Robust Gas Turbine Combustor with Acoustic Liner*

Kazufumi IKEDA**, Keisuke MATSUYAMA**, and Masaharu NISHIMURA***
** Takasago Research & Development Center, Mitsubishi Heavy Industries, ltd.,
2-1-1 Shinshama Ara’i-cho, Takasago, Hyogo 676-8686 JAPAN
E-mail: kazufumi_ikeda@mhi.co.jp
*** Department of Mechanical Engineering, Tottori University,
4-101 Koyama-cho South, Tottori, Tottori Pref., 680-8552 JAPAN

Abstract
Combustion oscillation of a self-excited thermo-acoustic phenomenon occurs inside the gas turbine combustor. Its excessive pressure fluctuation may impair the gas turbine engine operation, and could result in hardware severe damages. Therefore, combustion oscillation is one of the problems of the gas turbine development. In this paper, acoustic liner of an acoustic damping appending device to suppress the combustion oscillation is developed and discussed. Acoustic liner consists of a perforated plate, which the acoustic analysis models have been researched well at other papers referenced in this paper, and back cavity. The accuracy of these acoustic analysis models was verified by the laboratory model tests, and then the effectiveness that acoustic liner can suppress combustion oscillation at high frequencies was verified by the actual engine operation tests. This suppression method can be designed without the detailed combustion prediction. Hence the credible design is relatively easy. As the results of the actual engine operation tests shown in this paper, the gas turbine combustor with acoustic liner can be operating robustly without worrying about high frequency oscillation, and future more contributions to the combustor development can be expected.

Key words: Gas Turbine, Combustor, Combustion Oscillation

1. Introduction

Combustion oscillation of a self-excited thermo-acoustic phenomenon occurs inside the gas turbine combustor in operating at higher temperatures with reducing NOx and other gaseous emissions. In general, excessive pressure fluctuation of combustion oscillation may impair the gas turbine engine operation and could result in hardware severe damages. Therefore, combustion oscillation is one of the problems of the gas turbine development [1].

The mechanism of thermo-acoustic instability has been researched well, and many analytical and experimental researches have been discussed [2]-[9]. According to Rayleigh [2] who is referenced by many researchers, thermo-acoustic instability criterion is described as the following inequality called Rayleigh criterion

\[ \int_\tau \int_V p'q' \, dV \, dt > \int_\tau \int_V \Phi \, dV \, dt, \]

(1)

where \( p' \) and \( q' \) are perturbations of pressure and heat release, respectively, \( \tau \), \( V \) and \( \Phi \) are the period of oscillation, the combustor volume, and the acoustic energy dissipation,
respectively. The left hand side of the inequality (1) means the amplification rate of the pressure fluctuation and the heat release fluctuation, which are interacted. The other side means the acoustic damping rate. Since thermo-acoustic instability occurs when Rayleigh criterion (1) is satisfied, reduction of the left hand side of the inequality (1) can be considered as one of the instability suppression method. Therefore the accurate analysis of the interaction between the pressure fluctuation and the heat release fluctuation has been often discussed. But especially at high frequencies, the accurate analysis of this interaction is difficult in actually. On the other hand, increase of the right hand side of the inequality (1) can be considered as one of the instability suppression method as well. The estimation of the right hand side of the inequality (1) is relatively easier, since the discussion of dynamic combustion is not involved. Then, in this paper, the method of increasing the acoustic damping at enough wide frequency range is focused. This concept is discussed briefly in the following section.

The perforated plate with back cavity applied with the combustor wall shown in Figure 1 can be considered as one of the increasing method of the acoustic damping. This suppression method is called acoustic liner in this paper. The analysis method of acoustic property of a perforated plate has been well discussed [10]-[14]. For analyzing the acoustic absorption coefficient of acoustic liner at the high frequencies, the analysis model of Guess [14] is discussed in Section 3, then, the accuracy of the analysis model is verified by laboratory model tests in Section 4. Section 5 presents the results of the actual engine operation tests and the suppression effectiveness of acoustic liner against the instability. Conclusions are summarized in the final section.

