Radiative Heat Transfer Analysis in a Turbulent Natural Convection Obtained from Direct Numerical Simulation*

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Abstract

Thermal radiation in a turbulent natural convection plays an important role in a wide area of engineering and nature. The purposes of this study are to investigate the effects of turbulent fluctuation on radiative heat transfer, and to evaluate radiative heat transfer models applied to turbulent natural convection. The present radiative heat transfer analysis of a turbulent natural convection using direct numerical simulation (DNS) provides a useful fundamental data for the complete coupling simulation in the future.

Key words: Radiative Heat Transfer, Turbulence, Natural Convection

1. Introduction

Turbulence is the most common state of fluid flow in a wide area of engineering and nature. Turbulent natural convection occurs in high-temperature machines, indoor environment, atmosphere, etc. Thermal radiation in a turbulent natural convection plays an important role in many combustion systems, and solar thermal applications. Many radiation models have been proposed to simulate radiative heat transfer.

The nonlinear dependence of radiative heat transfer on temperature and radiative properties coupled with the fluctuations tends to enhance radiative transfer. Neglecting turbulent fluctuations may yield numerical errors⁽¹⁻⁴⁾. The problem is that turbulent flow and radiative transfer simulation itself are computationally extremely expensive. A complete coupling analysis of turbulence and radiation is important; however, such simulation is very difficult due to the limitation of computational resources. Hence, the radiation effects on turbulent natural convection have not been understood well. Furthermore, a new radiation model has to be proposed when a Reynolds averaged Navier-Stokes turbulence model is used.

A one-way coupling of turbulence and radiation is worth investigating for fundamental researches. Namely, effects of turbulent fluctuation on radiative heat transfer can be clarified. Direct numerical simulation (DNS) of fluid dynamics is recognized as the only computational technology for resolving fine turbulent eddies without any turbulence modeling⁽⁵⁻⁶⁾. The DNS of turbulent natural convection has been performed by Kerr⁽⁷⁾ and Groetzbach⁽⁸⁾. Therefore, a radiative heat transfer analysis of a turbulent natural convection using DNS provides a useful fundamental data for the complete coupling simulation in the future. Furthermore, a fast and accurate radiation model should be properly selected

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depending on conditions, and applied to the turbulent flow.

The purposes of this study are; 1) to validate the DNS of turbulent natural convection, 2) to evaluate four radiation models that are the Discrete Ordinate Radiation Element Method (hereafter DOREM)⁽⁹⁾, the Independent Column Approximation (ICA)⁽¹⁰⁻¹¹⁾, the Optically Thin Approximation (OTA)⁽¹²⁾ and the Spherical Harmonics Method (P1)⁽¹²⁾, 3) to investigate the effects of turbulent fluctuation on radiative heat transfer. The present study provides a basic guideline for the application to the complete coupling analysis with turbulent flow and the radiative heat transfer in future works.

Nomenclature

G	incident radiation, [W/m^2]	I	radiative intensity, [$\text{W}/\text{m}^2 \text{sr}$]
I_b	blackbody intensity, [$\text{W}/\text{m}^2 \text{sr}$]	$\mathbf{q}_R$	radiative heat flux, [W/m^2]
s	length [m]	T	temperature, [K]
<i>Greek symbols</i>			
β	extinction coefficient, [m^{-1}]	ε	emissivity
κ	absorption coefficient, [m^{-1}]	σ	Stefan-Boltzmann constant, [$\text{W}/\text{m}^2 \text{K}^4$]
σ_s	scattering coefficient, [m^{-1}]	τ	optical thickness
Ω	solid angle, [sr]	ω	scattering albedo

2. Physical Model

The instantaneous turbulent natural convective flow and temperature fields are derived from the DNS. Here, the physical model is simply described. The computational domain and the coordinate system are shown in Fig. 1. The top wall is cooled constantly at temperature T_c and the bottom is heated at temperature T_h . The following nondimensional parameters are assumed to be the Prandtl number, $Pr = 0.71$, and the Rayleigh number, $Ra = 8.0 \times 10^6$. The computational domain is a rectangular box with the nondimensional length ($L_x/\delta = 8.0$, $L_y/\delta = 2.0$, and $L_z/\delta = 8.0$). The periodic boundary condition is applied in x and z directions. Uniform grid mesh is used in x and z directions with non-uniform grid spacing in the y direction (finer grid points are used near the boundary walls). Symbol δ is the half-length between the top and the bottom walls. The number of grid ($N_x \times N_y \times N_z$) is set to $64 \times 61 \times 64$ or $128 \times 127 \times 128$. The fine grid is mainly employed in this study. The wall temperatures are defined as $T_h = 320\text{K}$ and $T_c = 300\text{K}$, and the half-length $\delta = 0.5\text{m}$ for the optical thickness $\tau = 0.1$ of typical atmosphere. If a medium involves post-combustion gases, the optical thicknesses become large.

