Entropy Analysis in Mixed Convection MHD flow of Nanofluid over a Non-linear Stretching Sheet *

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Abstract
This article deals with a numerical study of entropy analysis in mixed convection MHD flow of nanofluid over a non-linear stretching sheet taking into account the effects of viscous dissipation and variable magnetic field. The nanofluid is made of such nano particles as $SiO_2$ with pure water as a base fluid. To analyze the problem, at first the boundary layer equations are transformed into non-linear ordinary equations using a similarity transformation. The resultant equations are then solved numerically using the Keller-Box scheme based on the implicit finite-difference method. The effects of different non-dimensional governing parameters such as magnetic parameter, nanoparticles volume fraction, Nusselt, Richardson, Eckert, Hartman, Brinkman, Reynolds and entropy generation numbers are investigated in details. The results indicate that increasing the nano particles to the base fluids causes the reduction in shear forces and a decrease in stretching sheet heat transfer coefficient. Also, decreasing the magnetic parameter and increasing the Eckert number result in improves heat transfer rate. Furthermore, the surface acts as a strong source of irreversibility due to the higher entropy generation number near the surface.

Key words: Entropy Analysis, Nanofluid, Mixed Convection, Stretching Sheet, MHD

1. Introduction

Many recent studies have been focused on the problem of magnetic field effect on laminar mixed convection boundary layer flow over a vertical non-linear stretching sheet (1)-(3). Some industrial examples of the problem are extrusion processes, cooling of nuclear reactors, glass fiber production and crystal growing.

Malarvizhi et al. (4) have investigated free and mixed convection flow over a vertical plate with prescribed temperature and heat flux. MHD heat transfer mixed convection flow along a vertical stretching sheet studied by Mohebujjaman et al. (5) in the presence of magnetic field and heat generation. Kumaran et al. (6) studied the transition of MHD boundary layer flow past a stretching sheet. The study by Fadzilah et al. (7) has investigated numerically free convection boundary layer in a Viscous Fluid. Salleh et al. (8) studied forced boundary layer Flow at a Forward Stagnation Point. Prasad (9) has taken into account the effects of temperature dependent properties on the MHD forced convection over a
non-linear stretching plate. In recent years, convective heat transfer from nanofluids has been considered noticeably. To improve the conventional fluids transport properties such as conductivity, nano or micro particles are added. Chio (10) was the first person who utilized the nanofluid and Choi et al. (11) affirmed that the addition of one percent of nanoparticles by volume to the usual fluids increases the thermal conductivity of the fluid up to approximately twice. Recently, several modeling of the natural or mixed convection of nanofluids have been investigated numerically. Ho et al. (12) studied the effect of natural convection of nanofluid in an enclosure due to uncertainties in viscosity and thermal conductivity. The study by Ghasemi and Aminossadati (13) has presented the numerical solution of natural convection in an inclined enclosure filled with a water-Cuo nanofluid. The effect of nanofluid on forced convection heat transfer enhancement was studied by Maiga et al. (14). Numerical investigations, experiments and applications of nanofluids have been reported by Wang and Mujumdar (15)-(17) in a wide range.

The mixed convection heat transfer of nanofluid over a vertical stretching sheet in the presence of variable magnetic field and viscous dissipation effects has not been investigated yet. The importance of viscous dissipation term is intensified by the increase of friction coefficient in the case of nanofluids. Entropy generation due to the thermodynamic irreversibility results from heat transfer processes and viscous dissipation (18)(19). It has been studied by Saouli and Ai’boud-Saouli (20) in a falling liquid film along an inclined heated plate, Mahmud et al. (21) for mixed convection in a channel, and Ai’boud-Saouli et al. (22)(23) in a falling film considering the magnetic field and viscous dissipation effects.

In the present study, entropy generation in mixed convection MHD flow of a nanofluid along a non-linear stretching sheet with the presence of viscous dissipation and variable magnetic field is investigated. Boundary layer equations are transformed into the non-linear ordinary equations using similarity variables. The resultant equations are solved by the implicit finite-difference scheme known based on the Keller-Box algorithm. The entropy generation is calculated using the entropy relation by putting the temperature and velocity fields obtained from solving the momentum and energy equations.

