Study and Development of a Mini-Tube Gas Separator Utilizing the Soret Effect*

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Abstract

The Soret effect is a phenomenon in which a temperature gradient gives rise to a concentration gradient. In this study, we attempted a simple technique for the separation of hydrogen from a gas mixture by utilizing the Soret effect. A gas mixture of hydrogen and carbon dioxide was introduced into a metal mini tube in which a temperature gradient was established in the direction normal to the flow. The experiment demonstrated that hydrogen was actually separated from the gas mixture, although the separation was very small with the present apparatus. From results of the experiments and computational fluid dynamics (CFD) analysis, we believe that the layer of high hydrogen concentration that moved toward the heating wall was very thin. It would be necessary to extract the high concentration of hydrogen flowing near the wall to increase the separation efficiency.

Key words: Soret Effect, Hydrogen Separation, Separation Distance, Re-Mixing, CFD

1. Introduction

Many technological developments are currently being investigated to improve sustainability and protect the global environment. A society based on hydrogen is a serious possibility because deriving energy from hydrogen places a relatively small burden on the environment. However, existing hydrogen-production techniques cannot produce sufficient hydrogen to meet the demands of a future hydrogen-based society.

One method of obtaining hydrogen is steam reforming, a technique that produces a mixture of gases including hydrogen, carbon dioxide, carbon monoxide, and methane by adding oxygen or steam to standard hydrocarbon gas⁽¹⁾. However, the reformed gas cannot be used in low-temperature fuel cells, because the gas mixture, including the hydrogen generated, contains significant impurities. Moreover, the carbon monoxide generated in the reforming process damages the platinum electrodes used in low-temperature fuel cells and degrades their performance.

Another typical method of hydrogen production is the pressure swing adsorption (PSA) method⁽¹⁾. PSA is a technique for producing highly pure hydrogen by the adsorption and desorption of impure gases including hydrogen under increased pressure. However, although highly pure hydrogen gas of greater than 99% purity can be produced by the PSA method, this system would not be suitable for large-scale production because it requires large quantities of adsorbing materials.

Therefore, in this study, we propose a novel separation technique utilizing the Soret effect. The Soret effect is a phenomenon in which a temperature gradient gives rise to a

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concentration gradient. To enhance the Soret effect, the only change that is required is a larger temperature gradient. If a mini-tube system is used, a large temperature gradient can be prepared easily in the direction normal to the flow. Moreover, any heating and cooling sources can be applied to obtain the temperature gradient, which is another virtue of the technique. In this study, we attempted coarse gas separation in our test equipment.

Nomenclature

$j^{(s)}$: Mass flux based on Fick's diffusion law (kg/s)
 $j^{(T)}$: Mass flux based on the Soret effect (kg/s)
 ρ : Density (kg/m³)
 c : Mol concentration (mol/m³)
 D : Diffusion coefficient (m²/s)
 T : Temperature (K)
 k_T : Thermal diffusion ratio
 x : Hydrogen mol concentration (mol/m³)
 Δx : Mole fraction
 U : Flow velocity (m/s)
 t : Time (s)
 L : Diffusion length (m)
 L_s : Separation distance (m)
 M : Molecular mass (g/mol)
 C_p : Specific heat at constant pressure (kJ/(kg.K))
 λ : Heat conductivity (W/(m.K))
 w : Tube inner diameter (m)

Index

i : Chemical species
hot: Hot region
cold: Cold region

2. Preliminary studies

2.1 Closed separating system

The Soret effect is a molecular diffusion motion in which lighter molecules move toward a warm region and heavier molecules move toward a cold region in the gas or liquid mixture under a temperature gradient⁽²⁾, Enskog and Chapman first studied this phenomenon from the perspective of molecular diffusion motion⁽³⁾. Chapman demonstrated the Soret effect using a gas mixture of hydrogen and carbon dioxide⁽³⁾. Recently, Nakano et al. investigated in detail the behavior of oxygen and nitrogen caused by the Soret effect in a cryogenic environment^(4,5).