2. Instability criterion and conceptual design to suppress the instability

In this section, the concepts for suppressing of combustion oscillation at high frequencies are discussed, through the brief theoretical study of thermo-acoustic instability.
2.1. Instability criterion of combustion oscillation

For the simplicity of the discussion, the essential parts of the interaction between the pressure fluctuation and the heat release fluctuation are considered, but the other effects, as the nonlinearity, the mean flow effects, and the viscosity are considered to be negligible. The equations governing the combustion oscillation can be described as

\[
\frac{\partial \rho'}{\partial t} + \nabla \cdot \mathbf{u}' = 0, \\
\frac{\partial \mathbf{u}'}{\partial t} = -\frac{1}{\rho} \nabla p', \\
\rho \frac{T}{\gamma} \frac{\partial S'}{\partial t} = q', \\
\frac{\partial p'}{\partial t} - c^2 \frac{\partial \rho'}{\partial t} = (\gamma - 1) \rho \frac{T}{\gamma} \frac{\partial S'}{\partial t},
\]

where \( \rho, p, \mathbf{u}, S, T, \gamma \) and \( c \) are density, pressure, velocity, entropy, temperature, ratio of specific heats, and speed of sound, respectively. A combination of the equations above is

\[
\frac{\partial E}{\partial t} = \frac{\gamma - 1}{\rho c^2} p' q' - \nabla \cdot (p' \mathbf{u}'),
\]

where

\[
E = \frac{p^2}{2} \left( \left| \mathbf{u}' \right|^2 + \frac{1}{\rho^2} \frac{\rho'^2}{c^2} \right)
\]
is acoustic energy density. Integrating the equation (6) temporally over the period \( \tau \) of oscillation and spatially over the volume \( V \) of combustor, the increase degree of the instability can be estimated as

\[
\int_V E(t+\tau) \, dV - \int_V E(t) \, dV = \int_\tau \int_V \frac{\gamma - 1}{\rho c^2} p' q' \, dV \, dt - \int_\tau \int_A p' \mathbf{u}' \cdot \mathbf{n} \, dA \, dt,
\]

where Gauss integral formula has been applied to the second term of the right hand side. The notations of \( A \) and \( \mathbf{n} \) are the boundary surface of combustor volume \( V \) and the normal vector at its boundary surface \( A \), respectively. The first term of the right hand side of the equation (8), which the phase lag between the pressure fluctuation \( p' \) and the heat release fluctuation \( q' \) is important, means the amplification rate of these dynamic values which are coupled with each other. The second term means the acoustic energy absorbed at the boundary surface. Summarizing above, the right hand side of the equation (8) means the balance of the amplification with the damping of oscillation. Therefore, when the following criterion is satisfied, the combustion oscillation instability can occur:

\[
\int_\tau \int_V \frac{\gamma - 1}{\rho c^2} p' q' \, dV \, dt > \int_\tau \int_A p' \mathbf{u}' \cdot \mathbf{n} \, dA \, dt.
\]

2.2. Conceptual design to suppress the combustion oscillation

The combustor should be designed so that the instability criterion not satisfied. Thus, it should be that the left hand side of the inequality (9) is designed to be small, or the right
hand side is designed to be larger. Using the thermo-acoustic analysis, the left hand side of the inequality (9) can be analyzed. However especially at higher frequencies many acoustic modes are inside the combustor, and even if the huge and complex calculations can be done, the suppressing design and analysis of all modes is not easy. Whereas, the estimation of the right hand side of the inequality (9) can be analyzed easier without the discussion of the dynamic combustion. Combustion oscillation is expected to be suppressed robustly by designing an additional acoustic damping at wide frequencies.