The governing equations are the continuity, the Navier-Stokes, and the energy conservation equations. The buoyancy force is included by using the Boussinesq approximation. The time advancement is carried out by the fractional time step method, where the Crank-Nicolson method is used for the wall-normal second derivatives and the

Fig. 1 Computational domain

Fig. 2 Velocity vector and temperature profile on y - z slice at $x/\delta = 4.0$

Fig. 3 Velocity vector and temperature profile on x - z slice at $y/\delta = 1.0$

Fig. 4 Validation results for the DNS of natural convection

Adams-Bashforth method for other terms. The space difference is expressed by the forth-order central difference. The Poisson equation for the pressure is solved by the fast Fourier transform in the streamwise and the spanwise direction and by TDMA of compact scheme in wall-normal derivatives. The same computational strategy was employed in our previous DNS studies⁽¹³⁻¹⁵⁾.

Figures 2 and 3 show the instantaneous temperature and velocity vector profiles without radiation on y - z slice at $x/\delta = 4.0$ and x - z slice at $y^* = y/\delta = 1.0$. The upward stream can be found in the high temperature region, and the downward stream occurs in the low temperature region in Fig. 2. The small eddies are emerged near the high and low temperature regions. Figure 4 shows the result of the validation test for the DNS of natural convection without radiation. The present turbulence statistics for the temperature profile and the Nusselt number were presented at $Ra = 8.0 \times 10^6$. In Fig. 4(a), the Rayleigh number is the intermediate value between the references (Groetzbach⁽⁸⁾, $Ra = 3.81 \times 10^5$, and Kerr⁽⁷⁾, $Ra = 2.0 \times 10^7$). The present temperature profile exists between the reference values, so that the DNS code for natural convection was validated. Furthermore, Fig. 4(b) shows the result of Nusselt number (Nu). The present results are compared with the Kerr⁽⁷⁾'s DNS and the Fitzjarrald's experiment⁽¹⁶⁾. As shown in this figure, the DNS code with the fine grid ($128 \times 127 \times 128$) shows good accordance with the reference data.

3. Radiative Heat Transfer Models

3.1 Discrete Ordinate Radiation Element Method (DOREM)

In the present study, the discrete ordinate radiation element method (DOREM) is employed for the reference model. The method is an extension model of the radiation element method, and the detailed formulation of DOREM is available in the reference⁽⁹⁾.

The radiation element method was originally developed by Maruyama and Aihara⁽¹⁷⁻¹⁸⁾. The DOREM has been already well validated with the finite volume/ discrete ordinate method⁽¹⁹⁾, and the Monte Carlo method⁽¹²⁾.

The generalized radiative transfer equation (RTE) in its differential form may be expressed by^(12,18)

$$\frac{dI}{ds} = -\beta I(\hat{s}) + \kappa I_b + \frac{\sigma_s}{4\pi} \int_{4\pi} I(\hat{s}', \hat{s}) \Phi(\hat{s}', \hat{s}) d\Omega', \quad (1)$$

where $\beta [\text{m}^{-1}]$ is the extinction coefficient, $\kappa [\text{m}^{-1}]$ is the absorption coefficient, $\sigma_s [\text{m}^{-1}]$ is the scattering coefficient, $I_b [\text{W}/\text{m}^2\text{sr}]$ is the blackbody radiative intensity, $I [\text{W}/\text{m}^2\text{sr}]$ is the radiative intensity, $\hat{s}$ is the unit directional vector, Φ is the scattering phase function. Although nongray gas and light scattering can be included in this calculation, only gray assumption and nonscattering medium are considered in order to simplify the problem. Solving the RTE, and thus divergence of radiative heat flux can be derived by

$$\nabla \cdot \mathbf{q}_R = -\kappa \left(4\sigma T^4 - \int_{4\pi} I d\Omega \right) = -\kappa (4\sigma T^4 - G), \quad (2)$$

where $G [\text{W}/\text{m}^2]$ is the incident radiation. The RTE has the five dimensions as the space (x, y, z) and the direction (θ, ϕ) ; hence, it is impossible to derive analytical solution in this situation.

