Transient Behaviour of Natural Convection in a Reservoir Model Induced by Surface Heating*

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Abstract
We perform detailed numerical simulations of natural convection in a reservoir model induced by surface heating. The transient behaviour of the flow, including streamlines, isotherms, surface velocity profiles, volumetric flow rates, heat transfer rates, and temperature averaged over the local water depth are presented at different Rayleigh numbers. At an instantaneous time within the sloping bottom region, the surface velocity initially increases and then decreases with an increase of the offshore distance. Over a certain range of Rayleigh numbers, the location of the peak surface velocity may initially move in the offshore direction and then retract with the elapse of time. The point at which the rates of horizontal conduction and convection balance also retracts towards the tip region with the elapse of time. Some insight into understanding the mechanisms that drive the flow is provided according to the detailed transient behaviour of the flow.

Key words: Natural Convection, Reservoir, Unsteady Flow, Thermal Boundary Layer, Surface Heating

Nomenclature

\[ A \] slope inclination
\[ c_p \] specific heat
\[ g \] gravity
\[ h \] the maximum water depth
\[ H_0 \] \( I_0 (\rho C_p) \)
\[ H(X) \] horizontal heat transfer rate
\[ I_0 \] imposed surface heat flux
\[ L \] total length of reservoir
\[ p \] pressure
\[ Pr \] Prandtl number
\[ Q(X) \] volumetric flow rate
\[ Ra \] Rayleigh number
\[ t \] time
\[ t_c \] time scale for flow reaches the quasi-steady state
\[ t_d \] time scale for the thermal boundary layer to reach the sloping bottom
\[ T \] fluid temperature
\[ T_0 \] reference temperature
\[ u, v \] horizontal and vertical velocity components
\[ U, V \] dimensionless velocity components
\[ x, y \] horizontal and vertical coordinates
\[ X, Y \] dimensionless coordinates
\[ X_i \] intersecting position of horizontal conduction and convection curves
\[ X_m \] position of the peak velocity
\[ Y_b Y \] \( Y \)-coordinate of the bottom at a given horizontal position \( X \)

Greek
\[ \beta \] volume expansion coefficient
\[ \kappa \] thermal diffusivity
\[ \nu \] kinematic viscosity
\[ \rho_0 \] density
\[ \tau \] dimensionless temperature
\[ \tau^* \] temperature difference between the local and the spatially averaged temperatures

1. Introduction

The underlying mechanisms that govern natural convection in lakes and reservoirs are of great significance in the successful management of these resources, particularly in terms of nutrient and pollutant transport. The nearshore bathymetry of an increasing water depth...
in the offshore direction of lakes and reservoirs implies that, as the result of approximately uniform heat fluxes at the surface during daytime heating and night time cooling, the shallow region heats up or cools down more rapidly than the deeper region, setting up a horizontal temperature gradient which drives a natural convection flow. This flow exchanges nutrients and pollutants between the nearshore and the central regions, and thus influences the water quality. There have been a considerably large number of investigations of natural convection in nearshore lake waters, reservoir sidearms or other shallow water bodies with a sloping bottom\(^1-^3\), by using analytical, experimental, and numerical methods.

The field study of Monismith et al.\(^4\) has demonstrated a clear picture of this thermally driven flow, which greatly enhanced the rate of horizontal exchange between the sidearm and the body of the reservoir. Their data indicated that velocities of the order of 5 cm/s may be associated with this process, which suggests that over a typical 12 hour period, transport could occur over a distance of order 2 km. Many pollutants in reservoir systems are shore based, and this mechanism can therefore provide significant transport of these materials from the shore to the central part. The transport of pollutants or nutrients in the form of solutes or suspended particles between the littoral and pelagic zones plays a significant role in the ecological state of the reservoir. Clearly this has economic implications for subsequent water treatment and other management strategies for water storage reservoirs. Recently, the field experiment of Monismith et al.\(^5\) indicated that the thermally driven flow may also be a generic feature of the hydrodynamics of coral reefs and coastal oceans in general. It also helps to alleviate the stress of coral bleaching and enhance connectivity between the reef and the ocean\(^6\). The implications of this for coral reef health are clear, and the connection to a much wider tourism industry is apparent.

