Design and Construction of a Standing-Wave Thermoacoustic Engine with Heat Sources Having a Given Temperature Ratio*

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Abstract

A standing-wave thermoacoustic engine is essentially composed of a stack, two heat exchangers, and tubes; the stack has many narrow flow channels and is located inside one of the tubes. One end and the other end of the stack are thermally connected to hot and cold heat sources via the heat exchangers, respectively. When the ratio of temperatures of the heat sources exceeds a critical value, the gas inside the tubes spontaneously oscillates and the stack generates acoustical energy using heat from the hot heat source. In this study, the critical temperature ratio needed for exciting the spontaneous gas oscillation was numerically calculated by changing the stack's length, flow channel radius, and position. Further, the thermal efficiency with the critical temperature ratio was calculated. These calculations allowed us to design and construct the engine in accordance with the heat source temperatures. The constructed engine worked with the heat sources having the temperature ratio 1.7 and achieved 8% of the Carnot efficiency. These obtained values quantitatively agreed with the design ones.

Key words : Thermoacoustic Engine, Waste Heat, Second-Law Efficiency

1. Introduction

A standing-wave thermoacoustic engine essentially consists of tubes, a stack having many narrow flow channels, and hot and cold heat exchangers.[see Fig. 1] The stack is sandwiched by the heat exchangers and is located in one of the tubes. Heat is inputted through the hot heat exchanger. A part of the heat is converted into acoustical energy in the stack due to a spontaneously generated gas oscillation, and the rest of the heat is rejected through the cold heat exchanger. This engine is an external combustion engine, and because this engine has no moving parts, it can have a long lifetime and work with low maintenance costs. These facts have motivated researchers to apply the engine to utilize waste heat from other engines or energy systems.^{(1),(2)}

In design for thermoacoustic engines utilizing waste heat, the ratio of the temperature of a waste heat source to that of an ambient heat source is one of the most important factors.⁽³⁾ This is because there is a critical value of the temperature ratio for the operation of thermoacoustic engines: When the temperature ratio is below the critical value, the engines do not work at all. Further, it is difficult to control the temperature of a waste heat source and this temperature is usually low compared with the temperature of combustion. Therefore, one must design thermoacoustic engines in accordance with the temperatures of the heat sources that are to be used.

In this study, by using thermoacoustic theory⁽⁴⁾⁻⁽⁷⁾, we have computed the critical temperature ratio needed to operate a standing-wave thermoacoustic engine by changing its design parameters. We have also calculated the efficiency of the engine. These calculations show how

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Fig. 1 The calculation model of a standing-wave thermoacoustic engine

to design a standing-wave thermoacoustic engine in accordance with the temperature ratio of heat sources. Based on the calculations, we have designed and constructed a standing-wave thermoacoustic engine working by heat sources having the temperature ratio 1.6; this ratio is determined based on the temperatures of a typical low-grade waste heat source and of an ambient heat source. The constructed engine achieved 8% of the Carnot efficiency. This achieved efficiency quantitatively agreed with the value predicted by the calculations.

2. Numerical calculation

2.1. Model

Figure 1 shows the calculation model of the standing-wave thermoacoustic engine. It is composed of a core tube, a resonator tube, a stack, two heat exchangers, and an acoustical load, and it is filled with nitrogen at 500 kPa. The diameters of the core and resonator tubes are 100 mm and 57 mm, respectively. Their lengths depend on the stack position, which is one of the design parameters, but the total of them is fixed at 2.3 m. The core tube contains the stack sandwiched by the heat exchangers; the details of the stack are mentioned below. The heat exchangers are modeled as the parallelly-located flat plates with a spacing of 1 mm, and their length is 30 mm. One end of the core tube is closed by a rigid plate and the other end is attached to one end of the resonator tube. The acoustical load is connected to the other end of the resonator tube. In the calculation, the load is expressed by the acoustic impedance that is the ratio of acoustical pressure and velocity. Its absolute value is fixed at 2.4 kPa·s/m and its argument is fixed at -84° . These values are determined based on the calculation⁽⁸⁾ of the acoustic impedance of our constructed traveling-wave thermoacoustic refrigerator similar to the refrigerator in Ref. (9). These absolute and argument values of the acoustic impedance and the total length of the tubes (2.3 m) make the working frequency of the engine about 50 Hz.