3. Design method for acoustic damping appending to combustor

3.1. Acoustic analysis model of the perforated plate with back cavity

There have been many discussions about the acoustic absorption coefficient of a perforated plate shown in Figure 3. In this paper, the analysis model of Guess[14]

\[
\frac{Z_p}{\rho_L c_L} = \left[ \frac{\sqrt{8} \nu \omega (t + d)}{\sigma c_t d} + \frac{\pi^2 d^2}{2 \alpha L^2} + \frac{1 - \sigma}{\sigma} \left[ \left| M_0 \right| + 0.3 \left| M \right| \right] \right]
\]

is referenced, where \( Z_p = p' / u' \cdot n \), \( \rho \), \( \nu \), \( \omega \), \( t \), \( d \), \( \sigma \), \( M_0 \), \( M \), \( i \), subscript \( L \), and \( \delta \) are the acoustic impedance of the perforated plate, mean density, kinetic viscosity, angular frequency, thickness of plate, diameter of the aperture, porosity, Mach number of the velocity fluctuation at a circular aperture, Mach number of the mean flow, the imaginary unit, values inside back cavity, and open end collection of the aperture described as

\[
\delta = \frac{8d \left( 1 - 0.7 \sqrt{\sigma} \right) \left( 1 + 5 \times 10^3 M_0^2 \right)}{3\pi \left( 1 + 305 M^3 \right) \left( 1 + 10^4 M_0^2 \right)}.
\]

The acoustic impedance \( Z_b \) of back cavity can be described as

\[
\frac{Z_b}{\rho_L c_L} = - i \cot k b,
\]

where \( k = \omega / c \) and \( b \) are the wave number and the height of back cavity, respectively. Considering the difference between the low temperature inside back cavity subscripted \( L \) and the high temperature inside the combustor subscripted \( H \), the acoustic impedance \( Z_F \) of the perforated plate with back cavity can be described as

\[
\frac{Z_F}{\rho_H c_H} = \frac{c_L}{c_H} \frac{Z_p + Z_b}{\rho_L c_L}.
\]

The acoustic reflection coefficient and the acoustic absorption coefficient can be described respectively as follows, using the estimation of the acoustic impedance (13):

\[
R_F = \frac{Z_F - \rho_H c_H}{Z_F + \rho_H c_H},
\]

\[
\alpha_F = 1 - \left| R_F \right|^2.
\]
According to the estimation of the acoustic absorption coefficient mentioned above, the acoustic absorption coefficient can be maximized at the specific frequency range where the imaginary part of the impedance $Z_F$ comes near to zero. Since the perforated plate is generally enough thin, the impedance $Z_b$ of back cavity is dominant in the imaginary part of the impedance $FZ$. Therefore, the optimization condition at which the acoustic absorption coefficient is maximized can be described as $0 \cot k b = \pi/2 + n \pi$, where $n = 1, 2, \cdots$. This means the acoustic absorption coefficient can be tuned by the height of back cavity. In general case the incident propagation direction to the perforated plate is not the normal, so the optimization condition can be corrected as $0 \cot (kb \cos \phi) = \pi/2 + n \pi$, where $\phi$ is the incident angle to the perforated plate. This means the frequency property of the acoustic absorption coefficient of the perforated plate with back cavity can be varied by the incident direction of the sound. Although there can be various sound propagation directions especially at high frequency range, the analysis model mentioned in this section is assumed as the acoustic wave is propagating only to the normal direction to the perforated plate. Validity of this assumption is discussed in the next section.

### 3.2. Acoustic absorption coefficient of perforated cylinder with back cavity

The acoustic waves can be regarded as the superimposed solutions of Fourier transform of the wave equation, that is, 

\[
p' = \sum_m a_{H,m} \left[ H_m^{(2)}(k_H r) + R_{1,m} H_m^{(1)}(k_H r) \right] \cos m \theta \quad (0 < r < r_1 - \varepsilon)
\]

\[
\sum_m b_{L,m} \left[ H_m^{(2)}(k_L r) + R_{3,m} H_m^{(1)}(k_L r) \right] \cos m \theta \quad (r_1 - \varepsilon < r < r_2)
\]

\[
u' \cdot n = \frac{i}{\rho_H c_H} \sum_m a_{H,m} \left[ H_m^{(2)}(k_H r) + R_{1,m} H_m^{(1)}(k_H r) \right] \cos m \theta \quad (0 < r < r_1 - \varepsilon)
\]

\[
\frac{i}{\rho_L c_L} \sum_m a_{L,m} \left[ H_m^{(2)}(k_L r) + R_{2,m} H_m^{(1)}(k_L r) \right] \cos m \theta \quad (r_1 - \varepsilon < r < r_1)
\]