Studies of natural convection in a simplified reservoir model have been performed by many researchers\(^7-^10\). In the early studies, a triangular enclosure was often used to approximate the offshore region. For example, Horsch and Stefan\(^7\) and Horsch et al.\(^8\) investigated numerically and experimentally natural convection in a triangular enclosure with a constant cooling flux at the surface. The transient behaviour of the flow, including the formation of sinking thermals and the establishment of a full cavity scale circulation, was illustrated. Using scaling analysis, Lei and Patterson\(^11\) found that the flow can be classified broadly into a conductive, a transitional or a convective regime determined merely by the Rayleigh number. By recognizing the dependence of various flow quantities on the horizontal position with a variable length scale, Mao et al.\(^12\) revealed a more detailed feature of the flow i.e. the entire flow domain is separated into different subregions with distinct flow and thermal features; namely a conduction dominated region, a stable convection dominated region and an unstable convection dominated region. Two different sets of scaling incorporating the offshore-distance dependency have been derived for the conduction-dominated region and stable-convection-dominated region respectively, which are confirmed by numerical simulations.

However, the triangular model is a poor geometric representation of the real system as the presence of the end wall prevents the circulation extending into the central region of the reservoir. Recently, a more realistic reservoir model has been applied, which consists of two distinct regions: one with a bottom slope and the other with a uniform water depth. Using this model, Bednarz et al.\(^10, 13\) studied natural convection induced by constant temperature and uniform heat flux at the surface, respectively. Natural convection induced by periodic thermal forcing has also been studied experimentally\(^14, 15\) and numerically\(^16\).

The understanding of natural convection in reservoir nearshore regions has advanced significantly in recent years, but is still far from complete, particularly with respect to the transient behaviour of the flow. This has motivated the present numerical study which will reveal detailed transient behaviour of natural convection in reservoir nearshore regions with a constant heating flux at the water surface. This thermal forcing may represent the day-time heating case in cloudy weather, when no solar radiation is present. The structure of
the paper is as follows: Section 2 describes the governing equations and boundary conditions as well as the numerical techniques and grid-independence study. Section 3 presents the numerical results. Finally, Section 4 draws conclusions.

2. Model formulation and numerical aspects

Here we consider a two-dimensional reservoir model consisting of two distinct regions: one with a bottom slope and the other with a uniform water depth (Figure 1). This model is more realistic than the model of a triangular enclosure and has been adopted by Lei and Patterson (16) and Bednarz et al. (10, 13-15). The slope inclination is fixed at \( A = 0.1 \) and the maximum water depth is \( h \). The length of the flat part is two times that of the inclined part, giving the total length of the model \( L = 30h \). The Cartesian coordinate system \((x, y)\) is adopted with the origin located at the tip of the reservoir model.

Note that the length of the flat part in the present study is longer than those of Lei and Patterson (16) and Bednarz et al. (10, 13-15) to further minimize the effect of the end wall. The Navier-Stokes and energy equations governing the flow and temperature evolution are expressed as follows, in which the Boussinesq assumptions have been made:

\[
\begin{align*}
\partial_t u_x + u_x \partial_x u_x + v \partial_y u_x &= -\rho_0^{-1} p_x + \nu \nabla^2 u_x \\
\partial_t u_y + u_x \partial_x u_y + v \partial_y u_y &= -\rho_0^{-1} p_y + \nu \nabla^2 u_y + g \beta (T - T_0) \\
\partial_t T + u_x \partial_x T + v \partial_y T &= \kappa \nabla^2 T
\end{align*}
\]

where \( u_x \) and \( u_y \) are the velocity components in the horizontal and vertical directions respectively; \( x \) and \( y \) are the horizontal and vertical coordinates originating from the tip of the sloping bottom region; \( t \) is time; \( T \) is the fluid temperature; and \( p \) is the pressure. The parameters \( \rho_0, \kappa, \nu \) and \( \beta \) are respectively the density, thermal diffusivity, kinematic viscosity and thermal expansion coefficient of the fluid at a reference temperature \( T_0 \).

