Numerical Simulation of Non-Equilibrium Conjugate Heat Transfer in Tubes Partially Filled with Metallic Foams*

Zhiguo QU**, Huijin XU** and Wenquan TAO**

**Key Laboratory of Thermo-Fluid Science and Engineering of MOE, School of Energy and Power Engineering, Xi’an Jiaotong University, Xi’an, Shaanxi, 710049, China
E-mail: zgqu@mail.xjtu.edu.cn

Abstract

Numerical simulation with the Forchheimer flow model and local thermal non-equilibrium model for porous region is performed on forced convective heat transfer in a tube partially filled with metallic foams. Flow and heat transfer of fluid in the hollow region and those of fluid in the porous region are conjugated together via the coupling conditions at porous-fluid interface. A heat flow model is proposed with special numerical treatments employed for non-equilibrium conjugated heat transfer in foam-fluid system. Velocity and temperature profiles in the flow direction are obtained and validated with analytical results. Effects of porosity, pore density, dimensionless interfacial radius and fluid-to-solid thermal conductivity ratio on flow characteristics and thermal performance are examined. Accordingly, the entrance effect is analyzed through the numerical simulation in terms of both flow and heat transfer. The present tube exhibits more excellent heat transfer performance at the expense of moderate pressure drop compared with the tube without porous material. The numerical work is not only developed for forced convection in metal-foam partially filled tube, but can also be extended to similar problem with porous-fluid interface for other porous media with significant thermal non-equilibrium effect.

Key words: Numerical Simulation, Porous Media, Forced Convection, Heat Transfer Enhancement, Coupling Problem

1. Introduction

Recently, highly porous metallic foams become a research interest for their advantages of high thermal conductivity, light weight, excellent mechanical property, high surface area density and good flow mixing capability. They have potential applications in a number of fields including filters, cooling of nuclear reactors, chemical reactors, combustion chambers, reformers, bio-medical implants, compact heat exchangers and heat sinks (1)-(3). Utilizing metallic foam for heat transfer enhancement is not a new method, but the high cost for manufacturing metallic foams stops it being applied in industries. The development of metal-foam co-sintering techniques makes the use of metallic foam for heat transfer enhancement possible. Especially, the successful implementation of various types of heat sinks and heat exchangers with metallic foams offers attractive research outlook for convective heat transfer in metallic foams. Flow and heat transfer in a duct fully filled with porous media have been extensively investigated (2)-(13). Calmidi and Mahajan (4) presented experimental and numerical studies on forced convection in metallic foams with local...
convective heat transfer coefficient \( h_{sd} \) and thermal dispersion conductivity \( k_d \) determined. Zhao et al. \(^5\) performed experimental and numerical study on forced convection in metallic foam filled rectangular duct. Lu and Zhao et al. \(^6\) analytically investigated the metallic foam filled tube and annulus with the Brinkman-Darcy model. Dukhan and Chen \(^7\) conducted analytical and experimental study on forced convective heat transfer in a parallel-plate channel fully filled with metallic foams. DeGroot et al. \(^8\) undertook a numerical study to explore details of forced convection in finned aluminum foam heat sinks. Du et al. \(^9\) performed numerical study on heat transfer in metal-foam double-pipe heat exchangers with parallel-flow arrangement. Mahjoob and Vafai \(^3\) summarized different flow and heat transfer models for metallic foams appearing in existing literatures. Nakayama et al. \(^10\) analytically investigated the one-dimensional convective and conductive heat transfer problems in fluid saturated porous media. Yang et al. \(^11\) investigated the forced convective heat transfer in tubes fully filled with metallic foams and obtained the exact solution considering thermal dispersion. Kuwahara et al. \(^12\) presented exact solution for forced convective heat transfer in a parallel-plate channel fully filled with metallic foams and indicates that the local thermal non-equilibrium model should be considered for constant heat flux boundary conditions rather than constant temperature boundary conditions. Yang et al. \(^13\) provided the analytical solution for forced convective heat transfer in annuli fully filled with metallic foams with the local thermal non-equilibrium model. Overall, the heat transfer augmentation for metal-foam fully filled duct is attractive, but the pressure loss created by the porous structure, which is usually three to five orders of magnitude higher than that of empty duct or more, exceeds the acceptable scope for most engineering applications. To this end, the configuration of metallic-foam partially filled tube is investigated in this work.