The principle of the Soret effect is shown in Fig.1. In a system with the heating temperature T_{hot} and cooling temperature T_{cold} in regions 1 and 2, respectively, mass flux arises by virtue of the temperature gradient. The concentration gradient increases with the temperature gradient. On the other hand, in a system with a concentration gradient, mass flux based on Fick's diffusion law restricts the increase in the concentration gradient. In a binary system with chemical species A and B, of the Soret effect and Fick's diffusion law are explained using eqs.(1) and (2), respectively⁽²⁾. In the steady state, eq.(1) becomes equal to eq.(2). The resulting concentration difference caused by the balance of the Soret effect and Fick's diffusion can be represented by eq.(3).

Fig.1 Steady-state binary thermal diffusion

$$j_i^{(T)} = \frac{c^2}{\rho} M_A M_B D_{AB} \frac{k_T}{T} \frac{\Delta T}{L} \quad (1)$$

$$j_i^{(x)} = \frac{c^2}{\rho} M_A M_B D_{AB} \frac{\Delta x_i}{L} \quad (2)$$

$$\Delta x_i = x_{i2} - x_{i1} = -k_T \ln \frac{T_{hot}}{T_{cold}} \quad (3)$$

The value of the thermal diffusion coefficient k_T in the binary system is negative in lighter molecules and positive in heavier molecules. Therefore, lighter molecules move toward the heating region and heavier molecules move toward the cooling region. In this study, the value of the thermal diffusion coefficient in a gas mixture of hydrogen and carbon dioxide was 0.0899⁽⁶⁾. This value is relatively higher than other common gas mixtures⁽⁶⁾.

Before developing a mini-tube separation device, we considered it important to confirm the above theory concerning the Soret effect and the validity of the proposed value of k_T in the gas mixture of hydrogen and carbon dioxide. Therefore, we conducted a preliminary experiment, as shown in Fig.2. A gas mixture consisting of hydrogen (48%) and carbon dioxide (52%) was transferred into the experimental apparatus consisting of a stainless-steel tube and a temperature gradient was applied by heating and cooling regions of the tube. To avoid free convection caused by the temperature difference, a small connecting tube was used between the heating and cooling regions. Plastic tubes from which separated gases could be extracted were connected to both ends of the stainless steel tube. The ends of the plastic tubes were closed tightly to avoid gas leakage. A gas-lock syringe with 1-mL capacity was used to extract the separation gases. The outside of the separator tube was heated by a flexible heater in the heating region and cooled by a water ice bath in the cooling region. Thermocouples were positioned outside the separator tube to measure the heating and cooling temperatures. Gas concentrations were measured by a gas chromatograph (GC-14B, Shimadzu).

As shown in Table 1, the heating temperature was set at 200, 250, 300, 320, 420 and 450 °C, while the cooling temperature was fixed at 0 °C. According to computational fluid dynamics (CFD) analysis^(7,8), gas separation reached a steady state approximately 80 min after applying the temperature difference to the gas mixture. Therefore, the separation time was set at 120 min to ensure full gas separation. After 120 min, the separated gases were extracted from both ends of the stainless steel tube using syringes and analyzed by gas chromatography. As a measure of the separation ability, the concentration difference was defined as the difference in the hydrogen concentration between the heated and cooled regions.

Fig.2 Experimental apparatus for a closed separating system

Table1 Experimental condition for closed separation

Heating temperature (°C)	200–450
Cooling temperature (°C)	0
Temperature difference (°C)	200–450
Mole fraction (%) [H ₂]:[CO ₂]	48:52
Separation time(min)	120

We constructed a two-dimensional CFD model simulating the experimental apparatus used in closed separation because of its simplicity, as shown in Fig.3. Although the circular tube flow is three dimensional in reality, we considered a two-dimensional transfer of components to dominate in the separation regions in this situation. As shown in the experiment, the model had a connecting tube 4mm in diameter, as shown in Fig.3. The heating temperature was set to 200, 250, 300, 320, 420 and 450 °C in every experiment. The cooling temperature was fixed to 0 °C. A nonslip condition was imposed on the velocity of all inner surfaces of the tubes.