\[
\frac{i}{\rho_L c_L} \sum_m b_{L,m} \left[ H_m^{(2)}(k_L r) + R_{3,m} H_m^{(1)}(k_L r) \right] \cos m \theta \quad (r_1 < r < r_2)
\]

in cylindrical coordinates shown in Figure 4, where $R_{1,m}$, $R_{2,m}$, $R_{3,m}$, and $\mathbf{n}$ are the modal acoustic reflection coefficient at the boundary between high temperature and low temperature region $r = r_1 - \varepsilon$, the modal acoustic reflection coefficient at the perforated plate $r = r_1$, the modal acoustic reflection coefficient on the combustor wall $r = r_2$, and the normal unit vector, respectively. Hankel function denoted as $H_m^{(1or2)}$ means the sound wave propagating from the far field, or to the far field. The prime ‘ of Hankel function means the differential of Hankel function.
Considering the combustor wall to be rigid, that is, $u' \cdot n = 0$ at $r = r_2$, the acoustic reflection coefficients on the combustor wall can be described as

$$R_{3,m} = -\frac{H_m^{(2)}(k_L r_2)}{H_m^{(1)}(k_L r_2)},$$

(18)

Considering the thickness $\varepsilon$ of the wall to be enough small, and negligible, and taking account of the conservation of mass and momentum which are described as

$$C_1, m = \frac{C_{2, m} H_m^{(2)}(k_L r_1) - C_{3, m} H_m^{(2)}(k_L r_1)}{C_{2, m} H_m^{(1)}(k_L r_1) - C_{3, m} H_m^{(1)}(k_L r_1)},$$

(21)

$$R_{1, m} = \frac{H_m^{(2)}(k_H r_1) - C_{3, m} H_m^{(2)}(k_H r_1)}{C_{3, m} H_m^{(1)}(k_H r_1) - H_m^{(1)}(k_H r_1)},$$

(22)

where $Z_p$ is the acoustic impedance of the perforated plate (10), the acoustic reflection coefficient at the perforated plate can be estimated as

$$R_{2, m} = \frac{C_{1, m} H_m^{(2)}(k_L r_1) - C_{2, m} H_m^{(2)}(k_L r_1)}{C_{2, m} H_m^{(1)}(k_L r_1) - C_{3, m} H_m^{(1)}(k_L r_1)}.$$

(23)

$$C_{2, m} = H_m^{(2)}(k_L r_1) + R_{3, m} H_m^{(1)}(k_L r_1) + \frac{i Z_p}{\rho_c c_L} H_m^{(1)}(k_L r_1) + R_{3, m} \frac{i Z_p}{\rho_c c_L} H_m^{(1)}(k_L r_1),$$

(24)

$$C_{3, m} = \frac{c_L}{c_H} H_m^{(2)}(k_L r_1) + R_{2, m} H_m^{(1)}(k_L r_1).$$

(25)

Using these solutions, the modal acoustic absorption coefficients are as follows,

$$\alpha_m = 1 - \left| R_{1, m} \right|^2.$$

(26)
One example of analysis results is shown in Figure 5. As mentioned in the previous section, the acoustic absorption coefficient of the acoustic liner is maximized at the resonant frequency of back cavity. Here, the acoustic liner is designed using the normal incident perforated plate model (15) to become its acoustic absorption coefficient maximized at a certain target frequency $f_0$. In the actual engine temperature, pressure, flow rate, and more the actual engine operating conditions.

The solid line in Figure 5 is the acoustic absorption coefficient which is estimated by using the normal incident perforated plate model (15). The dotted lines in Figure 5 are the acoustic absorption coefficients which are estimated for each cylindrical acoustic mode by using the cylinder model (26). In the both cases of the solid line and the dotted lines, the height of back cavity, the thickness of the plate, the porosity, the diameter of the apertures, and the operating conditions are as same each other, respectively. The difference of conditions between each estimation case is just whether the acoustic liner structure is planar or cylindrical. The circumferential mode number corresponding to each dotted line shows as $m$ in Figure 5. Examples of these mode shapes are as shown in Figure 2. The number of $m$ means the number of the nodal diameters of the mode shape. Since the sound propagation directions of the cylinder model especially at higher modes are not the normal to the perforated plate, the frequency properties of the acoustic absorption coefficients of the cylinder model are shifted to higher frequency range than those of the plate model. This tendency is more pronounced in higher modes.