For conjugated problem for flow and heat transfer in a domain consisting of a porous-fluid interface, most previous studies are limited to either flow coupling \(^14\) or thermal equilibrium coupling at the porous-fluid interface. Recently, with the extensive use of highly porous metallic foams with high solid thermal conductivities, thermal performance estimation of related applications are urgently needed but the conventional equilibrium heat transfer model cannot provide precise prediction. Barletta and Rees \(^15\) investigated the Darcy-Benard instability with uniform heat flux condition using the linear stability analysis considering local thermal non-equilibrium model. Alazmi and Vafai \(^16\) numerically investigated the forced convective heat transfer through parallel-plates fully filled with porous media with the local thermal non-equilibrium model and discussed the effects of different parameters on heat transfer with variant boundary conditions for constant heat flux. Lee and Vafai \(^17\) indicated that the thermal non-equilibrium model is more accurate than the thermal equilibrium model when the difference between thermal conductivity of fluid and that of solid is significant, as is the case for air flow in metallic foams with high solid thermal conductivities. Thus, numerical simulation for conjugated non-equilibrium heat transfer in a porous-fluid circular duct is currently handled.

Only a few publications \(^18\)-\(^20\) can be found for comprehensive conjugated flow and non-equilibrium heat transfer in the presence of a porous-fluid interface. Ochoa-Tapia and Whitaker \(^18\) theoretically proposed a set of coupling conditions for flow and heat transfer in a porous-fluid domain with the local thermal equilibrium model and the local thermal non-equilibrium model. Phanikumar and Mahajan \(^19\) performed numerical simulation on free convection on a horizontal plate adhered with a porous layer. Lee and Anthony \(^20\) theoretically proposed a discretizing method for a domain involving a porous-fluid, porous-solid or fluid-solid interface. However, it was numerically treated in that paper only for Beavers and Joseph flow problem and thermal non-equilibrium problem for heat transfer in a porous medium fully fitted domain without any internal interface. Yang et al. \(^21\) numerically investigated the forced convective heat transfer in a parallel-plated channel
sintered with porous fins using the commercial software CFX10.0. However, interfacial conditions in that study were not strictly considered. Overall, numerical techniques for non-equilibrium conjugate heat transfer in the porous-fluid system needs to be specifically developed and implemented.

The aim of the present study is to present numerical model for forced convective heat transfer in a uniformly-heated tube partially filled with metallic foams based on Forchheimer flow model and local thermal non-equilibrium model, in which the thermal dispersion effect is considered. With some special numerical treatments, the numerical simulation was performed in condition of inequality between temperature variable numbers in the foam region and the hollow region. Thus, a new numerical method is then put forward for handling non-equilibrium conjugated heat transfer in a domain containing foam-fluid interface.

**Nomenclature**

- \(a_{sf}\) : specific surface area of metallic foam, \(m^{-1}\)
- \(C_i\) : inertial coefficient
- \(c\) : specific heat capacity, \(J \cdot kg^{-1} \cdot K^{-1}\)
- \(Da\) : Darcy number
- \(h\) : heat transfer coefficient, \(W \cdot m^{-2} \cdot K^{-1}\)
- \(h_f\) : convective heat transfer coefficient between fluid and solid, \(W \cdot m^{-2} \cdot K^{-1}\)
- \(K\) : permeability, \(m^2\)
- \(k\) : thermal conductivity, \(W \cdot m^{-1} \cdot K^{-1}\)
- \(k_d\) : dispersion thermal conductivity, \(W \cdot m^{-1} \cdot K^{-1}\)
- \(k_{ef}\) : effective thermal conductivity of fluid, \(W \cdot m^{-1} \cdot K^{-1}\)
- \(k_\varepsilon\) : thermal conductivity ratio \((k_\varepsilon = k_f/k_s)\)
- \(k_{es}\) : effective thermal conductivity of solid, \(W \cdot m^{-1} \cdot K^{-1}\)
- \(L\) : tube length, \(m\)
- \(Nu\) : Nusselt number
- \(\vec{n}_p\) : unit vector normal to the porous-fluid interface pointing to the fluid side
- \(Pr\) : Prandtl number
- \(p\) : pressure, \(N \cdot m^{-2}\)
- \(q\) : heat flux, \(W \cdot m^{-2}\)
- \(R\) : dimensionless radius
- \(Re\) : Reynolds number
- \(r\) : radius, \(m\)
- \(r_t\) : tube radius, \(m\)
- \(T\) : temperature, \(K\)
- \(U\) : dimensionless axial velocity component
- \(u\) : axial velocity component, \(m \cdot s^{-1}\)
- \(V\) : dimensionless radial velocity component
- \(v\) : radial velocity component, \(m \cdot s^{-1}\)
- \(X\) : dimensionless axial position
- \(x\) : axial position, \(m\)
- \(\Theta\) : dimensionless temperature
- \(\rho\) : density, \(kg \cdot m^{-3}\)
- \(\varepsilon\) : porosity
- \(\omega\) : pore density, \(PPI\) (pores per inch)
- \(\mu\) : dynamic viscosity, \(Pa \cdot s\)

**Subscripts**

- \(b\) : bulk
- \(d\) : dispersion
- \(f\) : fluid
ρ: effective value of fluid phase

i: interface

in: inlet

m: mean

s: solid

se: effective value of solid phase

w: wall

x: at x position

2. Numerical Procedure

2.1 Problem description

The geometry of the tube partially filled with metallic foam is shown in Fig. 1. The metal-foam layer is sintered on the inner surface of the tube, toward the center-line. Incompressible fluid with constant thermal-physical properties flows through a long tube partially and annularly filled with metallic foams, which is subjected to uniform surface heat flux \( q_w \). The tube radius is \( r_0 \) and tube length is \( L \). The radius of porous-fluid interface is \( r_i \). The fluid frontal velocity and temperature are \( u_m, T_{in} \) respectively. The metallic foams are assumed to be homogeneous and isotropic to simplify the numerical simulation of present problem.