Fig.3 Simulation model for a closed system

The well-known PHOENICS⁽⁹⁾ software was used in the CFD analysis. Because this software does not handle the mass flux created by the Soret effect, we programmed this function and added it to the user's routine. We used a circular tube in the experiment, but a rectangular tube was used in the CFD analysis. This was for simplicity and because it was difficult to obtain meaningful results by complicated three-dimensional calculations. The physical properties of the gas mixture shown in Table 2 are averaged values for hydrogen and carbon dioxide. In this analysis, a gas mixture of hydrogen (50%) and carbon dioxide (50%) was used. The flow state was assumed to be laminar.

Table2 Physical properties of gas mixture at 27 °C.

Kinetic viscosity ν [m^2/s]	1.55×10^{-5}
Thermal diffusion ratio k_T [-]	0.0899
Diffusion coefficient D [m^2/s]	6.34×10^{-5}
Specific heat C_p [$kJ/(kg.K)$]	1.456
Heat conductivity λ [$W/(m.K)$]	0.0685

The experimental results are shown in Fig.4. Number of times of experiments was difference depending on the experimental condition. We did an experiment only one time under the temperature difference of 220, 250 and 420°C. Also, in the cases of the temperature difference of 300 and 320, or 450°C, the experiments were done from 2 to 4 times. The range of the data scattering is also shown in Fig.4 with the symbol of averaged value. The change in the concentration difference as simulated by CFD showed good agreement with eq.(3), which indicates that the two-dimensional CFD model was fully valid. Furthermore, the experimental values also showed the same tendency. Thus, we could successfully confirm the Soret effect in gas separation with a change in the temperature difference. However, the experimental values obtained were approximately 60% less than those obtained from in eq.(3) or the CFD analysis. We believe the reason for this is related to the heat conductivity of the stainless steel tube, which was not considered in the CFD model in Fig.3. For example, in ideal case where the temperature difference was set to be 450°C, the temperature at the tube end of cooling side should be 0°C, and that of heating side should be 450°C. However, in our experimental apparatus, we were not able to place heating or cooling devices to the very end points of the connecting tube. We estimated the temperatures of the end points of the actual connecting tube by using a simple heat conduction analysis considering both of the stainless steel tube and the heating/cooling devices. Then we found that the temperature at the tube end of cooling side was 40°C, and that of heating side was 397°C. These estimated data with eq.(3) gave 6.8 (%) as Δx value. The experimental value was 6.6 (%) at maximum. We think that this could explain the major cause of the decrease from the theoretical value ($\Delta x = 8.8$ (%) in this case). Therefore, we considered heat loss around the connecting tube mainly weakened the gas separation in the preliminary experiment. Heat energy is transported in the stainless steel tube and the temperature gradient inside the tube was therefore distorted in the real experiment. As a result, the imposed temperature difference could have been decreased and the Soret effect weakened. We could obtain higher concentration difference close to theoretical value if heat conduction of the connecting tube was suppressed.

Fig.4 Experimental data by unit cell (4mm)

As for the value of k_T , we think the value of $k_T(0.0899)$ was reasonable because the dependence of concentration difference on temperature difference was very similar between the experiments and the theory as shown in Fig.4. However, we admit that the value $k_T(0.0899)^{(6)}$ was a very old datum, and we may need to consider its dependence on the change of gas mixture ratio.

2.2 Separation distance in the mini-tube system

In designing a mini-tube equipment, we next needed to investigate the distance from the inlet to the point at which the gas mixture separation reaches a steady state, because this determines the minimum length of the mini tube needed for the gas separation. The separation distance was analyzed using the two-dimensional CFD model in Fig.5. Slip conditions at the walls were applied here for comparison with the series solutions described later. The gas mixture enters the separator tube at a flow rate of 50–250 mL/min at the inlet where the temperature is 27 °C and the absolute pressure is 1 atm. The gas mixture emerges from the outlet. The gas separation was evaluated by ascertaining the distance at which the concentration distribution reached a steady state. The separation distance was defined as the distance at which the change in the concentration difference in the flow direction reached less than 0.1%. The concentration difference was defined as the difference in the hydrogen concentration between the heating and cooling surfaces; in other words, the difference between the values in the first cell and the last cell in the diffusion direction (see Fig. 6). Each cell measured 0.2mm in the X-direction and 0.1mm in the Z-direction. Heating and cooling temperatures were set to 450 and 0 °C, respectively.