The stack is assumed to have many narrow circular flow channels. The inner radius of the channels and the stack length are denoted as r_{stack} and L_{stack} , respectively. The distance from the closed end of the core tube to the center of the stack is $L_{c.s.}$. Since a stack is generally the most important part of a thermoacoustic engine and it is characterized by r_{stack} , L_{stack} , and $L_{c.s.}$, these three are chosen as the design parameters in the present calculation.

The heat exchangers are assumed to be ideal and thus, the temperatures of the hot and cold heat exchangers are assumed to be the same as those of hot and cold heat sources; they are denoted as T_H and T_C , respectively. The temperatures of the gas and the tube wall in the space between the closed end of the core tube and the hot heat exchanger are assumed to be T_H , whereas those in the resonator tube is T_C . [see Fig. 1] In the calculation, T_C is set to 300 K.

Fig. 2 The calculated α and η_2 as functions of r_{stack} .

2.2. Procedure for evaluation of performance

It is well known that when the ratio of the temperatures at the heat exchangers rises above a critical value, the gas inside a thermoacoustic engine becomes unstable and begins to oscillate.⁽⁷⁾ This gas oscillation results in the emission of acoustic power flow from the stack; acoustic power flow is defined as the rate of the energy transmission due to gas oscillation through a tube cross-section. In order to evaluate the performance of the engine modeled in Sec. 2.1, we focus on the temperature ratio, $\alpha = T_H/T_C$, and the thermodynamic second-law efficiency, η_2 , at the stability limit condition for the spontaneous gas oscillation. The efficiency, η_2 , is the thermal efficiency relative to the Carnot efficiency and can be defined as

$$\eta_2 = \frac{W_{out}/Q_H}{1 - T_H/T_C}, \quad (1)$$

where W_{out} is the absolute value of the acoustic power flow delivered to the resonator tube and Q_H is the absolute value of the acoustical heat flow at the hot end of the stack. We should note that the amplitude of the oscillation under a stability limit condition that we can calculate is infinitesimal, while it becomes a finite value in an experiment: Hence, the present method finds the approximate value of η_2 . In the calculation, the thermal conduction loss along the stack, that from the core tube to its surroundings, and heat loss due to radiation from the core tube to its surroundings are neglected.

The procedure for calculating α and η_2 is described below.

(1) The stability limit condition of the spontaneous gas oscillation is calculated as a function of r_{stack} , L_{stack} , and $L_{c.s.}$.⁽⁷⁾ As a result, $\alpha(r_{stack}, L_{stack}, L_{c.s.})$ is obtained. Here, the expression in the form of $g(x, y, z)$ means that g is a function of x , y , and z .

(2) By using the obtained stability limit condition and linear acoustic theory^{(10),(11)}, the acoustical pressure and velocity at the hot end of the stack are calculated as functions of r_{stack} , L_{stack} , and $L_{c.s.}$.

(3) By using the calculated values of pressure and velocity and the thermoacoustic theory⁽¹²⁾, the temperature, acoustical heat flow, and acoustic power flow along the stack are calculated. As a result of this calculation, $Q_H(r_{stack}, L_{stack}, L_{c.s.})$ and $W_{out}(r_{stack}, L_{stack}, L_{c.s.})$ are obtained.

(4) By substituting the calculated Q_H , W_{out} , and $\alpha = T_H/T_C$ for Eq. (1), the efficiency $\eta_2(r_{stack}, L_{stack}, L_{c.s.})$ is evaluated.

These calculations were performed under the conditions $50 \text{ mm} \leq L_{stack} \leq 100 \text{ mm}$, $0.2 \text{ m} \leq L_{c.s.} \leq 0.6 \text{ m}$, and $0.2 \text{ mm} \leq r_{stack} \leq 0.6 \text{ mm}$.

3. Calculation results

Figure 2 shows the calculated values of α and η_2 against the flow channel radius r_{stack} under the conditions $L_{c.s.} = 0.3 \text{ m}$ and $L_{stack} = 0.1 \text{ m}$. The dotted line in Fig. 2 represents the stability curve for the spontaneous gas oscillation. The oscillation becomes neutral when the value of T_H/T_C is on this line and it becomes unstable when the value of T_H/T_C is above the

Fig. 3 The contour maps of (a) the maximum η_2 , (b) the minimum α , and (c) the optimum r_{stack}/δ_H .

line. As seen in Fig. 2, both α and η_2 depend on r_{stack} . At $r_{stack} = 0.37$ mm, α and η_2 take the minimum and maximum values, respectively. In other words, when $L_{c.s.} = 0.3$ m and $L_{stack} = 0.1$ m, the optimum value of r_{stack} for reducing the critical temperature ratio α and increasing the efficiency η_2 is 0.37 mm.