In radiative intensity calculations along a line of sight (i.e., $\hat{s}$ direction), the numerical method solves the RTE in its integral form for absorbing-emitting media described as,

$$I(\vec{r}, \hat{s}) = I(\vec{r}_0, \hat{s}) \exp\left(-\int_0^s \kappa ds''\right) + \int_0^s I_b(\vec{r}', \hat{s}) \exp\left(-\int_{s'}^s \kappa ds''\right) \kappa ds', \quad (3)$$

where $I(\vec{r}_0, \hat{s})$ is the upstream radiative intensity. The computational space is discretized using numerous radiation elements. The following assumptions are introduced to simplify the problem; 1) each radiation element is at a constant uniform temperature, 2) the refractive index and the heat generation rate per unit volume are constant and are uniform over the radiation element, 3) radiative intensities in a radiation element are constant for each discrete ordinate independent of location in the element. The DOREM code solves Eq. (3) by similar way of the ray tracing technique for this problem. The S_8 quadrature scheme is used for the ordinates of ray emission and the angular discretization⁽¹²⁾. Although nongray gas and light scattering can be included in this calculation, we assume that the participating medium is gray and non-scattering in order to simplify the problem.

3.2 Spherical Harmonics Method (P1)

The spherical harmonics method provides an approximate solution by transforming the RTE into partial differential equation⁽¹²⁾. The radiative intensity is expanded by l th degree and m th order spherical harmonics function Y_l^m as

$$I(r, \hat{s}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l I_l^m(r) Y_l^m(\hat{s}), \quad (4)$$

where I_l^m are the position-dependent coefficients. If the series in this equation is truncated beyond $l = 1$, the lowest-order approximation can be derived. The approximation is especially

called as the P1-method⁽¹²⁾. Finally, the Eq. (2) can be transformed into

$$\nabla_{\tau} \cdot \left(\frac{1}{1 - A_l \omega / 3} \nabla_{\tau} G \right) = -3(1 - \omega)(4\pi I_b - G), \quad (5)$$

where ∇_{τ} is the spatial derivative nondimensionalized by $\tau = \beta ds$, A_l is the linear anisotropic scattering coefficient, ω is the scattering albedo, but these scattering parameters are not considered. The boundary condition may be expressed by

$$-\frac{2 - \varepsilon}{\varepsilon} \frac{2}{3 - A_l \omega} \hat{\mathbf{n}} \cdot \nabla_{\tau} G + G = 4\pi I_{bw}, \quad (6)$$

where ε is the emissivity, $\hat{\mathbf{n}}$ is the normal vector, and I_{bw} is the blackbody radiative intensity. In the present study, these equations are discretized by the second-order finite difference method, and the matrix equation is solved by the successive over relaxation method. The divergence of radiative heat flux in Eq. (2) can be obtained by solving G from Eqs. (5)-(6).

3.3 Independent Column Approximation (ICA)

The ICA⁽¹⁰⁾ is the simplest extension of plane-parallel radiative heat transfer. It ignores net horizontal photon transport, but includes horizontal inhomogeneities in scalar field. This approach has originated from remote sensing applications and atmospheric sciences⁽¹¹⁾. The ICA has been applied in the turbulent channel flows⁽²⁰⁾, so the detailed explanation can be found in the reference.

Figure 5 shows the comparison between 3D ray tracing and ICA. It represents schematic demonstration of photon trajectory. In the ICA algorithm, a computational domain ($N_x \times N_y \times N_z$ elements) is divided by $N_x \times N_z$ columns. The each column has N_y element. Plane-parallel radiative transfer analyses are essentially conducted inside each of the columns. Namely, photons are constrained in each of the columns. The 3D ray tracing for photon trajectory experiences each participating media elements ($N_x \times N_y \times N_z$ elements) in all of the computational domain. On the other hand, the ICA computes only $N_x \times N_z$ elements because the 1D plane-parallel radiative transfer simulations are performed with y -direction along each column. The ICA algorithm requires horizontal periodicity due to this computational algorithm. This is the one of the limitation. The objective of this study is to show numerical efficiency for calculating local divergence of radiative heat flux with the ICA. Although the ICA ignores 3D radiative effects, the algorithm is very simple. Thus it is feasible to considerably decrease the computational costs.