Fig.5 Calculating model for gas separation

Fig.6 Calculating cell

We also theoretically estimated the separation distance by considering a one-dimensional, unsteady, coupled-diffusion problem and derived its series solution. Assuming that forced convection is not essential to the separation achieved by the Soret effect, we studied a simplified problem, as shown in Fig.7, in which the flow was neglected and the mass transfer was considered only in the Z-direction. Equations (4) and (5) are governing equations concerning the changes in the temperature and hydrogen concentration distributions with time.

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho C_p} \frac{\partial^2 T}{\partial z^2} \quad (4)$$

$$\frac{\partial x}{\partial t} = \frac{D^*}{cM_A} \left(\frac{\partial^2 x}{\partial z^2} - \frac{k_T}{T} \frac{\partial^2 T}{\partial z^2} \right) \quad (5)$$

where $D^* = \frac{c^2}{\rho} M_A M_B D_{AB}$, and $\bar{T} = \frac{T_1 T_2}{T_2 - T_1} \ln \frac{T_2}{T_1}$.

As in CFD, the initial temperature was set at 27 °C, and the mole fraction was 0.5 at

time $t=0$. At both boundaries, the heating and cooling temperatures were 450 and 0 °C, respectively, as in CFD; whereas the no-flux condition was applied in the X-direction. This problem can be solved rigorously by the Fourier-series expansion as shown in the Appendix.

Figure 8 shows the changes in the hydrogen concentration on the heated and the cooled surfaces calculated by the series solution. The separation time, Δt , indicates the time when the difference in the hydrogen concentration between the heated and cooled surfaces reaches the steady state. We estimated Δt from Fig.8 as the time beyond which the difference stays within 0.1% over 0.5 ms. Then, assuming that the separated gases are simply transported in the downstream direction by the forced convection at a velocity U , the separation distance can be estimated using eq.(6):

$$L_x = U\Delta t \quad (6)$$

Fig.7 One-dimensional model

Fig.8 Result by the series solution (2mm dia.)

We next estimated the separation distance using both CFD and series solutions. In the experiment, the gas mixture was introduced into the tube at a flow rate ranging between 50 and 250 mL/min. As an example, the calculated result in the case of a flow rate of 150 mL/min is shown in Fig.9 in which the difference in maximum and minimum concentrations at particular cross sections is plotted. In the numerical analysis, hydrogen concentrations on the top and bottom walls inside the tube were measured. It was found that CFD and the series solutions fully reached a steady state in only a few centimeters. This is also clear from Fig.10, which indicates the distribution of the hydrogen concentration by CFD. Thus, the gas separation by the Soret effect inside a mini tube can be completed almost instantaneously. In an earlier study⁽¹⁰⁾, it was reported that the Soret effect generally required a long time to separating gas components. However, we believe that it would be possible to shorten the separation time using a mini-scale system.

It was also found that two types of analysis had a similar tendency for gas separation and that concentration differences converged to the theoretical value of eq.(3). Hereby, our application of CFD was proved to be appropriate by comparison with the series solutions.

To determine the diameter of the mini tube, we investigated the influence of the tube diameter on the separation distance L_x defined by eq.(6). The heating and cooling temperatures were set to be 450°C and 0°C, respectively, as in section 2.2. The result by CFD and the series solution are shown in Fig.11. Note that the concentration differences in all of three sizes of tube diameter converged to the same value of 8.3% after reaching a steady state in CFD calculation. This figure shows the separation distances L_x when the average velocity was changed from 0 to 2 m/s and the tube diameter was changed from 1 to 3 mm. For example, when the average velocity was 1 m/s and the tube diameter was 2 mm,

the flow rate became 188 mL/min. We found that the separation distance increased with the tube diameter, and the separation distance was almost proportional to the square of the diameter. This is related to the decrease in the temperature gradient with increasing tube diameter. Thus, increasing the tube diameter lowers the efficiency of gas separation.