Here, we should focus to the acoustic absorption coefficient of each cylindrical acoustic mode at each resonant frequency. These are shown as circles above the dotted lines in Figure 5. The resonant frequencies of the cylindrical acoustic modes can be estimated using the wave equation solutions (16) and (17) under the rigid wall condition. In the wave equation solutions (16), a condition for homogeneous solutions is $R_{r,m} = 1$. Considering the thickness $\varepsilon$ of the wall to be enough small, and negligible, and taking account of the rigid wall condition to the wave equation solution (17) at $r = r_1$, the eigen equation can be derived as $J_m'(\omega r_1/c_H) = 0$, where $J_m'$ is Bessel function and the prime ' of Bessel function means the differential of Bessel function. The subscripted number $m$ in the eigen equation means the circumferential mode number, which is corresponding to the number $m$ of each dotted line in Figure 5. The resonant frequencies of the cylindrical acoustic modes which are shown as circles in Figure 5 can be estimated as the solutions of the eigen equation. For each circumferential mode $m$, there are plural eigen solutions caused for the radial modes. Therefore, there are plural resonant frequencies above each dotted line in Figure 5. In Figure 5, circles are found to be agreed well with the trend of the solid line. Hence, the analysis using the normal incident perforated plate model can be regarded as one of the estimation method of the acoustic absorption coefficient of the cylinder problem.
4. Verification of acoustic liner effects and the accuracy of analysis

In this section, the verification of the analysis accuracy and the effectiveness of acoustic liner applied to the actual gas turbine engine are shown.

4.1. Accuracy verification of acoustic absorption coefficient analysis

In this section, the experimental results of the acoustic absorption coefficient at the normal temperature laboratory are shown. Figure 6 shows the experimental apparatus which has high pressure air source, sound source speaker, steel tube, and two microphones.

The acoustic absorption coefficient was analyzed by outputs of two microphones. How to estimate the acoustic reflection coefficient and the acoustic absorption coefficient, by separating the incident and reflected waves from the outputs of two microphones, has been proposed and verified by Seybert and Ross [15]. When considering the enough longer wave lengths than the diameter of the test tube, the sound waves inside the test tube can be regarded as the one dimensional planar waves propagating to the axial direction of the test tube. In such cases, the acoustic reflection coefficient and the acoustic absorption coefficient can be estimated by analyzing the outputs of two microphones as follows,

\[ R_{EX} = -\frac{H_{12} - \exp(-iks)}{H_{12} - \exp(iks)}, \]  \tag{27} \]

\[ \alpha_{EX} = 1 - |R_{EX}|^2, \]  \tag{28} \]

where \( H_{12} \equiv \frac{p'_2}{p'_1} \) is the transfer function between \( p'_1 \) of Microphone 1 and \( p'_2 \) of Microphone 2. The distance between two microphones is denoted as \( s \).

The experimental results were compared with the normal incident perforated plate analysis results of the equation (15) at the mean pressure conditions 0.1, 1.0 and 2.0MPa. The tests were carried out at the normal temperature conditions. The excitation level of the speaker was adjusted so that the pressure fluctuation level in the test tube was to be constant at the different mean pressure conditions. The dimensions of the acoustic liner applied to this laboratory tests were designed not for the actual combustors, but designed to be easier of verifying the analysis accuracy at the laboratory conditions, and, designed so that the acoustic absorption coefficient became relatively large at the atmospheric pressure 0.1MPa condition.