We checked the calculated α and η_2 with some other combinations of $L_{c.s.}$ and L_{stack} . It was found that when the values of $L_{c.s.}$ and L_{stack} are fixed, there always exists the optimum r_{stack} at which the minimum α and the maximum η_2 are simultaneously realized. Hence, we plot the maximum $\eta_2(L_{c.s.}, L_{stack})$ and the minimum $\alpha(L_{c.s.}, L_{stack})$ at the optimum $r_{stack}(L_{c.s.}, L_{stack})$ as contour maps in Figs. 3(a) and (b), respectively; the horizontal axis is $L_{c.s.}$, and the vertical axis is L_{stack} .

First, let us focus on the second-law efficiency, η_2 . Figure 3(a) shows that as the values of L_{stack} and $L_{c.s.}$ are increased and decreased, respectively, η_2 becomes high in the calculated regions of $L_{c.s.}$ and L_{stack} . We consider that this tendency of η_2 to depend on L_{stack} and $L_{c.s.}$ can be attributed to the reduction of the heat flow Q_D caused by the dream-pipe effect⁽⁶⁾ for the following reason. According to the theory⁽⁶⁾ established by Tominaga, Q_D becomes the loss of heat and is proportional to the square of the velocity amplitude and the temperature gradient. As the stack is moved toward the closed end where the velocity amplitude is zero, the velocity amplitude in the stack becomes smaller. Further, the temperature gradient becomes smaller as the stack length becomes long. As a result, high efficiency is obtained, when a long stack is located near the closed end, as shown in Fig. 3(a).

Next, the critical temperature ratio α is focused. As shown in Fig. 3(b), the value of α takes the minimum value in the region $0.4 \text{ m} < L_{c.s.} < 0.6 \text{ m}$ with any given value of L_{stack} and increases with increasing L_{stack} and decreasing $L_{c.s.}$ ($< 0.4 \text{ m}$). This can be explained as follows. Under the stability limit condition, the dissipation due to the gas oscillation, $\Delta W_{diss.}$, is equal to the amount of the acoustic power flow generated in the stack, $\Delta W_{gene.}$. According to Tominaga⁽⁶⁾, $\Delta W_{gene.}$ of a standing wave thermoacoustic engine is proportional of the temperature gradient and the product of the velocity and pressure amplitudes. The value of this product was found to become maximum in the region $0.4 \text{ m} < L_{c.s.} < 0.6$ for the case of the present engine due to the difference between the cross-sectional area of the core tube and that of the resonator tube. Since the value of $\Delta W_{diss.}$ hardly depends on $L_{c.s.}$, so as to realize the condition $\Delta W_{diss.} = \Delta W_{gene.}$, α increases when $L_{c.s.}$ decreases from 0.4. The tendency of α to depend on L_{stack} can be simply explained; because to reduce L_{stack} results in the increase in the temperature gradient when α is constant, a short stack can reduce the value of α .

Finally, we see Fig. 3(c) that shows the contour map of the optimum $r_{stack}(L_{c.s.}, L_{stack})$ normalized by the thermal boundary layer thickness $\delta_H = \sqrt{2\kappa/(c_p\rho_m\omega)}$; ω is the angular frequency of the gas oscillation; κ , ρ_m , and c_p are the thermal conductivity, mean density, and isobaric specific heat of the working gas at T_H , respectively. This figure shows that the optimum r_{stack}/δ_H takes a value between 1.3 and 1.6. This optimum value of r_{stack}/δ_H is almost the same as that obtained in the other system⁽¹³⁾. Moreover, as shown in Fig. 2, when the value of r_{stack} is chosen between 90% and 110% of its optimum value, the values of η_2 and α hardly change and take almost the maximum and minimum values, respectively. Therefore, to design a standing-wave thermoacoustic engine, the radius can be determined independently of the heat exchanger temperatures so that r_{stack}/δ_H is approximately 1.4. Note that δ_H depends on the hot temperature T_H .