2.2 Governing equations

For metallic foam region, the Forchheimer flow model was adopted to establish the momentum equation and the local thermal non-equilibrium model was used to represent the heat exchange between solid and fluid in porous region. In the metallic foam region, the continuity equation, the momentum equations and the two energy equations are shown in the following.

\[
\nabla \left( \rho \langle \bar{u} \rangle \right) = 0 \tag{1a}
\]

\[
\frac{1}{\varepsilon^2} \left( \rho \langle \bar{u} \rangle \right) = -\nabla \langle \rho \rangle_i + \frac{H_0}{\varepsilon} \langle \bar{u} \rangle - \frac{H_0}{K} \langle \rho \rangle_i \nabla \langle \bar{u} \rangle - \frac{H_0}{K} \langle \rho \rangle_i \nabla \langle \bar{u} \rangle \tag{1b}
\]

\[
\nabla \left[ k_u \nabla \langle T_r \rangle \right] - h_a a_e \langle \langle T_r \rangle - \langle T_i \rangle \rangle = 0 \tag{1c}
\]

\[
\langle \rho \rangle c_i \langle \bar{u} \rangle \nabla \langle T_r \rangle = \nabla \left[ (k_u + k_s) \nabla \langle T_r \rangle \right] + h_a a_e \langle \langle T_r \rangle - \langle T_i \rangle \rangle \tag{1d}
\]

where the angle brackets represent volume-averaging values. For the fluid region, the classical Navier-Stokes equation was employed for momentum equation, as

\[
\nabla \left( \rho \langle \bar{u} \rangle \right) = 0 \tag{2a}
\]

\[
\left( \bar{u} \cdot \nabla \right) \rho \langle \bar{u} \rangle = -\nabla p_i + \mu \nabla^2 \langle \bar{u} \rangle \tag{2b}
\]
\[ \rho c \bar{u} \nabla T_i = \nabla (k_i \nabla T_i) \]  

(2c)

The boundary conditions for numerical simulation in the cylindrical coordinate system are presented in Eqs.(3a)-(3d).

\[ x = 0: \ u = u_n, \ v = 0, \ T_i = T_{in}, \ \frac{\partial T_i}{\partial x} = 0 \]  

(3a)

\[ x = L: \ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial T_i}{\partial x} = 0 \]  

(3b)

\[ r = 0: \ \frac{\partial u}{\partial r} = 0, \ v = 0, \ \frac{\partial T_i}{\partial r} = \frac{\partial T_s}{\partial r} = 0 \]  

(3c)

\[ r = r_h: \ u = v = 0, \ q_s = k_u \frac{\partial T_i}{\partial r} + k_s \frac{\partial T_s}{\partial r}, \ T_i = T_s \]  

(3d)

As indicated in Eq.(3d), the total heat flux includes heat condition by solid and that by fluid. The value of each part is determined by the interface energy balance and can be obtained through numerical solution. Also, the commonly-used thermal equilibrium condition \( T_f = T_s \) was applied at \( r = r_h \). Empirical correlations of important parameters, permeability \( K \), inertial coefficient \( C_I \), specific surface area \( a_s \), dispersion thermal conductivity \( k_d \), local convective heat transfer coefficient \( h_l \), effective thermal conductivities of solid and fluid \( k_{se} \) and \( k_{fe} \), can be found from corresponding literature.

2.3 Interfacial coupling conditions

For simplicity, the angle brackets for volume-averaged variables in the metallic foam region are dropped. The no-slip interfacial coupling conditions of velocity and shear stress are well accepted for fluid flow, as shown below:

\[ u \bigg|_r = u \bigg|_r \]  

(4)

\[ \mu \bigg[ \frac{\partial u}{\partial r} \bigg]_r = \mu \bigg[ \frac{\partial u}{\partial r} \bigg]_r \]  

(5)

With non-equilibrium heat transfer condition, the continuities of fluid temperature and heat flux across the porous-fluid interface are well guaranteed in the numerical procedure.

\[ T_i \bigg|_r = T_s \bigg|_r \]  

(6)

\[ \left[ k_s \frac{\partial T_i}{\partial r} + (k_u + k_s) \frac{\partial T_i}{\partial r} \right]_r = k_f \frac{\partial T_s}{\partial r} \]  