Fig.9 Analytical result for separation distance

Fig.10 Hydrogen concentration distribution near the inlet
(CFD result, flow rate=150 mL/min)

Fig.11 Relationship between separation distance and flow velocity

The above results allow us to consider the appropriate apparatus geometry. We decided to use a commercially available stainless steel tube 2.4mm in inner diameter to achieve a

higher efficiency of gas separation. A 1-mm inner-diameter tube was too thin to be prepared for our experimental set-up. The above results also indicated that a tube length of more than a few centimeters is sufficient to ensure full gas separation. In the real set-up, we needed to install connections and tube heating and cooling devices using in-house tools. We found that the heating and cooling temperature could not always be strictly constant. Therefore, it was necessary to utilize a longer tube length than that suggested by the analytical results. We decided to use a 120-mm-long tube, assuming this to be long enough for gas separation.

The result in Fig.11 could be used as a guide for device design. However, it considers the separation process only inside the tube, and it does not consider the influence of the outlet structure for emitting the separated gas in real experiment. Therefore, if we discuss the final (or actual) hydrogen concentration at the device outlet, the analysis considering the method of outlet flow or outlet shape should be done.

3. Study of gas separation in the mini-tube flow

3.1 Experimental procedure for gas separation in the mini-tube flow

The experimental set-up is shown in Fig.12. The gas mixture flowed inside the stainless steel tube and a given temperature gradient was applied in the direction normal to the flow direction. The stainless steel tube was 2.4mm in inner diameter and 3mm in outer diameter with the upper surface cooled and the lower surface heated for giving a temperature gradient. A 300-W cartridge heater was applied directly to the bottom surface of the tube. On the other hand, in the cooling system, an antifreeze fluid cooled at -20°C flowed inside the brass tube, which was set on the top surface of the stainless tube, using a constant temperature reservoir “ADVANTEC TBF230DB (cooling power max. 400W)”, which can supply an antifreeze fluid by up to 6.5L/min. Thermocouples were set on the cartridge heater directly and on the side surfaces of the brass tube. Therefore, the measured temperature values in experiment were the temperature of outer surface of the tube, not temperature of inner surface of the tube. As discussed in the previous section, the tube length for the temperature gradient was set to 120mm. The temperature was measured using thermocouples attached to the outer surface of the tube. To avoid heat loss, an insulator material was placed around the separator tube.

The outlet branch is shown in Fig.13. The aluminum plate was set inside the tube to divide the separated gases. This plate was incorporated to restrict the loss of the concentration gradient that could occur when the temperature gradient was diminished outside the heating or cooling sections. The temperature difference was given between the top and bottom surfaces of the tube. Experimental conditions are shown in Table 3. The heating temperature was 450°C on the bottom surface, and the cooling temperature was set to 0°C on the top surface. The gas mixture of hydrogen (48%) and carbon dioxide (52%) was introduced into the tube with the given temperature gradient. The in-flow rate ranged from 50 to 250 mL/min. The separated gases from the two outlets entered the gas chromatograph allowing the species concentration to be measured. In this study, the concentration difference was defined as the difference in the hydrogen concentration of the two sampled gases.

3.2 Analysis of gas separation in the mini-tube flow

We used a three-dimensional model of a rectangular tube for gas separation. Although the mini tube in the experiments was round, it was technically difficult to construct this geometry considering the Soret effect accurately in the CFD software we used. In this analysis, the three-dimensional velocity distribution and the heat conductivity of the mini tube (in other words, the temperature distribution in the tube material) were included to represent the Soret effect as realistically as possible. The region under the large temperature

gradient was the entire length in which the gas mixture flowed. Heating and cooling temperatures were set to 450 and 0°C on the outer surface of the tube, respectively. In this model, nonslip conditions at the inner surface of the tube were imposed.