Fig. 5 Schematic diagram demonstrating the mechanisms for 3D ray tracing and ICA

3.4 Optically Thin Approximation (OTA)

To solve the Eq. (2), it is not easy to determine the incident radiation. The easiest way of solving this problem is to use the OTA. The incident radiation can be simply approximated by using the mean quantities with the radiosities at the top and bottom walls⁽¹²⁾,

$$\nabla \cdot \mathbf{q}_R = -\kappa(4\pi I_b - 2J_h - 2J_c), \quad (7)$$

where J_h and J_c are the radiosities. Radiative intensity leaving from the surfaces is attenuated by absorption and scattering; however, the strength of radiosities is diminished very little due to the optically thin medium. Since the extinction coefficient is too small, emission term is assumed to be unattenuated by self absorption. Although the OTA model is limited for an optically thin media, the computational load is very small. When walls are blackbody, radiosities can be simply described as blackbody emitters.

4. Results and Discussion

4.1 Effect of Turbulent Fluctuation for Radiative Heat Transfer

The effects of turbulent fluctuation for radiative heat transfer are investigated. Figure 6 shows the comparisons of the divergence of radiative heat fluxes between the cases with and without turbulent fluctuation using the DOREM. The symbol, $\langle \nabla \cdot \mathbf{q}_R \rangle$, means an ensemble average of divergence of radiative heat flux, and the symbol, $\nabla \cdot \mathbf{q}_R(\langle T \rangle)$, means divergence of radiative heat flux using the ensemble average of temperature with varying optical thickness. As shown in these figures, the differences between the cases with and without turbulent

Fig. 6 Comparison of divergence of radiative heat fluxes between the cases with turbulent fluctuation, $\langle \nabla \cdot \mathbf{q}_R \rangle$, and without turbulent fluctuation, $\nabla \cdot \mathbf{q}_R(\langle T \rangle)$

fluctuation cannot be found for all the optical thicknesses. When the optical thickness is equal to 1.0, the absolute value of the $\langle \nabla \cdot \mathbf{q}_R \rangle$ is slightly larger than that of $\nabla \cdot \mathbf{q}_R(\langle T \rangle)$. This is attributed to assume a non-reacting flow in this simulation. Although the effect of turbulent fluctuation is relatively small in this case, the effect is important in practical combustion systems. Understanding of a non-reacting turbulent flow in the presence of radiation has academic significance. Since the time scale of radiative transfer is quite shorter than that of turbulent flow, accurate solutions of radiative transfer play an important role in an instantaneous scalar field. In the next two subsections, radiative heat transfer analyses are performed in the instantaneous temperature field using the different radiative heat transfer models.

4.2 Comparison of Radiative Heat Transfer Models

4.2.1 Divergence of Radiative Heat Flux in Optically Thin Medium

The first test shows the differences between the DOREM, the P1, the ICA, and the OTA results in an optically thin medium. The optical thickness is set to 0.1.

Figure 7(a) shows the result of the divergence of radiative heat flux on y - z slice at $x/\delta = 4.0$ and $\tau = 0.1$ using the DOREM. The value is nondimensionalized by $\kappa\sigma T_c^4$. In the vicinity of the hot wall, the values of the divergence of radiative heat flux are negative. This is originated from the radiative emission from the hot fluid. The upstream region corresponds to the high temperature region, so that the emission from the hot fluid may suppress the buoyant flows. On the other hand, the downstream region corresponding to the low temperature region at the right hand side of Fig. 7(a) has positive values. The radiative effects suppress natural convective flows.

Figures 7(b)-7(d) indicates the difference of the divergence of radiative heat flux between the DOREM and the other models on y - z slice at $x/\delta = 4.0$. The values of the both ends of the color bar indicate the maximum/ minimum difference by comparing with the DOREM. In Fig. 7(b), the difference with the ICA is relatively small, but the longitudinal distribution can be observed. This is attributed to the plane-parallel strategy of the ICA.

Fig. 7 Divergence of radiative heat flux on y - z slice at $x/\delta = 4.0$ and $\tau = 0.1$ by (a) DOREM, and differences; (b) with ICA, (c) with OTA, and (d) with P1

Fig. 8 Divergence of radiative heat flux on x - z slice at $y/\delta = 1.0$ and $\tau = 0.1$ by (a) DOREM, and differences; (b) with ICA, (c) with OTA, and (d) with P1

On the other hand, Figs. 7(c) and 7(d) show the similar distributions because the three-dimensionality of radiative transfer can be taken into account in the OTA and the P1 method. The OTA result shows good agreement; however, the P1 method shows very large discrepancy because the P1 method approximates the RTE into partial differential equation, so that the divergence of radiative heat flux in each of elements strongly affected by the radiative transfer from the neighboring elements. Accordingly, the P1 method is not suitable for optically thin medium.