(7)

According to Ochoa-Tapia and Whitaker \(^{(18)} \), the heat transfer between the solid phase and the fluid phase in the neighborhood of the clear fluid region can be expressed as:

\[ \bar{n}_{ip,f} \left( k_u \nabla T_i \bigg|_r \right) = h_{ip} \left( T_i \bigg|_r - T_s \right) \]  

(8)

Owing to the fact that the unit vector on the left hand side is normal to the porous-fluid interface pointing to the fluid side, Eq.(8) can be simplified as Eq.(9) for the present problem shown in Fig.1.

\[ k_u \frac{\partial T_i}{\partial r} = h_{ip} \left( T_i \bigg|_r - T_s \right) \]  

(9)

For the reason that the solid ligaments are discontinuous at the foam-fluid interface, heat conduction through the solid phase is totally transferred to the fluid in the manner of convective heat transfer across the porous-fluid interface. Hence, the physical meaning of Eq.(21b) is the convective heat transfer at the foam-fluid interface from solid ligament to the fluid nearby.

The schematic diagram for heat flow in the foam partially filled tube is shown in Fig.2. It should be indicated that heat is mainly transferred from the tube-wall to the center-line since the axial heat conduction either by fluid or by solid is very low. It can be seen that the
heat conducted by solid phase in the foam region from tube-wall to foam-fluid interface is dissipated by the fluid flowing through the foam region in the manner of the local convective heat transfer. In the fluid region, heat is only transferred by fluid from the foam-fluid interface to the center-line.

\[ q_w \]

\[ \text{fluid flow} \]
\[ \text{foam-fluid interface} \]

\[ \mathbf{r} \]

2.4 Normalization

Governing equations can be normalized with the following dimensionless qualities:

\[ X = \frac{x}{r_0}, \quad R = \frac{r}{r_0}, \quad U = \frac{u}{u_m}, \quad V = \frac{v}{v_m}, \quad \frac{\theta_{0i}}{\theta_m} = \frac{T_{0i} - T_m}{q_{s0}/k_t}, \quad P = \frac{p}{\rho_1 u_m^2}, \quad D_R = \frac{K}{r_0^2} \]  

\[ (10a) \]

\[ \alpha = \frac{k_0}{k_t}, \quad \alpha_a = \frac{k_1}{k_t}, \quad \alpha_s = \frac{k_s}{k_t}, \quad N_a = \frac{h_a k_0}{k_t}, \quad Re = \frac{\rho_1 u_m^2 r_0}{\mu_1}, \quad Pr = \frac{\mu_1}{\rho_1 c_1} \]  

\[ (10b) \]

Thus, the normalized momentum and energy equations are shown below. In foam region,

\[ \frac{1}{\varepsilon} \left( \frac{\partial^2 (U)}{\partial X^2} + \frac{1}{R} \frac{\partial (RUV)}{\partial R} \right) = \frac{\partial P}{\partial X} + \frac{2}{\varepsilon} \left[ \frac{\partial^2 U}{\partial X^2} + \frac{1}{R} \frac{\partial (RUV)}{\partial R} \right] \frac{2}{Da - Re} U - \frac{C_s}{\varepsilon} \frac{\partial (q_w)}{\partial X} \]  

\[ (11a) \]

\[ \frac{1}{\varepsilon} \left( \frac{\partial (U V)}{\partial X} + \frac{1}{R} \frac{\partial (R V^2)}{\partial R} \right) = \frac{\partial P}{\partial X} + \frac{2}{\varepsilon} \left[ \frac{\partial^2 V}{\partial X^2} + \frac{1}{R} \frac{\partial (RUV)}{\partial R} \right] \frac{2}{Da - Re} V - \frac{C_s}{\varepsilon} \frac{\partial (q_w)}{\partial X} \]  

\[ (11b) \]

\[ 0 = \alpha_a \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{1}{R} \frac{\partial (R \theta)}{\partial R} \right) - \frac{1}{\varepsilon} \right) \frac{\partial (q_w)}{\partial X} \]  

\[ (11c) \]

In clear fluid region,

\[ \frac{\partial^2 (U)}{\partial X^2} + \frac{1}{R} \frac{\partial (RUV)}{\partial R} = \frac{\partial P}{\partial X} + \frac{2}{\varepsilon} \left[ \frac{\partial^2 U}{\partial X^2} + \frac{1}{R} \frac{\partial (RUV)}{\partial R} \right] \]  

\[ (12a) \]

\[ \frac{\partial (U V)}{\partial X} + \frac{1}{R} \frac{\partial (R V^2)}{\partial R} = \frac{\partial P}{\partial X} + \frac{2}{\varepsilon} \left[ \frac{\partial^2 V}{\partial X^2} + \frac{1}{R} \frac{\partial (RUV)}{\partial R} \right] \]  

\[ (12b) \]

\[ \frac{\partial (U \theta)}{\partial X} + \frac{1}{R} \frac{\partial (R \theta)}{\partial R} = \frac{2}{Re \cdot Pr} \left[ \frac{\partial^2 \theta}{\partial X^2} + \frac{1}{R} \frac{\partial (R \theta)}{\partial R} \right] \]  

\[ (12c) \]