In concluding from this chapter, we can say that in order to design an efficient standing-wave thermoacoustic engine, we need to determine the values of L_{stack} and $L_{c.s.}$ in accordance with the temperatures of heat exchangers i.e. those of heat sources. On the other hand, r_{stack}/δ_H should be always set to be approximately 1.4. Consider the case when $T_H = 480\text{K}$ (i.e. = 207°C) and $T_C = 300\text{K}$ (i.e. = 27°C); these values are temperatures of a typical low-grade waste heat source and an ambient heat source. This corresponds to $T_H/T_C = 1.6$. The dotted line of $\alpha = 1.6$ shown in Fig. 3 (b) is superimposed on Fig. 3(a). This line shows that under the condition of $\alpha = 1.6$, the optimum values of L_{stack} and $L_{c.s.}$ to make the efficiency as high as possible are about 0.08 m and 0.32 m, respectively, while the optimum r_{stack} is 0.37 mm. With these values of L_{stack} , $L_{c.s.}$, and r_{stack} , η_2 takes the highest value of approximately 0.13.

4. Construction and test of the designed thermoacoustic engine

4.1. Constructed thermoacoustic engine

Based on the results described in Sec. 3, we designed, constructed, and tested a standing wave thermoacoustic engine. We chose the temperature ratio T_H/T_C to be 1.6; if the temperature T_C is kept at 27°C, the hot heat exchanger temperature becomes 207°C. As described previously and shown in Fig. 3, the second-law efficiency, η_2 , with $T_H/T_C = 1.6$ takes approximately the maximum value 0.13 when $L_{stack} = 0.08 \text{ m}$, $L_{c.s.} = 0.32 \text{ m}$, and $r_{stack} \sim 1.4\delta_H \sim 0.4 \text{ mm}$. However, the length of the stack we had was 0.09 m, which is slightly longer than the optimum value. Since we could not easily change the length, we used the stack with $L_{stack} = 0.09 \text{ m}$. With this value of L_{stack} and $\alpha = 1.6$, the best $L_{c.s.}$ is 0.37 m, and the maximum η_2 is approximately 0.12 as can be seen from Fig. 3(a).

The line drawing of the constructed thermoacoustic engine is shown in Fig. 4. The distance between the closed end and the center of the stack, $L_{c.s.}$, was set to 0.37 m, and the length of the stack, L_{stack} , was 90 mm. The diameter of the core tube was 100 mm. A ceramic honeycomb with square channels was used as the stack, although the stack used in

Fig. 4 The constructed experimental setup.

the calculation was assumed to have circular channels. The length of one side of the square cross section of the channels, $2r_{exp}$, was 0.79 mm. The stack was sandwiched by two heat exchangers. They were comprised of flat brass plates set in parallel with the spacing of 1 mm. The thickness of the plates was 1 mm, and their axial length was 30 mm. The hot heat exchanger was connected to electrical heaters. The total electrical power supplied to the heaters is denoted as $Q_{H,ex}$, while the temperature of the hot heat exchanger is denoted as $T_{H,ex}$. The cold heat exchanger was cooled by chilled water. However, as described below, its temperature, $T_{C,ex}$, could not be maintained near an ambient one.

The diameter of the resonator tube was 57 mm, and it was joined with the core tube via a tapered tube whose length was 100 mm. The total length of the three tubes was 2.3 m. At the end of the resonator tube, a traveling-wave thermoacoustic refrigerator was attached as an acoustical load. In this paper, the performance of the refrigerator is not described. On the wall of the resonator tube, two pressure sensors were mounted which allowed us to measure the acoustic power flow delivered to the resonator tube.^{(10),(11)}

4.2. Experimental results

In Fig. 5(a), $T_{H,ex}$ and $T_{C,ex}$ are shown as functions of $Q_{H,ex}$ by open squares and open circles, respectively. Since the gas oscillation did not occur, the temperature $T_{H,ex}$ monotonically increased with increasing $Q_{H,ex}$ below 200 W, while $T_{C,ex}$ was kept at about 10°C. When $Q_{H,ex}$ was 214 W, the temperature $T_{H,ex}$ exceeded 200°C and the gas in the engine spontaneously oscillated with the frequency of 48 Hz. However, the oscillation was stopped after several seconds from the onset, and then, after several seconds, it restarted. These onset and offset of the oscillation was repeated. This may be attributed to the fact that $T_{H,ex}/T_{C,ex}$ fluctuated due to the change in the amplitude of the oscillation.^{(14),(15)} Under the condition $214 \text{ W} \leq Q_{H,ex} \leq 580 \text{ W}$, the oscillation was not stably maintained and became intermittent.