**Nomenclature**

- $b$: stretching rate, positive constant
- $B(x)$: magnetic field, Tesla
- $B_0$: magnetic rate, positive constant
- $Br$: Brinkman number
- $C_f$: skin friction coefficient
- $C_p$: specific heat at constant pressure of the basic fluid, J/kg.K
- $Ec$: Eckert number
- $f$: dimensionless velocity variable
- $g$: gravitational acceleration, m/s
- $Gr$: Grashof number
- $Ha$: Hartman number
- $k$: thermal conductivity, W/m.K
- $m$: index of power law velocity, positive constant
- $Mn$: magnetic parameter
- $N_f$: Entropy generation number
- $Nu$: Nusselt number
- $Pr$: Prandtl number
- $Re$: local Reynolds number
2. Mathematical Analysis

The entropy analysis for a steady state two dimensional mixed convection MHD flow of a nanofluid along a vertically moving stretching sheet with variable magnetic field and viscous dissipation effect is considered. A quiescent incompressible and electrically conducting fluid in the presence of a magnetic field \( B(x) \) perpendicular to the sheet is taken into account in the presence of the dissipation effect. Fig. 1 shows the schematic view of the physical model and coordinates of the system.

The \( x \)-axis is assumed to be in the direction of the flow and the \( y \)-axis to be perpendicular to it. The temperature at the sheet (\( T_w \)) surface is larger than the ambient temperature (\( T_\infty \)). The base fluid is water and the considered nanoparticles include \( CuO, Cu, Al_2O_3, TiO_2, Ag \) and \( SiO_2 \). For an incompressible viscous fluid flow with variable properties, the governing equations based on the Boussinesq approximation can be written as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]
\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho_{ul}} \left( \mu_{ul} \frac{\partial^2 u}{\partial y^2} + g \left( \rho \lambda \right)_{ul} (T - T_u) - \sigma B^2(x) u \right)
\]
(2)

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{ul}}{(\rho C_p)_{ul}} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{ul}}{(\rho C_p)_{ul}} \left( \frac{\partial u}{\partial y} \right)^2
\]
(3)

Fig. 1 Schematic of the physical model and coordinate system.

The boundary conditions are:

\[
u = u = b y^m, \quad v = 0, \quad T = T_u, \quad \text{at} \quad y = 0
\]
\[
u \to 0, \quad T \to T_u, \quad \text{at} \quad y \to \infty
\]
(4)

where \( b \) is the constant parameter, \( u \) and \( v \) the velocity components along the \( x \) and \( y \) directions, respectively, \( \sigma \) the electric conductivity, \( B(x) \) the variable magnetic field, \( \rho_{nf} \), \( \mu_{nf} \) and \( \alpha_{nf} \) are the effective density, effective dynamic viscosity, and effective diffusivity, respectively (24).

\[
p_{ul} = (1 - \phi) \rho_f + \phi \rho_s
\]
(5)

\[
\mu_{ul} = \frac{\mu_f}{(1 - \phi)^{2.5}}
\]
(6)

\[
(\rho \lambda)_u = (1 - \phi) (\rho \lambda)_f + \phi (\rho \lambda)_s
\]
(7)

\[
\alpha_{ul} = \frac{k_{ul}}{(\rho C_p)_{ul}}
\]
(8)

where \( \phi \) is the solid volume fraction, \( \mu_f \) the dynamic viscosity of the basic fluid, \( \lambda_f \) and \( \lambda_s \) are the thermal expansion coefficients of the base fluid and nanoparticle, respectively, \( \rho_f \), \( \rho_s \), \( (C_p)_f \) and \( (C_p)_s \) are the density of basic fluid, the density of the nanoparticle, the heat capacity of the basic fluid and the heat capacity of the nanoparticle, respectively. \( k_{nf} \) is the thermal conductivity of the nanofluid. The following expressions are applicable.

\[
(\rho C_p)_{ul} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_s
\]
(9)

\[
\frac{k_{ul}}{k_f} = \frac{k_f + 2k_s - 2\phi(k_f - k_s)}{k_f + 2k_f + 2\phi(k_f - k_s)}
\]
(10)
where \( k_f, k_s \) are the thermal conductivity of the base fluid and nanoparticles, respectively.

