_H \) is defined as the difference between the heat flow transferred from the regenerator to the hot heat exchanger and the heat flow entering the thermal buffer tube from the hot heat exchanger. It can be written as: \( Q_H = Q(x = L_H - L_{\text{HHX}}) - Q(x = L_H) \). Here, \( L_{\text{HHX}} \) is the hot heat exchanger length and \( Q \) is the heat flow produced by the acoustic wave which is defined as: \( Q = H - W \).

Finally, the \( \text{COP} \) of the travelling wave thermoacoustic heat pump working as a heater is the ratio of the heating power to the input acoustic power: \( \text{COP} = \frac{Q_H}{W_{in}} \).

3. Numerical results and discussion
3.1 Heating performance

In order to validate our numerical code, the heating power without considering the heat losses, \( Q_{H \text{ without loss}} \), is calculated and the obtained results are compared to those obtained experimentally by Kikuchi et al. (2015). Both heating powers are plotted in Fig. 2 as a function of the hot heat exchanger temperature \( T_H \).

We note that in order to estimate \( Q_{H \text{ without loss}} \), Kikuchi et al. (2015) proposed the following method: Before turning on the acoustic driver, they measured the heat losses caused by thermal conductivity and by radiation. For this, heated oil was circulated in a flow channel around the hot heat exchanger. Then, the oil temperatures at the inlet and outlet points of the flow channel around the hot heat exchanger were measured using calibrated thermocouples. Because of thermal losses, the temperature at the outlet point is found to be lower than that at the inlet one. The heat loss from the hot heat exchanger to the surrounding air is determined as: \( Q_{\text{loss}} = mC\Delta T \). Where, \( m \) is the mass flow rate of the circulating oil, \( C \) is the specific heat capacity of the oil, and \( \Delta T \) is the difference between the oil temperatures at the inlet and outlet points. We note also that the oil temperature at the outlet point of the flow channel around the hot heat exchanger is the hot heat exchanger temperature.

Besides, Kikuchi et al. (2015) measured the oil temperatures at the inlet and outlet points of the flow channel around the hot heat exchanger when 40 W of acoustic power is delivered to the looped tube. The heating power \( Q_H \) is determined as \( Q_{\text{loss}} \). However, due to the heat production, the temperature at the outlet point is found to be higher than that at the inlet one.

The heating power without considering the losses is then expressed as: \( Q_H + Q_{\text{loss}} \).
In Fig. 2, the experimental results obtained by Kikuchi et al. (2015) are represented by open triangles with error bars, whereas the simple full triangles show the calculation results obtained in this work. The experiment and the calculation were performed by maintaining the acoustic power $W_{in}$ at 40 W. As seen in Fig. 2, the calculated and the experimental heating powers are in qualitative agreement. They both decrease with increasing the temperature of the hot heat exchanger. However, a quantitative discrepancy is observed especially at high temperatures of the hot heat exchanger. We consider that the main reason of this quantitative discrepancy is the thermal nonlinear effects caused by acoustic streaming (Biwa et al., 2007), such as Gedeon streaming (Gedeon, 1997; Backhaus and Swift, 1999; Tijani and Spoelstra, 2011; Tang et al., 2017). These effects are estimated in experiment and not considered in the numerical calculation. Furthermore, the heat exchangers are assumed to have constant temperatures during calculation while in reality, there is a difference between the internal and external temperatures of the heat exchangers. Moreover, in the calculation the regenerator is estimated to be perfectly insulated which is not possible in experiments. On the other hand, in the experiment (Kikuchi et al., 2015; Bassem et al., 2011b), an elastic membrane was inserted in the looped tube to minimize the acoustic streaming. This makes the quantitative discrepancy between experiment and calculation almost constant in the temperature range $50 \degree C – 160 \degree C$ as shown in Fig. 2. However, the relative large discrepancy observed at $T_H =160 \degree C$ is attributed to the fact that the loss caused by the thermal non linear effect increases with increasing temperature. In addition, temperatures are measured by self calibrated thermocouples which explain the low precision observed in Fig. 2 by the large error bars.