The results of the tests and the analysis shown in Figure 7 are agreed well with each other. The horizontal axis of Figure 7 shows the relative value of the frequency \( f_0 \) where the acoustic absorption coefficient has been designed to be the maximum in the actual engine operating conditions as shown in Figure 5. Regarding both of the experimental results and the analysis results in Figure 7, the acoustic absorption coefficient is reduced against the mean pressure rise. Since the temperature is constant but the mean pressure is different, the mean density can be considered different at each test conditions. Since the pressure fluctuation with a constant level despite the different mean density conditions, according to the equation of motion (3), the velocity fluctuation can be considered different depending on the mean pressure. For the velocity fluctuation level has been changed at each mean pressure conditions, the acoustic absorption coefficient can be considered changing, according to the acoustic property of the perforated plate (10) involving the velocity fluctuation parameter \( M_0 \). Summarizing above, because the test model has been designed to obtain a large damping performance at the atmospheric pressure condition, the decreasing velocity fluctuation level due to a rise in the mean pressure can be considered as the cause of decreasing the acoustic absorption coefficient as shown in Figure 7.
4.2. Verification of damping effects at laboratory tests

The damping effects of the acoustic liner are examined experimentally by the cylindrical apparatus which is simulated the outline shape of the actual combustor as shown in Figure 8. The mean flow velocity was adjusted to agree with Mach number of the actual engine operating conditions. The speaker was placed at the position corresponding to the combustion region of the actual combustor where was regarded as an acoustic source position. The acoustic liner structure of this test, which was same as Figure 5, was designed in the actual engine operating conditions. The test was carried out at the normal temperature and the atmospheric pressure conditions. Figure 9 shows the pressure fluctuation measured near the driving source speaker developed for simulating a loud like the actual engine. The pressure fluctuation level excited by the speaker was adjusted that the ratio of the pressure fluctuation level and the mean pressure as to match to those of the actual engine operating conditions. The thin line in Figure 10 shows the acoustic absorption coefficient of the test acoustic liner estimated at the laboratory conditions of the normal temperature and pressure. The sound speed in the laboratory conditions is slower than that of the actual high temperature conditions. The horizontal axis of Figure 10 has been converted to the frequency of the actual conditions using the ratio of the sound speed. The frequency denoted as $f_0$ in Figure 10 means the same frequency discussed in Figure 5. The bold line in Figure 10 shows the acoustic absorption coefficient of the test acoustic liner estimated at the actual engine operating conditions.

Figure 11 is the comparison of the experimental results of acoustic lining wall conditions and rigid wall conditions. Comparing the experimental results and the designed acoustic absorption coefficient, the effects of the acoustic liner appending the sufficient acoustic damping as shown in Figure 11 can be found at wide frequency range where the acoustic absorption coefficient has been designed to be large as shown in Figure 10.
Fig. 8  Schematic of the experimental apparatus of speaker test

Fig. 9  Pressure fluctuation near the driven speaker in the open fields

Fig. 10  Acoustic absorption coefficient estimated at the laboratory condition (thin line) and the actual engine condition (bold line)

Fig. 11  Comparison of the measured pressure fluctuation inside the cylinder model of acoustic lining wall conditions (red line) and rigid wall conditions (black line)
4.3. Actual engine operation tests

The experimental results of the acoustic liner applied to the actual gas turbine combustor at the actual operating conditions are presented in this section. In the operation without acoustic liner, namely in the conventional operation, a self-excited pressure fluctuation at high frequency range was measured. However, there was not excessive pressure fluctuation in operation with the acoustic liner, as shown in Figure 12.

Figure 13 shows the maximum pressure fluctuation levels measured for each of all combustors of the gas turbine over the whole of the operating conditions. The suppression effects of acoustic liner against high frequency combustion oscillation can be found at each of all combustors as shown in Figure 13. Moreover, as the results, there are found uniformization effects of combustion oscillation in each combustor.

![Comparison of the measured pressure fluctuation inside the actual combustor without (left hand side) or with (right hand side) acoustic liner](image)

![Pressure fluctuation inside each combustor consist of the whole of gas turbine engine](image)

5. Conclusions

Acoustic liner can suppress combustion oscillation at high frequencies. This suppression method can be designed without the detailed combustion prediction. Hence the credible design is relatively easy. As the results of the actual engine operation tests in this paper, the gas turbine combustor with acoustic liner can be operating robustly without worrying about high frequency oscillation, and future more contributions to the combustor development can be expected.
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