Here, the functional form of magnetic field is as
\[
B(x) = B_0 \sqrt{x^{m-1}} \quad (23)(26)
\]

The following dimensionless similarity variable is used to transform the governing equations into the ordinary differential equations
\[
\eta = \frac{y}{x} \sqrt{\frac{m+1}{2}} (Re_\eta)^\frac{1}{2}
\]
where
\[
Re_\eta = \frac{\rho_f \mu_w(x)}{\mu_1} x.
\]

The dimensionless stream function and dimensionless temperature are:
\[
f(\eta) = \frac{\psi(x,y)(Re_\eta)^\frac{1}{2}}{u_w(x)}
\]
\[
\theta(\eta) = \frac{T - T_w}{T_e - T_w}
\]
where the stream function \( \psi(x,y) \) satisfies the Eq. (1)
\[
u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}
\]

By applying the similarity transformation parameters, the momentum Eq. (2) and energy Eq. (3) can be reduced to
\[
f_{\eta\eta} + (1 - \phi)^2 \left\{ \left[ (1 - \phi) + \phi \left( \frac{\rho_f}{\rho_t} \right) \right] f_{\phi\eta} - 2\phi f_{\phi\phi} \right\} - Mnf_{\phi} + \left\{ [(1 - \phi) + \phi \left( \frac{\rho_f}{\rho_t} \right)] (\frac{Gr}{Re_\eta}) \phi \right\} = 0
\]
\[
\frac{1}{(1 - \phi) + \phi \left( \frac{\rho_f}{\rho_t} \right)} \left( \frac{k_w}{k_t} \frac{\theta_{\eta\eta}}{\theta_{\eta\eta}} + \frac{1}{(1 - \phi)^2} EcPrf_{\phi\phi} + Prf \theta_{\phi\phi} \right) = 0
\]

Therefore, the transformed boundary conditions are:
\[
f_{\eta}(0) = 1, f(0) = 0, \theta(0) = 1
\]
\[
f_{\eta}(\infty) = 0, \theta(\infty) = 0
\]

The dimensionless parameters of \( \beta, Mn, Gr(Re_\eta)^2, Pr, Ec, Nu \) and \( C_f \) are the stretching parameter, magnetic parameter, Richardson, Prandtl, Eckert, Nusselt numbers and stretching sheet friction coefficient, respectively. Their definitions are
\[
Pr = \frac{\mu_t(C_v)_T}{k_t} \quad \beta = \frac{2m}{m+1} \quad Mn = \frac{2\sigma B_0^2}{\rho_\infty b(m+1)}
\]
\[
Ec = \frac{u_w(x)^2}{C_k(T_w - T_\infty)} \quad Gr = \frac{g(T_w - T_\infty) \lambda}{u_w(x)^2 x^2} \quad Nu = \sqrt{\frac{m+1}{2} Re_\eta \theta_{\eta}(0)}
\]
\[ C_r = -\frac{2(m+1)}{\text{Re}_x} f_{m}(0) \]  

(19)

3. Entropy generation analysis

The local volumetric rate of entropy generation in the presence of a magnetic field for nanofluids can be expressed as (27)

\[ S_G = \frac{k_{nf}}{T_e} \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \frac{\mu}{T_e} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_s^2}{T_e} u^2 \]  

(20)

The contributions of three sources of entropy generation are considered in Eq. (20). The first term indicates the entropy generation due to heat transfer across a finite temperature difference, the second term the local entropy generation due to viscous dissipation, and the third term the local entropy generation due to the effect of the magnetic field. A dimensionless number for entropy generation rate \( N_s \) is defined as the ratio of the local volumetric entropy generation rate \( (S_{G,nf}) \) to a characteristic entropy generation rate \( (S_{G,0}) \).