Kikuchi et al. (2015) measured experimentally the coefficient of performance of the constructed heat pump and obtained a low $COP$ of 0.21 when a temperature of 160 °C is generated. Since the energy conversion efficiency is considerably affected by the characteristics of the regenerator, we consider that the reason of the low $COP$ obtained by Kikuchi et al. (2015) could be the choice of the regenerator characteristics. In order to elucidate this supposition, the acoustic power distribution along the looped tube is calculated and illustrated in Fig. 3. It is shown that $W_{reg}$, which is the acoustic power converted into heat inside the regenerator, is much bigger than the acoustic power dissipated outside the regenerator. This means that the low value of $COP$ is due to the low energy conversion efficiency inside the regenerator.

It is known that the main parameter affecting the energy conversion efficiency is the regenerator radius and position (Ueda et al., 2010). For this reason, the regenerator characteristics are optimized numerically and the results are shown in section 2. We remind that the regenerator used by Kikuchi et al. (2015) in their experiments has a radius equal to 0.033 mm.

![Fig. 2 Heating power, without heat losses, as a function of the hot heat exchanger temperature.](image-url)
4. Regenerator optimization

We numerically optimized the parameters characterizing the regenerator, which plays an important role in the operation of the thermoacoustic heat pump. Ueda et al. (2010) showed that the flow channel radius and the position of the regenerator inside the looped tube are the most two important parameters that should be optimized. They showed also that the regenerator length has no big effect on the performance of the thermoacoustic devices. The regenerator length \( L_{\text{reg}} \) is hence chosen to be 50 mm.

In Fig. 4, the calculated \( \text{COP} \) is shown as a function of the regenerator flow channel radius normalized by the thermal penetration depth (Bassem et al., 2011b) \( \frac{r_{\text{reg}}}{\delta_{\kappa}} \) and the regenerator position normalized by the looped tube length \( \frac{L_{H}}{L_{\text{loop}}} \). The simulation was performed for three hot heat exchanger temperatures. The pressure amplitude at the junction between the branched and looped tubes was kept at 20 kPa which is about 4% of the mean pressure of the working gas, a value used by Bassem et al. (2011b) in their experiments. As seen in Fig. 4, the \( \text{COP} \) depends on the values of the regenerator flow channel radius and position. Further, the optimum values of the regenerator parameters depend strongly on the hot heat exchanger temperature. Figure 4 (a) shows that at 200 °C, the highest \( \text{COP} \) is obtained when a regenerator having a normalized flow channel radius of 0.28 (\( r_{\text{reg}} = 0.07 \) mm) is placed at \( \frac{L_{H}}{L_{\text{loop}}} = 0.72 \). At these two optimum values \( \text{COP} \) reaches 1.13 which is about 41% of Carnot \( \text{COP} \). The optimum regenerator radius determined by calculation (\( r_{\text{reg}} = 0.07 \) mm) is much different to that used in experiments by Kikuchi et al. (2015) (\( r_{\text{reg}} = 0.033 \) mm). This confirms the reason of the low \( \text{COP} \) found by Kikuchi et al. (2015).