When $Q_{H,ex}$ was 585 W, the amplitude of the oscillation was maintained stably. Under this stable condition, $T_{H,ex}$ and $T_{C,ex}$ became 306°C and 64°C, respectively, and the acoustic power flow outputted to the resonator tube, $W_{out,ex}$, became 20 W. With the increase in $Q_{H,ex}$, both temperatures ($T_{H,ex}$ and $T_{C,ex}$) and the output power flow $W_{out,ex}$ increased as shown in Fig. 5(a). When $Q_{H,ex}$ reached 830 W, $T_{H,ex}$ and $T_{C,ex}$ became 360°C and 98°C, respectively, and $W_{out,ex}$ reached to 30 W. Note that each of data shown in Fig. 5(a) was taken under the condition that the amplitude of the oscillation was maintained stably.

4.3. Comparison between experimental and calculation results

To compare the experimental results with the calculation ones, we have plotted $T_{H,ex}/T_{C,ex}$ in Fig. 5(b) as a function of $Q_{H,ex}$. As mentioned above, when the gas oscillation began, the temperatures $T_{H,ex}$ and $T_{C,ex}$ were 200°C and 10°C, respectively. Hence, $T_{H,ex}/T_{C,ex} = (200 + 273)/(10 + 273) \sim 1.7$. This value of $T_{H,ex}/T_{C,ex}$ is just above the design value of 1.6. Moreover, $T_{H,ex}/T_{C,ex}$ was almost independent of $Q_{H,ex}$ ($>214 \text{ W}$) as shown in Fig. 5(b) by open squares though both $T_{H,ex}$ and $T_{C,ex}$ increased with increasing $Q_{H,ex}$.

By substituting $T_{H,ex}/T_{C,ex}$, $Q_{H,ex}$, and $W_{out,ex}$ into Eq. (1), the experimentally obtained second-law efficiency, $\eta_{2,ex}$, was calculated. It was found that $\eta_{2,ex}$ hardly depends on $Q_{H,ex}$ ($>580 \text{ W}$) and takes the value 0.08 as shown in Fig. 5(b) by open triangles. This value is low

Fig. 5 (a)The experimentally obtained T_H , T_C , and W_{out} . (b)The experimentally obtained T_H/T_C and η_2 . They are shown as a function of the input power Q_H .

compared with the design value of 0.12. We think that the discrepancy in η_2 obtained by the calculation and experiment can be attributed to the fact that the thermal loss was neglected in the calculation. The total heat loss in the experiments can be estimated to be approximately 210 W because the electrical power $Q_{H,ex}$ of about 210 W was needed for maintaining $T_{H,ex}/T_{C,ex}$ at 1.7 without the occurrence of gas oscillation. If this loss is neglected in the experiment, i.e., if

$$\eta'_2 = \frac{W_{out}/(Q_H - 210)}{1 - T_H/T_C}, \quad (2)$$

is used instead of Eq. (1) for the evaluation of the experimentally obtained second-law efficiency, $\eta'_{2,ex}$ becomes 0.12 as shown in Fig. 5(b) by closed triangles. This value is in quantitative agreement with the design value of η_2 . Therefore, based on the comparison between the experimental and calculated results of T_H/T_C and η_2 , we can say that the calculation performed in this paper is useful for designing a standing-wave thermoacoustic engine in accordance with the temperature ratio of heat sources.

5. Summary

By using the thermoacoustic theory, we numerically calculated the critical temperature ratio needed for the operation of a standing-wave thermoacoustic engine and its efficiency. This was done by changing three design parameters; the length, position, and flow channel radius of the stack. By using the results of this calculation, the standing-wave thermoacoustic engine operating under heat sources having the temperature ratio of 1.6 was designed and constructed. The constructed engine worked with the ratio of 1.7 and achieved 8% of the Carnot efficiency, which agreed well with the design value.

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