For a prescribed boundary condition the characteristic entropy generation rate is

\[ (S_{G,0}) = \frac{k_{nf} (\Delta T)^2}{x T_e} \]  

(21)

Hence, the entropy generation number is

\[ N_s = \frac{S_G}{(S_{G,0})} \]  

(22)

Here using Eqs. (11), (12), (13) and (20), it can be expressed as

\[ N_s = \frac{Br \text{Re}}{\Omega} \left( \frac{y}{T_e} \right)^2 + \frac{Br (Ha)}{\Omega} \left( \frac{y}{T_e} \right)^2 + \frac{Re \theta_T^2}{\Omega} \]  

(23)

where \( \text{Re}_e \) denotes the Reynolds number, \( Br_{nf} \) the Brinkman numbers, \( \Omega \) the dimensionless temperature difference, and \( Ha_{nf} \) the Hartman number. These numbers can be written as

\[ Br = \frac{\mu u^2}{k_{nf} \Delta T}, \Omega = \frac{\Delta T}{T_e}, Ha = B_s x \left( \frac{\sigma}{\mu} \right)^{\frac{1}{2}} \]  

(24)

4. Numerical Method

Here three non-dimensional equations including momentum, energy, and entropy generation equations for mixed convection of nanofluid over a non-linear stretching sheet are considered in the presence of viscous dissipation and variable MHD. The equations are
transformed into three non-linear coupled ordinary differential equations using similarity variables. The resultant system of first-order differential equations are solved numerically using an efficient implicit finite-difference scheme based on the Keller-Box algorithm. The non-linear discretized system of equations is linearized using Newton’s method. Since the system of obtained equations is a block-tridiagonal, the block-tridiagonal-elimination technique is carried out for solution. A step size of \( \Delta \eta = 0.005 \) is selected to satisfy the convergence criterion of \( 10^{-4} \) in all cases. In this solution, \( \eta_\infty = 5 \) is sufficient to satisfy the boundary layer thickness.

5. Results and Discussions

Here, entropy generation for MHD mixed convection heat transfer of nanofluid along a vertical non-linear stretching sheet has been considered. The effect of volume fraction of nano particles, stretching parameter, Eckert number, Richardson number, Reynolds number, Brinkman number and Hartman number are considered.

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>Fluid Phase (water)</th>
<th>SiO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>997.1</td>
<td>3970</td>
</tr>
<tr>
<td>( C_p ) (J/kg.K)</td>
<td>4179</td>
<td>765</td>
</tr>
<tr>
<td>( \lambda \times 10^5 ) (1/K)</td>
<td>21.0</td>
<td>0.63</td>
</tr>
<tr>
<td>( k ) (W/m.K)</td>
<td>0.613</td>
<td>36.0</td>
</tr>
</tbody>
</table>

In Table 1, thermal properties of nano particle used in the present work are indicated. Also, the values of Nusselt number and stretching sheet friction coefficient for different physical parameters are shown in Table 2. As already mentioned, in the numerical results obtained here, \( f_\eta \) denotes the non-dimensional velocity and \( \theta \) indicates the non-dimensional temperature.

![Fig.2 Dimensionless velocity profiles for different values of Richardson number and magnetic parameter, \( Ec=0.20, \beta=1.0, \phi=0.20, Pr=6.20 \).](image-url)
Table 2 Skin friction coefficient and Nusselt number for different values of the physical parameters. \( SiO_2\)-Water, \( Pr=6.2 \)