Dimensionless boundary conditions are as follows:

\[ X = 0 : \quad U = 1, \quad V = 0, \quad \theta = 0, \quad \frac{\partial \theta}{\partial X} = 0 \]  

\[ (13a) \]

\[ X = L/r_0 : \quad \frac{\partial U}{\partial X} = \frac{\partial V}{\partial X} = \frac{\partial \theta}{\partial X} = 0 \]  

\[ (13b) \]

\[ R = 0 : \quad \frac{\partial U}{\partial R} = 0, \quad V = 0, \quad \frac{\partial \theta}{\partial R} = 0 \]  

\[ (13c) \]

2.5 Numerical method

With SIMPLE algorithm for pressure-velocity coupling, the present problem was solved numerically with a full-field method, i.e. the one-domain method. The finite volume
method was adopted for discretizations of governing equations in the two regions. The normalized governing equations were discretized with the power-law scheme. Harmonic mean formula for computing diffusive coefficients at the co-interface of two adjacent control volumes was employed. The parameters kept constant in the calculation are shown in the Table 1. Tube length was selected as $L = 100r_0$. Variation range of $Re$ is from 250 to 2000. The grid dense independence for numerical solutions can be guaranteed when grid number is $152 \times 62$ from Fig. 3(a). Non-uniform grid system (Fig. 3(b)) was employed, in which grid is made dense in positions with tremendous changes of physical qualities, such as at region near wall and that near porous-fluid interface. Numerical iteration can be treated as convergent when relative variations in pressure drop and Nusselt number between successive iterations are less than $10^{-7}$.

### Table 1 Constant parameters in numerical simulation

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<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
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<td>kg·m$^{-3}$</td>
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<td>$C_1$</td>
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<td>J·kg$^{-1}$·K$^{-1}$</td>
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<td>W·m$^{-1}$·K$^{-1}$</td>
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<td>$\mu_1$</td>
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<td>kg·m$^{-1}$·s$^{-1}$</td>
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</tr>
<tr>
<td>$T_w$</td>
<td>303</td>
<td>K</td>
</tr>
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</table>

Fig. 3 Grid independence check and the non-uniform grid: (a) Grid independence check; (b) non-uniform grid.

#### 2.6 Numerical treatments

Special numerical treatments were employed to unify the conjugate non-equilibrium heat transfer problem including a porous-fluid interface with the grid system of SIMPLE algorithm. For the temperature field in the foam region, there are two variables, fluid temperature and solid temperature. However, in the clear fluid region, the solid temperature does not exist for the case of full-field solving procedure with one-domain approach. To this end, the values of solid temperature were extended to the whole computational domain as follows. The values of solid temperature in the clear fluid region are artificially set. The virtual diffusion term and source term for solid temperature were set to zero. Thus the heat conducted through the solid matrix can be thoroughly transferred to the fluid in the foam region and the virtual values of solid temperature in the clear fluid region have no influence on the final computational results.

Overall, in the fluid region, the advection term, the diffusion coefficient, and the source term are all set to zero for the extended solid energy equation. Thus, even though the solid energy equation is extended to the fluid region, it is actually diminished in the fluid region, which is in accordance with the physical process. The present problem
consists of a porous-fluid interface at which the quasi-third boundary condition exists. To take the heat transferred from the edge of solid ligaments to the fluid nearby at the porous-fluid interface into account, the additional source term method was used, converting the porous-fluid interface area to additional specific area in the adjacent control volume, expressed as:

\[ a_{d,\text{total}} = \frac{A_d + A_{\text{total}}}{\Delta V} = \frac{a_d + \frac{r \Delta r \Delta x + 1}{r \Delta r \Delta x}}{r \Delta r} = \frac{a_d + 1}{\Delta r} \]

(14)

2.7 Parameter definitions

Some other parameters for flow and heat transfer, such as Reynolds number, Local Nusselt number, bulk mean fluid temperature at the cross section and Average Nusselt number are determined in this section, respectively shown in Eqs.(15)-(18).

\[ Re = \frac{\rho_l u_m 2r_0}{\mu_l} \]  

(15)

\[ Nu_x = h_x \frac{2r_0}{k_x} = \frac{q_x}{l_x - \bar{T}_c} \]  

(16)

\[ T_{\text{wall}} = \frac{\int_0^L \int_0^1 T r dr d\phi}{\int_0^L \int_0^1 r dr d\phi} \]  

(17)

\[ Nu_m = h_m \frac{2r_0}{k_m} = \frac{1}{L - L_1} \int_{0}^{L_1} h_d r dr \]  