Fig.12 Experimental apparatus for gas separation in the mini-tube flow

Fig.13 Outlet branch

Table 3 Experimental conditions for tube flow separation

Heating temperature (°C)	450
Cooling temperature (°C)	0
Temperature difference (°C)	450
Mole fraction(%) [H ₂]:[CO ₂]	48:52
Inflow rate(mL/min)	30–250

Fig.14 Calculating model for tube flow separation

3.3 Results and analysis

Figure15 shows the results obtained. We extracted and analyzed 2 to 3 times, the separated gas from each outlet of heating and cooling sides in order to consider errors. The experimental value was defined as the difference of the averaged data of hydrogen

concentration in each gas. The data range (maximum and minimum) were shown in Fig.15 with the symbol of the averaged value. The concentration difference in the experiment was a maximum of 0.96% when the flow rate was 200 mL/min. The difference in CFD was 1.23% and remained almost constant irrespective of the flow rate in CFD. We were able to confirm the gas separation caused by the Soret effect in the mini-tube gas flow, but the separation capability was as small as $\sim 1.0\%$ compared with a separation of approximately 9% in the closed separation system discussed in Section 2.1.

Fig.15 Experimental result

Fig.16 Temperature distribution
(flow rate = 150 mL/min)

Fig.17 Hydrogen concentration distribution
(flow rate = 150 mL/min)

Fig.18 Hydrogen concentration distribution for a new separating method
(flow rate = 50 mL/min)

We are of the view that this difference was because the separated gases were re-mixed in

each branch path going to the outlet. This is clearly shown in Figs.16 and 17, which depict the temperature and hydrogen concentration distributions, respectively, simulated by CFD. The thin layer formed on the inner heated surface of the tube contained a higher concentration of hydrogen, while a lower concentration of gas flowed around the center of the tube. In this apparatus, separated gases were divided upward and downward by an aluminum plate, and in each path of the gas, the higher and lower concentration gases were re-mixed with the final concentration being averaged.

Furthermore, as mentioned above, Fig.17 shows that the higher concentration layer near the heated surface was extremely thin. We found from the CFD analysis that the difference in the hydrogen concentration between the vicinity of the heating and cooling surfaces was in agreement with the value in the closed separation system; in other words, the value represented in eq.(3). Thus, the small separation efficiency in this experiment was caused by the re-mixing of separated gases and not through forced convection itself. Therefore, extracting the higher concentration gas near the inner surface of the tube was very important to improve the separation efficiency.

Next, as a new separating method, we developed a separator as shown in Fig.18. In this method, separated gases are divided in two directions, and the heating section is attached locally before branching of the path. The size of the heating section could be reduced because gas separation can reach a steady state after a few millimeters at a very low flow rate, as shown in Fig.11. Gas pressure at outlet-1 and outlet-2 were simply set to be 1atm in CFD simulation. The CFD results suggested that the difference in the hydrogen concentration between outlet-1 and outlet-2 increased to about 3% (data not shown), and this was found to show good potential for increasing the efficiency of gas separation. Recently, we obtained approximately 2.5% of the difference in hydrogen concentration in our current experiment⁽¹¹⁾. Moreover, we found from CFD result that the ratio of outflow rate from outlet-1 and from outlet-2, to inflow rate were 10% and 90%, respectively. Thus, efficiency of flow separation should be considered as a next problem in this new apparatus. We will report the above new data in the near future. We are presently developing a new separator based on those results.

4. Conclusion

In this study, a hydrogen separation technique based on the Soret effect was studied experimentally and theoretically. We initially performed experiments in a closed separating system to confirm the Soret effect. Gas separation in the closed system was detected and the experimentally observed change in the concentration with temperature was similar to CFD results and theoretical calculations. Before designing the mini-tube flow equipment, we studied the required length of the tube for gas separation using CFD and the analytical series solution. From the study, it was found that gas separation reached a steady state in the mini tube at the distance of a few centimeters from the inlet. The test equipment consisting of the mini-tube flow was then constructed and we actually detected the gas separation caused by the Soret effect. However, owing to re-mixing of the once-separated gases, the gas separation efficiency was still small. To improve the separation efficiency, it will be necessary to extract the higher concentration gas in the vicinity of the inner surface of the tube.