Initially \((t \leq 0)\), the fluid is at rest and isothermal, that is

\[
\begin{align*}
u_x &= 0, \quad \nu_y = 0 \\
T &= T_0
\end{align*}
\]

The heating is introduced with a constant heat flux at the water surface as

\[
T_y = I_0 / \kappa \rho_0 C_p = H_0 / \kappa, \quad y = 0, \quad t > 0
\]

where \( I_0 \) is the imposed surface heat flux and \( C_p \) is the specific heat.

A non-dimensional form of the governing equations can be expressed as:

\[
\begin{align*}
\partial_t U_x + V_U x + V_V y &= 0 \\
U_x + U U_x + V U_y &= -(Pr Ra) P_x + Pr \nabla^2 U \\
V_x + U U_y + V V_y &= -(Pr Ra) P_y + Pr \nabla^2 V + (Pr Ra) \tau \\
\tau_x + U \tau_x + V \tau_y &= \nabla^2 \tau
\end{align*}
\]

with the non-dimensional surface heat flux condition:

\[
\tau_y = 1, \quad Y = 0
\]

All the quantities above have been normalized by the following scales: \( x, y \sim h; t \sim h^2 \kappa^{-1} \); \( T \sim H_0 h \kappa^{-1} \); \( u, v \sim kh^{-1} \); \( p \sim \rho_0 g \beta h \kappa^{-1} \). In Eqs (10) and (11), \( Pr \) and \( Ra \) denote the Prandtl number and Rayleigh number respectively, which are defined as:

\[
Pr = \nu / \kappa
\]
As the heat flux continuously enters from the surface and all of the other boundaries are adiabatic, the temperature of the water body continues to increase. In this sense, there is no steady-state with respect to the water temperature. However, a quasi-steady state may be reached in which the temperature gradient and flow velocity become steady. At the quasi-steady state, temperature increases at the same rate everywhere, and the difference between the local temperate and the spatially averaged temperate becomes steady\cite{11, 12}.

The above non-dimensional governing equations along with the specified boundary and initial conditions are solved numerically using a finite-volume method. The second-order central difference scheme is applied for the spatial derivatives in the governing equations. The second-order backward scheme is applied for time discretization in calculating the transient flow. The SIMPLEC method is adopted for pressure and velocity coupling. The detailed numerical procedures on discretization can be found in the works of Ferziger and Perić\cite{10} and Yu et al.\cite{11}.

For the present numerical simulations, the dimensionless height of the flow domain is $h = 1$, the bottom slope is $A = 0.1$ and the total dimensionless length of the reservoir is $L = 30$. To avoid a singularity at the tip, a very small tip region ($X = 0 - 0.16$) was cut off and an extra vertical adiabatic, no slip wall was assumed at $X = 0.16$. The cut off region is less than 0.01% of the entire domain and thus has a negligible effect on the flow, except at the very beginning of the flow development. The simulation is performed with a fixed Prandtl number of $Pr = 7$, which is typical for water. Before carrying out the simulation, a mesh and time step dependency test has been conducted using three different meshes, $601 \times 31$, $851 \times 46$, and $1151 \times 61$, for $Ra = 2.1 \times 10^7$, which is the highest Rayleigh number among all the simulation cases. All the meshes are non-uniform, with higher grid density near the boundaries. The time step is adjusted for the different meshes so that the Courant-Friedrichs-Lewy (CFL) number remains approximately the same for different meshes.

Figure 2 compares the horizontal velocity profile along the surface of the reservoir at the quasi-steady state for $Ra = 2.1 \times 10^7$.