(18)

3. Results and Discussion

3.1 Validation of numerical method

Based on analytical solution in open literature (25), the present numerical simulation with the heat flow model is validated in terms of velocity and temperature profiles. Fig. 4(a) shows the comparison of velocity profile between result with the present numerical model and that of analytical solution (25). It is found that the velocity in the present numerical simulation coincides with the analytical result very well. Fig.4(b) presents the contrast of the numerical solution and the analytical solution (25) for temperature profile. It is noted that the present numerical result is slightly lower than the analytical solution, which is attributed to the fact that the 2-D numerical solution with axial heat conduction and thermal entrance effect is quite different from the 1-D analytical solution. Thus, the present numerical method for the metal-foam partially filled tube is validated.

3.2 Velocity distribution

2-D dimensionless axial velocity distributions for Re=250 and Re=2000 are respectively shown in Fig.5(a) and Fig.5 (b). It can be seen that at the entrance, the axial
velocity varies sharply for both cases. However, the velocity for \( Re = 250 \) comes to the fully-developed state more easily than that for \( Re = 2000 \). This implies that the flow entrance length is positively increased with Reynolds number augmentation.

Fig.6(a) and Fig.6(b) respectively presents the isolines of dimensionless axial velocity for \( R_i = 0.3 \) and \( R_i = 0.7 \). It is noted that the velocity in the center-line for \( R_i = 0.3 \) is much higher than that for \( R_i = 0.7 \). From the velocity isolines in the foam region, it can be concluded that the flow entrance length for \( R_i = 0.3 \) is longer than that for \( R_i = 0.7 \).

Fig.5 Axial velocity distribution for different \( Re (R_i=0.5, r_0=0.01m, \varepsilon=0.95, \omega=10PPI): \)

(a) \( Re=250 \); (b) \( Re=2000 \).

Fig.6 Axial velocity distribution for different dimensionless interfacial radii \( (Re=1000, r_0=0.01m, \varepsilon=0.95, \omega=10PPI): \)

(a) \( R_i=0.3 \); (b) \( R_i=0.7 \).

Fig.7 Axial velocity distribution for different porosities \( (R_i=0.5, r_0=0.01m, Re=1181, \omega=10PPI): \)

(a) \( \varepsilon=0.8 \); (b) \( \varepsilon=0.94 \).

Fig.8 Axial velocity distribution for different pore densities \( (R_i=0.5, r_0=0.01m, \varepsilon=0.9, Re=1181): \)

(a) \( \omega=5PPI \); (b) \( \omega=20PPI \).

Fig.7(a) and Fig.7(b) show the effect of porosity on the 2-D dimensionless axial velocity \( (\varepsilon=0.8 \text{ and } 0.94) \). It can be seen from these figures that the velocity in the center-line for \( \varepsilon=0.8 \) is slightly higher than that for \( \varepsilon=0.94 \). This can be attributed to that for metal-foam partially filled pipe, the increase in porosity will make permeability of
foam region increased, which thus leads to the relieved flow resistance and the improved flow heterogeneity. Simultaneously, due to the fact that the effect of porosity on the foam permeability is limited, the difference between the velocity isolines of $\varepsilon=0.8$ and $\varepsilon=0.94$ is mild.

Fig.8 shows the effect of pore density on the dimensionless axial velocity distribution (5PPI and 10PPI). It is revealed from Fig.8(a) and Fig.8(b) that the flow field for 5PPI reaches the fully-developed state with a shorter axial distance than that for 10PPI does. This means that the increase in pore density can lead to a shortened entrance for metal-foam partially filled tubes. Moreover, the pore density increase will result in the increased velocity in the fluid region and the decreased velocity in the foam region. This is due to the fact that the increased pore density can make the permeability of foam region decreased significantly.

3.3 Pressure drop

![Graph showing the effect of dimensionless interfacial radius on the global pressure drop.](a)

![Graph showing the effect of Reynolds number on the global pressure drop.](b)

![Graph showing the effect of pore density on the global pressure drop for different tube radii.](c)

![Graph showing the effect of porosity on the global pressure drop.](d)

Fig.9 Effects of important parameters on the global pressure drop: (a) effect of dimensionless interfacial radius; (b) effect of Reynolds number; (c) effect of pore density; (d) effect of porosity.

The effect of dimensionless interfacial radius on the global pressure drop is shown in Fig.9(a). As $R_i$ increases with other variables fixed, the hollow region with low flow resistance is enlarged and the area of foam region with high flow resistance is decreased, which leads to that the pressure drop gradually decreases. From $R_i=0.1$ to $R_i=0.9$, the pressure drop decreases in about two orders of magnitude, which makes possible the concept of using metal-foam to enhance heat transfer with relatively low pressure loss.