Fig. 4 shows that when hot heat exchanger temperature is increased from 200 °C to 400 °C, the regenerator optimum parameters; i.e. the channel radius and the position, are changed. The values of \( \frac{r_{\text{reg}}}{\delta_{\kappa}} \) decreases from 0.28 at \( T_{H} = 200 \) °C to 0.2 at \( T_{H} = 300 \) °C, and reaches 0.15 when \( T_{H} = 400 \) °C. The normalized position \( \frac{L_{H}}{L_{\text{loop}}} \) increases from 0.72 when \( T_{H} = 200 \) °C to 0.82 when \( T_{H} = 300 \) °C, and reaches 0.95 when \( T_{H} = 400 \) °C. The value of \( \text{COP} \) decreases from 1.13 at \( T_{H} = 200 \) °C to 0.9 at \( T_{H} = 300 \) °C, and reaches 0.8 at \( T_{H} = 400 \) °C.
Fig. 4  \( \text{COP} \) as a function of the regenerator normalized flow channel radius and normalized position. The mean temperature at the hot end of the regenerator is: (a) 200 °C, (b) 300 °C, (c) 400 °C.
In order to elucidate the effect of the generated temperature on the thermoacoustic performance, we calculate the COP as a function of hot heat exchanger temperature for the three optimized regenerators. The first regenerator (rege1) has a normalized flow channel radius \( r_{\text{rege}}/\delta_k \) equal to 0.28 and positioned at \( L_H/L_{\text{loop}} = 0.72 \). The second (rege2) has a normalized flow channel radius \( r_{\text{rege}}/\delta_k \) equal to 0.2 and positioned at \( L_H/L_{\text{loop}} = 0.82 \). The third one (rege3) has a normalized flow channel radius \( r_{\text{rege}}/\delta_k \) equal to 0.15 and positioned at \( L_H/L_{\text{loop}} = 0.95 \). The radius and position of the first, second, and third regenerators are the optimized parameters at 200 °C, 300 °C and 400 °C, respectively.

![COP as a function of hot heat exchanger temperature for different optimized regenerators.](image)

The COP values for rege1, rege2, and rege3 are illustrated in Fig. 5. At \( T_H = 200 \) °C, a COP of 1.13 is obtained using rege1. When the temperature increases from 200 °C to 400 °C, the COP obtained, using the same regenerator, is about 0.65. At this same temperature, a higher value of COP (0.8) could be obtained if rege3 is used instead of rege1. On the other hand, at \( T_H = 300 \) °C, the rege2 allows to obtain a COP higher than that obtained by rege1 and rege3. Therefore, in order to obtain higher COP, we should optimize the regenerator characteristics for each increasing temperature. This result is not considered in all our previous studies (Ueda et al., 2010; Bassem et al., 2011a, 2011b; Kikuchi et al., 2015) and could be important in the optimization of the thermoacoustic devices.

Furthermore, we notice that the regenerator optimum flow channel radius decreases with increasing temperature while the regenerator optimum normalized position increases with increasing temperature. In fact, the heating performance decreases with increasing temperature. However, decreasing the regenerator flow channel radius contributes to increase the pressure amplitude and also to decrease the velocity amplitude inside the regenerator. Consequently, the heat exchange between gas parcels and the regenerator flow channels walls become more efficient which could improve the heat pumping and hence the thermoacoustic performance. However, when the flow channel radius becomes too small, viscous losses inside the regenerator increase and consequently the performance of the heater becomes very low. Moreover, increasing the regenerator position contributes not only to increase the heating power but also to increase the input power, which affects the performance of the thermoacoustic heater. Hence, the optimum position corresponds to the optimum COP ratio obtained, which is equal to the heating power divided by the input acoustic power.

5. Conclusions

The numerical performance of the thermoacoustic heat pump was compared to the experimental results obtained previously by Kikuchi et al. (2015) and good agreement was found. Besides, the coefficient of performance was numerically calculated by optimizing the regenerator flow channel radius and position inside the looped tube.

Moreover, it was found that the regenerator optimum parameters change significantly with changing temperature. More the temperature increases, more the regenerator optimum flow channel radius decreases and the optimum
regenerator normalized position inside the looped tube increases. For this reason, it is recommended that the regenerator parameters should be optimized at each given temperature.

This thermoacoustic heat pump could be driven by a thermoacoustic engine driven by waste heat or solar energy. In fact, according to Tijani et al. (2013), 40 W of acoustic power can be generated by a thermoacoustic Stirling engine using hot air at about 300 °C. The thermoacoustically driven heat pump could hence generate thermal power over 400 °C using a temperature gradient of about 270 °C.

References

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