We conclude that because of the virtue of the enlarged temperature gradient in the mini-tube system, the Soret effect was found to be applicable in this gas separation technique. Although the present efficiency was not large, our new ideas for improvement could make the technique more promising for practical use in future.

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Appendix

The forms of eqs. (4) and (5) together with the boundary conditions suggest the following expressions for the temperature T and the hydrogen concentration x .(The form can also be derived from Green's function.) W is the width of the channel in z direction.

$$T(z,t) = \frac{T_1 + T_2}{2} + \frac{T_2 - T_1}{W} z + \sum_{m=1}^{\infty} b_m(t) \sin(\lambda_m z) + \sum_{m=1}^{\infty} d_m(t) \cos(\sigma_m z)$$

$$x(z,t) = x_0 + c_0(t) + \kappa_3 \left[T(z,t) - \frac{T_1 + T_2}{2} \right] + \sum_{m=1}^{\infty} a_m(t) \sin(\sigma_m z) + \sum_{m=1}^{\infty} c_m(t) \cos(\lambda_m z)$$

where

$$\lambda_m = \frac{2m\pi}{W}, \sigma_m = \frac{(2m-1)\pi}{W},$$

and

$$\kappa_1 = \frac{\lambda}{\rho C_p}, \kappa_2 = \frac{D^*}{cM_A}, \text{ and } \kappa_3 = \frac{k_T}{T}.$$

Substitute into eqs.(4) and (5), then take Galerkin projections of these using weight functions $1, \sin(\lambda_m z), \cos(\lambda_m z), \sin(\sigma_m z),$ and $\cos(\sigma_m z)$. The generated equations are all satisfied when

$$b_m(t) = \frac{T_2 - T_1}{m\pi} (-1)^m \exp(-\kappa_1 \lambda_m^2 t), d_m(t) = \frac{4y_0}{(2m-1)\pi} (-1)^{m+1} \exp(-\kappa_1 \sigma_m^2 t),$$

$$a_m(t) = \sum_{n=1}^{\infty} \alpha_{mn} \left\{ \exp(-\kappa_1 \lambda_n^2 t) - \exp(-\kappa_2 \sigma_m^2 t) \right\},$$

$$c_m(t) = \sum_{n=1}^{\infty} \gamma_{mn} \left\{ \exp(-\kappa_1 \sigma_n^2 t) - \exp(-\kappa_2 \lambda_m^2 t) \right\}$$

For $m=1, 2, \dots$, and

$$c_0(t) = -\frac{8\kappa_3 y_0}{\pi^2} \sum_{m=1}^{\infty} \frac{\exp(\kappa_1 \sigma_m^2 t)}{(2m-1)^2} \text{ and } y_0 = T_0 - \frac{T_1 + T_2}{2}.$$

Here,

$$\alpha_{mn} = \frac{f_{mn}}{\kappa_2 \sigma_m^2 - \kappa_1 \lambda_n^2}, \gamma_{mn} = \frac{g_{mn}}{\kappa_2 \lambda_m^2 - \kappa_1 \sigma_n^2},$$

$$f_{mn} = \frac{32\kappa_1 \kappa_3 (T_2 - T_1) (-1)^m}{W^2} \times \frac{n^2}{4n^2 - (2m-1)^2},$$

and

$$g_{mn} = \frac{16\kappa_1 \kappa_3 y_0 (-1)^m}{W^2} \times \frac{(2n-1)^2}{(2n-1)^2 - 4m^2}.$$

As can be easily seen, all time-dependent modes $a_m(t)$, $b_m(t)$, $c_m(t)$, and $d_m(t)$ decay exponentially as $t \rightarrow \infty$. The slowest modes decay at time scales

$$t = O(\kappa_1^{-1} \sigma_1^{-2}, \kappa_2^{-1} \sigma_1^{-2}) = O\left(\frac{W^2}{\pi^2 \kappa_1}, \frac{W^2}{\pi^2 \kappa_2}\right).$$