Figure 9 (b) illustrates the effect of Reynolds number of the global pressure drop for different $R_i$. Apparently, the curves exhibit no n-linear increasing trend as $Re$ increases, which is attributed to the inertial terms in momentum equations.

Figure 9 (c) presents the effect of pore density on global pressure drop for different tube radii ($r_0=0.01$ m, 0.02 m, and 0.03 m). The result shows that pressure drop increases with an increase in pore density, which is resulted from the gradually decreased permeability with
the continuously increasing pore density. Simultaneously, for a fixed pore density, the larger the tube radius, the lower the global pressure drop. This is attributed to the fact that the cross-sectional area of fluid region increased in proportion greatly eases the flow resistance when the tube radius increases.

Figure 9(d) shows the effect of porosity on the global pressure drop for different pore densities (5 PPI, 10 PPI, 20 PPI and 30 PPI). Obviously, as porosity increases, pressure drop is decreased for the reason that the increase in porosity can lead to the volume fraction of the obstructing solid. As expected, with a fixed porosity, the larger the pore density, the higher the pressure drop.

3.4 Temperature profile

To examine the effect of Reynolds number on the fluid temperature with the present numerical method, Figure 10 presents the constant temperature line for fluid in the condition of \( Re=250 \) and \( Re=2000 \). It can be seen that the isothermal line of fluid for \( Re=250 \) is much gentler than that for \( Re=2000 \). This implies that as Reynolds number increases, the flow field is intensified and thereby governs the temperature field.

Fig.11 gives the constant temperature line of fluid for two dimensionless interfacial radius (\( R_i=0.3 \) and \( R_i=0.7 \)), in which the foam-fluid interface can be roughly estimated. It can be seen both from Fig.11(a) and Fig.11(b) that in the foam region, the fluid temperature variation in the radial direction is not significant while it changes monotonically in the fluid region. Fluid temperature in the fluid region for \( R_i=0.3 \) is much more inhomogeneous than that for \( R_i=0.7 \) in the same region. This is attributed to the fact that the highly-porous fluid region for \( R_i=0.3 \) with smaller cross-sectional area leads to the fluid flow in this region is strongly intensified compared with \( R_i=0.7 \). Thus, temperature field for \( R_i=0.3 \) owns a larger relatively homogeneous foam region than that for \( R_i=0.7 \).

![Fig.10 Fluid temperature for different Re (Re=250, Re=2000).](image1)

![Fig.11 Fluid temperature for different dimensionless interfacial radii (Re=1000, Re=2000).](image2)

To examine the effects of metal-foam morphology parameters (porosity and pore density) on temperature field, Figs.12-13 give the related isothermal line of fluid. Fig.12 presents the fluid constant temperature line for \( \varepsilon =0.8 \) and 0.94. It can be seen that the difference between Fig.12(a) and Fig.12(b) is mild except that the fluid temperature variation in the foam region for \( \varepsilon =0.94 \) is slightly obvious than that for \( \varepsilon =0.8 \). This implies that the increase in porosity can increase the permeability of the foam region,
resulting in the increased fluid velocity in the foam region, but the effect of porosity on temperature field is mild. Figure 13 shows the contours of fluid temperature distribution for different pore densities (ω = 5PPI and 20PPI). It is found that in the foam region, the fluid temperature for 20PPI is more uniform than that for 5PPI, while in the fluid region, the former is much more inhomogeneous than the latter. With an increase in pore density, the permeability in the foam region is decreased, thereby leading to the decreased velocity in the foam region and increased velocity in the fluid region. Thus, the flow heterogeneity in the two regions leads to the result predicted in Fig.13.

![Figure 12 Fluid temperature for different porosities](image1)

![Figure 13 Fluid temperature for different pore densities](image2)

3.5 Heat transfer analysis

In this section, the effect of important parameters, such as porosity, pore density, dimensionless interfacial radius and thermal conductivity ratio $k_r$, on the global Nusselt number of forced convective heat transfer in metal-foam partially filled tubes are examined from Fig.14 (a) to Fig.14(d).