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \beta )</th>
<th>( Ec )</th>
<th>( Mn )</th>
<th>( Gr/(Re_c)^2&lt;&lt;1 )</th>
<th>( Gr/(Re_c)^2=1 )</th>
<th>( Gr/(Re_c)^2&gt;&gt;1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_f )</td>
<td>( Nu )</td>
<td>( C_f )</td>
<td>( Nu )</td>
<td>( C_f )</td>
<td>( Nu )</td>
<td>( C_f )</td>
</tr>
<tr>
<td>0.2</td>
<td>1</td>
<td>0.02</td>
<td>0.50</td>
<td>0.0949</td>
<td>6.6746</td>
<td>0.0696</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.75</td>
<td>0.1009</td>
<td>7.9201</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>1.0</td>
<td>0.1065</td>
<td>9.1075</td>
<td>0.0849</td>
</tr>
<tr>
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<td>1.0</td>
<td>0</td>
<td>0.1065</td>
<td>9.6732</td>
<td>0.0850</td>
</tr>
<tr>
<td></td>
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<td>0.0849</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( Ec )</td>
<td>( Mn )</td>
<td>( \phi )</td>
<td>( C_f )</td>
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<td>( C_f )</td>
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<tr>
<td>1</td>
<td>0.02</td>
<td>1.0</td>
<td>0.0</td>
<td>0.1065</td>
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<td>0.0849</td>
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<tr>
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<td>0.0831</td>
<td>5.8316</td>
<td>0.0716</td>
</tr>
</tbody>
</table>

Fig.2 illustrates the effect of magnetic parameter on the non dimensional velocity at three different Richardson numbers. As can be seen from the figure, by increasing the Richardson number, the hydrodynamic boundary layer thickness increases, which results in lower shear stress on the wall. The reason is due to the fact that the higher the Richardson numbers, the more the natural convection effect occurs, which follows the reduction in the velocity gradient. Also, the reduction in magnetic parameter causes an increase in the boundary layer thickness due to the reduction of Lorentz effect.

Fig.3 Dimensionless velocity profiles for different values of Richardson number and nanoparticles volume fraction, \( Ec=0.20, \beta=1.0, Mn=1.0, Pr=6.20, Gr/(Re_c)^2=1 \).

In Fig. 3, the non dimensional velocity profile is shown at different Richardson numbers for different volume fraction of nano particles. As is evident from the figure, when the
Richardson number is lower than or equal to one, the increase in nano particle volume fraction causes the increase in the hydrodynamic boundary layer thickness. While, at high Richardson numbers, the increase in nano particles volume fraction near the wall causes the reduction of boundary layer thickness.

As can be seen from Fig. 4, when the stretching parameter varies from 1 to -1, the hydrodynamic boundary layer thickness thickens. In addition, it can be understood that the stretching parameter has more effects on the velocity profile for Richardson numbers higher than unity. Furthermore in the case natural convection \((Gr/Re)^2 \gg 1\) when the stretching parameter gets a minus value a peak is appeared in the dimensionless velocity. This peak shows that the velocity of nanofluid near the sheet is more than the stretching velocity of the sheet. It is physically reasonable because as the Richardson number increases the buoyancy effect that invigorates the fluid movement, overcomes to the viscous forces. Fig. 5 shows the effect of Eckert number at different Richardson numbers on the velocity profile. As is evident from the figure, the effect of Eckert number on the boundary layer thickness is negligible at the Richardson numbers lower than or equal to unity. The reason associated to this is that the convection is dominated by the forced convection flow and the order of
magnitude of inertia forces on the velocity is more than shear forces, while at Richardson numbers higher than unity, the increase in $Ec$ number causes the boundary layer thickness becomes thicker.

Fig.6 Dimensionless temperature profiles for different values of Richardson number and magnetic parameter, $\beta=1.0$, $\varphi=0.20$, $Ec=0.20$, $Pr=6.20$.

Fig.7 Dimensionless temperature profiles for different values of Richardson number and nanoparticles volume fraction, $\beta=1.0$, $Mn=1.0$, $Ec=0.20$, $Pr=6.20$.

Fig.8 Dimensionless temperature profiles for different values of Richardson number and stretching parameter, $\varphi=0.20$, $Mn=1.0$, $Ec=0.20$, $Pr=6.20$. 
In Fig. 6, the non dimensional temperature profiles versus variation of magnetic parameter and Richardson numbers are plotted. The numerical results obtained indicate that an increase in the magnetic parameter causes the increase in the thermal boundary layer thickness due to the Laurent forces effect. The Laurent force increases the nanofluid resistance which causes the temperature increase. Also, an increase in the Richardson number causes the decrease in the thermal boundary layer thickness, which results in a sharp temperature gradient at the wall and higher heat transfer rate.