Figure 8(a) shows the result of the divergence of radiative heat flux on x - z slice at $y/\delta = 1.0$ and $\tau = 0.1$ using the DOREM. The other figures show the differences with other models. As described above, the OTA and the P1 method can take into account the horizontal inhomogeneity, resulting in similar distributions (Figs. 8(c) and 8(d)), but the value is not reasonable for the P1. The difference with the ICA distributes with fine structure because the method cannot resolve horizontal radiative transport. Except for this problem, the result is generally better than the P1 result.

4.2.2 Divergence of Radiative Heat Flux in Optically Thick Medium

The second test shows the differences between the models in an optically thick medium. The optical thickness is set to 10.0.

Figures 9(a) and 10(a) show the result of the divergence of radiative heat flux on y - z slice at $x/\delta = 4.0$ and x - z slice at $y/\delta = 1.0$ using the DOREM. As shown in Fig. 9(a), the positive

Fig. 9 Divergence of radiative heat flux on y - z slice at $x/\delta = 4.0$ and $\tau = 10.0$ by (a) DOREM, and differences; (b) with ICA, (c) with OTA, and (d) with P1

Fig. 10 Divergence of radiative heat flux on x - z slice at $y/\delta = 1.0$ and $\tau = 10.0$ by (a) DOREM, and differences; (b) with ICA, (c) with OTA, and (d) with P1

Fig. 11 Ensemble average of divergence of radiative heat flux (L: left column), and two-point correlation coefficients (R: right column) with varying optical thickness; (a) $\tau = 0.01$, (b) $\tau = 1.0$, and (c) $\tau = 50$

value of the divergence of radiative heat flux distributes near the hot wall, except for the hot fluid plumes because the optically thick radiative gases absorb radiative energy from the hot wall. By contrast, the radiative cooling can be clearly observed near the cold wall. In the optically thick medium, the divergence of radiative heat flux in each of elements tends to be affected by the neighboring elements. Therefore, the OTA result shows the large discrepancy (Fig. 9(c)). That can be also found in Fig. 10(c). On the other hand, the P1 and the ICA results are generally good as shown in Figs. 9(b) and 9(d). However, the values of the difference with the ICA are generally small, but the maximum and the minimum values are relatively large in Fig. 10(b). The locations of the error peaks correspond to that of the peaks of the divergence of radiative heat flux in Fig. 10(a) because the horizontal radiative

transport is important in the local gradient of the divergence of radiative heat flux. The P1 method shows excellent agreement in the optically thick medium in Fig. 10(d).

4.2.3 Ensemble Average and Two-point Correlation Coefficients

In order to clarify the differences of the radiation models, the ensemble averages of divergence of radiative heat flux and the two-point correlation coefficients are presented here. The left hand sides of Fig. 11 show the ensemble average of the divergence of radiative heat flux with varying the optical thickness. As shown in these figures, the differences between the radiative heat transfer models are clarified. When the optical thickness is larger than 1.0, the divergence of radiative heat flux is negative near the bottom wall; however, the value is positive just above the bottom layer. The inversion phenomenon can be also found near the top wall. This phenomenon displays a characteristic tendency of a wall turbulent heat transfer in the presence of radiation.

The right hand sides of Fig. 11 show the x - z mean the two-point correlation coefficients between the DOREM results and the others. Two-point correlation coefficients for turbulent fluctuating part of the divergence of radiative heat flux is defined by

$$R = \frac{\overline{(\nabla \cdot \mathbf{q}_{R,DOREM})' (\nabla \cdot \mathbf{q}_{R,n})'}}{\sqrt{\overline{(\nabla \cdot \mathbf{q}_{R,DOREM})'^2}} \sqrt{\overline{(\nabla \cdot \mathbf{q}_{R,n})'^2}}}, \quad (8)$$

where n indicates the ICA, the OTA, or the P1. When the optical thickness is equal to 0.01, the results of the three models are nearly equal to 1.0. Although the P1 shows large error for the divergence of radiative heat flux in the optically thin medium, the turbulent fluctuation parts show good agreement with the DOREM because the effect of wall radiative heat flux is large. When the optical thickness is equal to 1.0, the ICA and the OTA results slightly drop down to lower values because the effect of horizontal radiative transport becomes important in the moderately optically thick medium. The effect is clearly found in the optically very thick medium ($\tau = 50$). The P1 model especially provides robustness for predicting the turbulent fluctuation on radiative heat transfer analysis in the optically thick medium.