Fig.14(a) and Fig.14(b) respectively give the effect of metal-foam morphology parameters, porosity and pore density, on the global Nusselt number. From Fig.14(a), as porosity increases, the Nusselt number gradually increases for small porosity and then begins to decrease after the maximum $Nu$ maximized at a certain porosity (around 0.94) . As porosity increases, the volume fraction of solid accounting for the vast majority of heat conduction is decreased. However, the fluid velocity in the foam region increases due to the increased permeability of foam region with an increase in porosity. The former plays a negative role for heat transfer while the latter is a positive factor when porosity increases, which jointly leads to the maximized $Nu$. In Fig.14(b), the global Nusselt number sharply decreases for small pore density and then is gradually reduced to a constant value when the pore density is further increased. This is attributed to that the permeability of foam region is sharply decreased when pore density is small and then nearly reduced to zero when pore density is big. This implies that the fluid velocity in the foam region for small pore density is relative high and is reduced to about zero for big pore density, at which the local convective heat transfer from solid to fluid is mild. At this condition, the global Nusselt number keeps constant regardless of the increase of pore density.
The effect of thermal conductivity ratio on the global Nusselt number for different pore densities is shown in Fig. 14(c). As can be seen, for high $k_r$ with the lower solid thermal conductivity, the slope of $Nu-k_r$ curve is nearly zero while it is also nearly zero for small $k_r$ with relative higher solid thermal conductivity. For moderate thermal conductivity ratio, the varying amplitude of $Nu$ with the change of $k_r$ is considerable. The reason is analyzed here. Heat transfer in metallic foams consists of two mechanisms: conductive heat transfer by solid and fluid phases and convective heat transfer from solid to fluid. When thermal conductivity ratio $k_r$ is high (greater than $10^{-1}$), the conductive thermal resistance is responsible for the majority of total thermal resistance since the solid thermal conductivity is relatively low. However, when the thermal conductivity ratio $k_r$ is low (greater than $10^{-3}$), the conductive thermal resistance can be neglected and the convective thermal resistance accounts for the majority of total thermal resistance due to the fact that solid thermal resistance is very high. The two competing factors jointly lead to result predicted from $k_r=10^{-4}$ to $10^{-1}$ in Fig. 14(c).

Amongst all the parameters affecting the forced convective heat transfer performance of metal-foam partially filled tubes, the dimensionless interfacial radius $R_i$ is the most important one since its variation directly influences the flow and heat transfer characteristics. Fig. 14(d) shows the effect of dimensionless interfacial radius on the global Nusselt number for thermal conductivity ratios $k_r$. It can be found that Nusselt number first decreases for small $R_i$ and is reduced to a minimized value for a certain $R_i$, after which the Nusselt number begins to rise. The range of $R_i$ is merely taken as 0.1 to 0.9 in the present study. It can be predicted from the varying trend of Fig. 14(d) that the global Nusselt number for $R_i=1.0$ is greater than 4.36 for fully-developed heat transfer in empty tube, owing to the fact that the global Nusselt number involving entrance effect.

Fig. 15 presents the effect of porosity, pore density and thermal conductivity ratio on the global Nusselt number. The effect of porosity on the Nusselt number is shown in Fig. 15(a). The Nusselt number decreases as the porosity increases for all $k_r$ values. The effect of pore density on the Nusselt number is shown in Fig. 15(b). The Nusselt number increases as the pore density increases for all $k_r$ values. The effect of thermal conductivity ratio on the Nusselt number is shown in Fig. 15(c). The Nusselt number decreases as the thermal conductivity ratio increases for all $R_i$ values. The effect of dimensionless interfacial radius on the Nusselt number is shown in Fig. 15(d). The Nusselt number increases as the dimensionless interfacial radius increases for all $k_r$ values.
the global Nusselt number under the constraint of identical pumping power. The thermal performance of metallic foam partially tubes with different parameters can be easily estimated. The Nusselt number at smaller porosity 0.85 is slightly higher than that at higher porosity 0.9 with fixed pore density and thermal conductivity ratio. With the similar condition, the Nusselt number at smaller pore density 5PPI is much higher than that with higher pore density 10PPI. Likewise, the Nusselt number for $k_r=10^{-3}$ is higher than that for $k_r=10^{-2}$. Overall, the decrease in pore density and the decrease in thermal conductivity ratio can significantly increase the global Nusselt number. Hence, the tube pattern with low porosity and pore density was recommended. For example, the Nu number for tube ($\varepsilon=0.85, 5\text{PPI}$) is 2.06 and 1.94 times than that of tube ($\varepsilon=0.9, 10\text{PPI}$) at $Re=1500$ and 4000 respectively.

Fig.15 The effect of key parameters on the global Nusselt number under the constraint of identical pumping power.

4. Conclusions

The conjugated heat transfer for metallic foam partially-filled tubes was numerically investigated based on the local thermal non-equilibrium model of porous medium. The present numerical heat flow model can be applied for the evaluation of flow and heat transfer in porous-fluid systems. The increase in Reynolds number can lead to the increased entrance length for both flow and heat transfer. Pressure drop increases with increases in pore density and $Re$, and decreases with increases in porosity and dimensionless radius. As $Re$ increases, pressure drop increases non-linearly due to the Forchheimer term. The global Nusselt number can be increased by decreasing pore density and fluid-solid thermal conductivity ratio. An optimal porosity (around 0.94) exists corresponding to maximized Nu number, while a minimum Nu occurs at a certain $R_i$ for $k_r=10^{-1}$ and $10^{-2}$. Decreasing the pore density or the fluid to thermal conductivity ratio can significantly increase the global Nusselt number. The porous media with low porosity and pore density was recommended under the constraint of identical pumping power.

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