![Fig. 2 Horizontal velocity profiles along the surface of the reservoir model at the quasi-steady state for $Ra = 2.1 \times 10^7$.](image)

Figure 3 compares the horizontal velocity profile along the surface of the reservoir at the quasi-steady state for all meshes. Clearly, all the meshes produce almost identical solutions. Comparisons of other flow properties yield similar results. It is also expected that the difference between the solutions are smaller at lower Rayleigh numbers. However, to ensure the accuracy of the simulation, mesh 3 is used for the present simulations and the dimensionless time step is fixed at $10^{-6}$.
For the present configuration, the heat flux is imposed at the surface and all other boundaries are insulated. Thus, the energy input from the surface is totally absorbed by the water, and the increase of the spatially averaged temperature in the reservoir is linear with time, which gives, in a dimensionless form:

\[ \Delta T = 6t/5 \] (16)

Figure 3 shows the increase of the spatially averaged temperature obtained from numerical simulations at different Ra. The plot indeed demonstrates that the computed spatially averaged temperature increases linearly with time as specified by Eq (16). In the following sections, the presented temperature (or isotherms) is the temperature difference between the local and the spatially average temperatures. This only affects the absolute value of the temperature but not the temperature gradient.

3. Numerical Results

The present study focuses on the transient behaviour of natural convection in the reservoir model induced by surface heating. Apart from the velocity and temperature, two additional parameters that have practical significance are calculated from the numerical data: The volumetric flow rate \( Q(X) \) and the horizontal heat transfer rate \( H(X) \). These two parameters are defined as:

\[
Q(X) = \frac{1}{2} \int_{Y_b}^{b} \left| \int_{0}^{Y_b} U \right| dY
\] (17)

\[
H(X) = \frac{1}{2} \int_{0}^{b} \left( U \tau^* - \tau_b^* \right) dY
\] (18)

where \( Y_b \) is the corresponding \( Y \)-coordinate of the bottom at a given horizontal position \( X \) and \( \tau^* \) is the temperature difference between the local and the spatially averaged temperatures. Eqs (17) and (18) have been successfully used in the previous studies (11, 12) to characterize the steady-state flow behaviour. The present study will further demonstrate the transient behaviour of these flow properties.

It is worth noting that, although the present heat flux condition at the surface is opposite to that in Lei & Patterson (11) and Mao et al. (12), the underlying mechanism that drives the flow is the same, i.e. the unequal heat gain or loss. It is expected that the present flow behaviour would confirm the scales developed in Mao et al. (12) except for those associated with a Rayleigh-Bénard type instability occurring in the surface layer. However, it will be shown below that the transient behaviour of the flow demonstrates certain new features that are not captured by those scales. We believe that one of the major causes of the variations is the result of the different geometric configurations used in the two studies, i.e., an infinite triangular domain without the endwall assumed in the scaling of Mao et al. (12) versus a finite triangular domain followed by a flat bottom part with an endwall in the present simulation. In the following, the new flow phenomena are highlighted and additional insight into understanding the mechanisms that drive the flow is provided.

For the case of surface cooling, the scaling analysis of Mao et al. (12) provided detailed features of the flow: After cooling is initiated, a thermal boundary layer grows underneath the water surface until it reaches the sloping bottom and becomes indistinct. This indistinct thermal boundary layer region extends in the offshore direction with the elapse of time. Near the tip region, a buoyancy induced horizontal pressure gradient develops due to the unequal heat loss, which results in a horizontal flow along the surface towards the tip region. The velocity increases with time, resulting in enhanced convection. When the heat conducted into the boundary layer balances that convected away, the thermal boundary layer stops growing and becomes distinct, and the flow reaches steady state.

According to the scaling analysis of Mao et al. (12), the flow can be classified into two flow regimes depending on the global Rayleigh Number: i.e. (I) \( Ra < A^2 \) and (II) \( Ra > A^2 \). It is worth noting that, unlike the case considered in Mao et al. (12), in the present case with
Rayleigh-Bénard type instability under the surface cooling condition. It is also worth mentioning that the Rayleigh numbers in Mao et al.\(^{(12)}\) and the present study differ by a factor of \(A^4\) because different characteristic length scales are used to define the Rayleigh number. The simulations performed in the present study involve a wide range of \(Ra\) to cover the two flow regimes, which are shown in Table 1. The presentation of the numerical results is thus divided into two sections, with each covering one of the two flow regimes.