Fig. 7 shows the temperature distribution at different Richardson numbers for various SiO$_2$ volume fractions. As can be seen from the figure, the increase in volume fraction results in the thermal boundary layer thickens due to the increase of nanoparticles momentum.

The effect of stretching parameter ($\beta$) on the thermal boundary layer is represented in Fig. 8 considering different Richardson numbers. As is evident from the figure, the larger stretching parameter causes thicker thermal boundary layer. Fig. 9 shows that as the Eckert number increase the thermal boundary layer thickens. This phenomenon occurs because increasing the Eckert number means the increase of viscous dissipation that is reason for the temperature enhancement in the boundary layer.

The entropy generation is also studied at different non-dimensional parameters involved in this study to show how the thermal boundary develops. In Fig. 10 the effect of magnetic parameter on the entropy generation number is depicted. As is evident from the figure, the
Fig. 10 Dimensionless entropy generation number profiles for different values of magnetic parameter, $\varphi=0.20$, $Gr(Re_x)^{\beta}<1$, $\beta=1.0$, $Pr=6.20$.

Fig. 11 Dimensionless entropy generation number profiles for different values of nanoparticles volume fraction, $Mn=1.0$, $Gr(Re_x)^{\beta}<1$, $\beta=1.0$, $Pr=6.20$.

Fig. 12 Dimensionless entropy generation number profiles for different values of Eckert number, $\varphi=0.20$, $Gr(Re_x)^{\beta}<1$, $\beta=1.0$, $Pr=6.20$, $Mn=1.0$. 
entropy generation is mostly intensified near the wall at which the magnetic parameter causes higher dissipation energy than farther from the wall. Also as the magnetic parameter increases the entropy generation near the wall increases while the entropy generation far from the wall is not affected seriously. The reason for this phenomenon is that increasing the magnetic parameter causes the resistant forces against the fluid movement and then heat transfer rate in the boundary layer enhances. Fig. 11 indicates the effect of the nanoparticles volume fraction on the entropy generation number. As the volume fraction decreases, the entropy generation increases near the wall due to the higher dissipation energy resulted from the sharper velocity gradient near the wall. However, this reverses far enough from the wall. The Eckert number effect on the entropy generation number indicates that the heat transfer plays more remarkable role in thermodynamic irreversibility than the viscous dissipation. This can be seen in Fig. 12 at wall which is the main source of irreversibility due to the presence of sharp temperature gradient. When the Eckert number reduces from 0.3 to 0 the entropy generation near the sheet increases and the entropy at far from this reduces. Fig. 13 shows that the entropy generation number increases with the increase of Hartman number. This is due to the increase of Lorentz forces which strengthen the dissipation energy as a source of irreversibility. The effects of the non-dimensional parameter $Br\Omega^{-1}$ and the Reynolds number on the entropy generation number are represented in Figs. 14 and 15 respectively. The increase of both non-dimensional parameters results in the entropy generation increase. The reason for increasing the entropy generation with Reynolds number is that the increase of Reynolds number disturbs the fluid and then the chaos appears in the fluid movement.

Fig. 13 Dimensionless entropy generation number profiles for different values of Hartman number, $\varphi=0.20$, $Gr/(Re_c)^{2}<<1$, $\beta=1.0$, $Pr=6.20$, $Mn=1.0$. 

![Fig. 13](image-url)
6. Conclusions

In this work, a numerical study of entropy generation for MHD mixed convection of nanofluid over a non-linear stretching sheet including viscous dissipation is considered. The nanofluid is assumed to be made of nano particles as $SiO_2$ with pure water as a base fluid. The values of Nusselt and drag coefficients at different non dimensional parameters consisting of stretching and magnetic field effects, Eckert and Richardson numbers, and volume fraction of nano particles are investigated in detail. The results show that adding nano particles to the base fluids in forced, natural, and mixed convection would cause a reduction in shear force and a decrease in stretching sheet heat transfer coefficient. Also, in nanofluid the decrease in magnetic parameter and increase in Eckert number cause better thermal conditions. In addition, the entropy generation number is higher near the surface at which a strong irreversibility occurs.
References