4.2.4 Wall Radiative Heat Flux

Figure 12 shows the x - z averaged wall radiative heat flux with varying optical thicknesses. The absolute values of the DOREM and the ICA become small with increasing the optical thickness because the thermal radiation attenuates in the participating medium.

Fig. 12 x - z averaged wall radiative heat flux with varying optical thicknesses

The P1 provides good results when the optical thickness is larger than 1.0, on the other hand, has poor accuracy in the optically thin medium.

Note that if a convergence algorithm of the P1 is improved, the result will be better. The OTA results show good accordance with the DOREM, so that the OTA is reasonable in the optically thin medium.

4.2.5 Computational Performance

All the computations are performed using a normal desktop computer (CPU: Intel(R) Xeon(R) X5570, 2.93GHz), and the g95 Fortran compiler. The present source codes are optimized in order to save a memory size and an access speed to the CPU memory. Since a computational speed depends on the programming technique, note that the present CPU time gives an indication of the computational speed.

Table 1 shows the CPU time for solving the RTE and the root mean square (RMS) error with varying optical thicknesses.

The reference values for RMS are achieved from the DOREM, thus RMS is defined by

$$RMS = \sqrt{\frac{\sum_{i=1}^N [(\nabla \cdot \mathbf{q}_R)_i - (\nabla \cdot \mathbf{q}_R)_{i,DOREM}]^2}{N}}, \quad (9)$$

where N is the total number of computational elements. The solution criterion is changed in the P1 method. Note that the S_8 quadrature scheme is employed for the DOREM.

When τ is equal to 0.1, the P1 method is very time-consuming, and has a problem in the computational accuracy. The ICA result is best in this case. Although the computational accuracy of the OTA is relative lower than that of the ICA, the difference may be within acceptable range. Furthermore, the computational load of the OTA is very small, so that the OTA is available if an optical thickness is less than 0.1. If an optical thickness is greater than 1.0, the error of the OTA may be beyond the allowable range. When τ is equal to 1.0, the ICA result is best for the CPU time and the RMS. In the optically thick medium ($\tau > 10.0$), the P1 method is superior compared with the ICA. Even though the solution criterion is optimistic, the computational accuracy is better than the ICA.

Table 1 Comparison of the CPU time for solving RTE and the root mean square error

$\tau = 0.1$				$\tau = 1.0$		
	Criterion	CPU(s)	RMS	Criterion	CPU(s)	RMS
DOREM	-	6489	-	-	6505	-
ICA	-	108.8	5.20×10^{-5}	-	74.30	3.71×10^{-4}
OTA	-	0.880	2.66×10^{-3}	-	0.623	2.93×10^{-2}
P1	10^{-4}	2391	11.4	10^{-4}	608.2	3.47×10^{-1}
	10^{-5}	9041	1.03	10^{-5}	1450	4.57×10^{-3}
	10^{-6}	28638	1.64×10^{-2}	10^{-6}	2924	3.46×10^{-4}
$\tau = 10.0$				$\tau = 50.0$		
	Criterion	CPU(s)	RMS	Criterion	CPU(s)	RMS
DOREM	-	6367	-	-	4278	-
ICA	-	106.3	3.56×10^{-4}	-	108.9	5.16×10^{-5}
OTA	-	0.623	2.82×10^{-2}	-	0.69	3.03×10^{-2}
P1	10^{-4}	58.37	3.02×10^{-4}	10^{-4}	6.63	1.74×10^{-5}
	10^{-5}	81.82	1.93×10^{-4}	10^{-5}	8.21	1.78×10^{-5}
	10^{-6}	127.9	1.93×10^{-4}	10^{-6}	12.0	1.78×10^{-5}

5. Conclusions

The obtained conclusions are summarized as follows;

1. Although the effect of turbulent fluctuation for radiative heat transfer is relatively small for non-reacting turbulent natural convection, the obtained fundamental results with the DNS data will be useful for practical problems for future works.
2. Since computational load is very small, the OTA is available with the acceptable accuracy if an optical thickness is less than 0.1.
3. Although the horizontal radiative transport cannot be exactly resolved for the local estimation, the ICA is generally provides better computational performance for all the optical thicknesses.
4. The P1 is not suitable for the optically thin medium ($\tau < 0.1$); however, the computational performance is best for the optically thick medium ($\tau > 10.0$). The P1 provides robustness for predicting the turbulent fluctuation on radiative heat transfer analysis in the optically thick medium.

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