### 3.1 Flow behaviour in the regime \(Ra < A^2\)

Figure 4 shows the simulation results for \(Ra = 70\) at the quasi-steady state. A clockwise flow is formed as shown in Figure 4a. At the quasi-steady state, the maximum flow velocity for \(Ra = 70\) is 3.5 (see Figure 5a). Figures 4b and 4c show isotherms in the whole reservoir and in the sloping bottom region, respectively. It is worth mentioning that the \(x\)-axis and the \(y\)-axis in all the contour plots, except for those in Figure 4c, are not on the same scale. The nearly vertical isotherms in Figure 4c indicate the presence of an indistinct thermal boundary layer. This is confirmed by the calculated horizontal heat transfer rates as shown in Figure 4d. Note that the first and the second terms of the integral in Eq (18) denote the contributions of convection and conduction, respectively. These two terms are calculated separately as shown in Figure 4d. The positive sign of the heat transfer rate indicates that heat is transferred away from the shore. Near the tip region, conduction initially increases along the offshore direction, which is due to the cut-off of the tip and the implementation of an adiabatic wall there. A sudden change in the conduction curve occurs around the end of the sloping bottom region (\(X = 10\)). Outside of the sloping region conduction is much weaker and more uniform across the region. This indicates that the unequal heat gain only occurs at the sloping bottom region.

Figure 5a shows the horizontal velocity profile along the water surface at different times for \(Ra = 70\). At a given time, the velocity initially increases and then decreases in the offshore direction. Near the quasi-steady state, the peak velocity is located at the end of the sloping bottom region (\(X = 10\)). This is because the unequal heat gain only occurs at the sloping bottom region, and thus the driving force is terminated at the end of the sloping bottom, resulting in the peak velocity there. At the early stage, the surface velocity is almost zero when \(X\) is larger than a certain value, confirming the non-existence of driving force in
the flat region. The velocity increases with the elapse of time. Figure 5b shows the variation of the volumetric flow rate along the offshore direction at different times. The behaviour of the volumetric flow rate is rather similar to that of the velocity profile shown in Figure 5a.

![Fig. 5(a) and 5(b)](image)

**Fig. 5** (a) Horizontal velocity along the water surface and (b) volumetric flow rate at different times for $Ra = 70$.

Mao et al.\(^{12}\) proposed a velocity scale for the early stage of the flow, which shows that the velocity at an instantaneous time decreases with the offshore distance. Although the velocity profile along the surface obtained in the present simulation indeed shows that the velocity decreases with the offshore distance in the offshore region, it clearly demonstrates that in the tip region, the velocity (for $t < 1$) increases with the offshore distance as shown in Figure 5a (refer also to Figure 12a). As the flow initially develops in the tip region and then extends to the whole reservoir, an additional velocity scale may be needed for the tip region.

We also record the time histories of the peak horizontal velocity along the water surface and its corresponding position for $Ra = 0.14$ and 70. It is expected that the velocity is linearly dependent on $Ra$ in the conduction-dominated region according to Mao et al.\(^{12}\). Thus, the peak velocity presented in Figure 6a is further normalized by $Ra$. The peak velocity initially increases rapidly and then slowly approaches a constant value with the elapse of time. The two curves initially collapse together and then deviate from each other with the elapse of time. Figure 6a also indicates that the flow reaches the quasi-steady state faster for higher Rayleigh numbers.

The transient behaviour of the position of the peak velocity $X_m$ is more interesting (see Figure 6b). For the two Rayleigh numbers simulated, at the initial stage $X_m$ increases approximately linearly with time at a very fast rate until $t \approx 1$. After that, $X_m$ slowly
decreases with $t$ until $t \approx 3$, indicating that the position of the peak velocity retracts towards the tip region. For $t > 3$, $X_m$ increases with time until it reaches a constant value of 10 at $t \approx 50$. By comparing Figure 6a and 6b, it is found that even when $X_m$ has reached a constant value, the peak velocity continues to increase for a certain period of time.

Mao et al.\(^{(12)}\) provided a time scale $t_d$ for the thermal boundary layer to reach the sloping bottom, which reads,

$$t_d \sim A^2 x^2 \kappa^{-1}$$  \hspace{1cm} (19)

They assumed that once the thermal boundary layer attaches to the sloping bottom, the flow reaches a quasi-steady state. Thus, $t_d$ is also regarded as the time scale for the flow in the conduction-dominated region to reach the quasi-steady state. For $Ra < A^{-2}$, the thermal boundary layer attaches to the sloping bottom in the whole reservoir at a dimensionless time $t_d \approx 1$. This implies that the flow in the entire reservoir should reach the quasi-steady state at $t_d \approx 1$. However, both Figures 5 and 6 show that the flow continues to develop for $t \gg 1$. For the conduction-dominated region, it is expected that the horizontal temperature gradient would be constant at the quasi-steady state. However, Mao et al.\(^{(12)}\) assumed the flow stops developing as soon as the thermal boundary layer attaches to the sloping bottom, and the development of the horizontal temperature gradient in the conduction-dominated region is not considered in their scaling analysis. The present simulations suggest that a re-consideration of the criterion for the flow to reach the quasi-steady state in the conduction-dominated region is necessary. Since the flow reaches the quasi-steady state at $t \sim O(100)$, the quasi-steady state time may be determined by the horizontal diffusion time scale instead of the vertical diffusion time scale $t_d$ in the conduction-dominated region.

The velocity scale in the conduction-dominated region at the quasi-steady state provided by Mao et al.\(^{(12)}\) indicates that the position of the peak horizontal velocity moves in the offshore direction with time and the peak velocity at the steady state is proportional to $Ra$. However, Figure 6a shows complex features of the transient behaviour of the peak location and that the time histories of the peak velocity (re-normalized by $Ra$) at different $Ra$ deviate from each other eventually. Note that different geometric configurations are used in the present study and Mao et al.\(^{(12)}\), which may be responsible for the deviations between the two studies.

### 3.2 Flow behaviours in the regime $Ra > A^{-2}$

Figure 7 shows the simulation results for $Ra = 2.1 \times 10^4$ at the quasi-steady state. Similar to Figure 4a, a clockwise circulation is formed as shown in Figure 7a, which occupies the whole reservoir. However, in the present case the circulation becomes much stronger, and the maximum flow velocity at the quasi-steady state is around 32 (refer to Figure 13). The isotherms in Figure 7b clearly show that the nearly vertical isotherms near the shore gradually transfer into horizontal isotherms as the offshore distance increases. The transitional region indicates an increasing effect of convection in horizontal heat transfer.

![Fig. 7 Flow properties at the quasi-steady state for $Ra = 2.1 \times 10^4$ ($t = 15$). (a) Clockwise streamlines at an interval of 35. (b) Isotherms with an interval of 0.4. (c) Profile of the horizontal heat transfer rate averaged over the local water depth.](image-url)
transfer. Figures 7c shows the horizontal heat transfer profiles. As the offshore distance $X$ increases, horizontal conduction initially increases rapidly and then decreases. Horizontal convection is nearly zero in the tip region, and increases sharply and then decreases as the offshore distance $X$ increases. At a certain horizontal position, horizontal convection is equal to horizontal conduction, which approximately indicates the dividing position between the indistinct and distinct thermal boundary layer regions.

The heat transfer rates in Figure 7c are comparable to those of Figures 7d in Mao et al.\(^{(12)}\). Note that the Rayleigh numbers in these two studies differ by a factor of $A^4$ as the horizontal length of the sloping bottom region is used as the length scale in their study whereas the height $h$ is used in the present study. It is worth mentioning that the dividing position predicted by the present study is at $X = 2.37$, which agrees well with that ($X = 2.5$) of Mao et al.\(^{(12)}\). This confirms that the fundamental mechanisms that drive the flows in the heating and cooling flux conditions are the same, i.e. the unequal heat acquisition or loss as proposed by Lei & Patterson\(^{(11)}\). It is also worth noting that the presently calculated results of the dividing positions are slightly further towards the tip than those of Mao et al.\(^{(12)}\). An enclosed triangular model is applied in the study of Mao et al.\(^{(12)}\). It is expected that the presence of the end wall reduces the horizontal velocity towards the wall and in turn reduces the strength of convection. Therefore, the dividing positions obtained by the previous simulations are slightly further out from the tip.

The transient behaviour of the heat transfer profiles for $Ra = 2.1 \times 10^3$ and $2.1 \times 10^4$ are shown in Figures 8a and 8b, respectively. For any instantaneous time, both horizontal conduction and convection initially increase, and then decrease with an increase in the
offshore distance. In the tip region, horizontal conduction is always stronger than convection. Horizontal conduction reaches its peak value at a position ahead of that for the peak convection. Both horizontal conduction and convection are enhanced with the elapse of time until the flow reaches the quasi-steady state. With the elapse of time, the position of the peak horizontal conduction moves in the offshore direction, whereas that of the peak convection retracts towards the tip region. At a given time, the horizontal conduction and convection profiles intersect with each other at a certain position between the two peak positions of the horizontal conduction and horizontal convection respectively. The intersection point also retracts towards the tip region with the elapse of time. Since the intersection point indicates the balance between horizontal conduction and convection, the transient behaviour of the intersection point implies that the conduction-dominated region may shrink with time.

Figure 9 summarizes the time histories of the positions of the intersection points at \( Ra = 2.1 \times 10^3 \) and \( 2.1 \times 10^4 \). It is seen in this figure that the intersection points initially rapidly move towards the tip regions and then slowly approach a constant value with the elapse of time. At any instantaneous time, the intersection point for the higher \( Ra \) case is closer to the tip region compared to that for the lower \( Ra \) case. This indicates that the conduction-dominated region (i.e. the indistinct thermal boundary layer region) shrinks with the increase of \( Ra \). The \( Ra \)-dependent behaviour is consistent with that reported by Mao et al.\(^{(12)}\).

To further demonstrate the transient behaviour of the flow, the time series of streamlines and isotherms for \( Ra = 2.1 \times 10^5 \) are shown in Figures 10 and 11, respectively. The streamline contours show that the flow initially develops at the tip region. With the elapse of time, the flow becomes stronger and expands to the offshore region. The circulation almost occupies the entire reservoir at \( t = 0.5 \). The flow stops developing when \( t > 2 \) (refer to Figure 13a). Figure 10f thus presents the typical streamline contours at the quasi-steady state.

Figure 11a shows that at the very
early time $t = 0.0125$, the isotherms are horizontal and parallel in most of the reservoir except for the tip region. This is due to the uniform flux of heat through the surface and subsequent conduction of the heat into the body. The isotherms curl downwards in the tip region in order to satisfy the no-flux condition on the slope, indicating that the thermal boundary layer attaches to the bottom and the indistinct thermal boundary layer region forms. With the elapse of time, the curved isotherms at the tip region gradually become vertical. The nearly vertical isotherms indicate that the flow is conduction-dominated in the tip region. The conduction-dominated region extends in the offshore direction with time. We also marked the zero contour lines in Figures 11a to 11f. It is shown that the zero contour line initially moves towards the bottom and then moves slightly back towards the surface. This zero line divided the whole reservoir into ‘hot’ and ‘cold’ subregions relative to the linear growth of the spatially averaged temperature. In the hot region (near the tip region), the density of the isotherms increases with time, indicating an increase of the horizontal temperature gradient. In the cold region (near the flat bottom region), the temperature drops slightly with time until the quasi-steady state is reached.

Figure 12a shows the horizontal velocity profile along the water surface at different times for $Ra = 2.1 \times 10^5$. At a given time, the velocity initially increases and then decreases in the offshore direction. The velocity increases with time across the entire domain. Figure 12a shows that except for the tip region, a further increase of the surface velocity is observed for $t > 1$. Figure 12b shows the
variation of the calculated volumetric flow rate along the offshore direction at different times. The behaviour of the volumetric flow rate is rather similar to that of the horizontal velocity profile.

Figure 12c shows the horizontal profile of the temperature averaged over the local water depth, which clearly demonstrates the development of the horizontal temperature gradient in the reservoir. At the initial stage, the horizontal temperature gradient only develops near the tip region. With the elapse of time, the gradient increases rapidly near the tip region but slowly in the region far away from the tip. This also reflects the transition from the conduction-dominated region (with a large horizontal temperature gradient) to the convection-dominated region (with a small horizontal temperature gradient).

Figures 13a and 13b summarize the time histories of the peak horizontal velocity along the water surface and its corresponding position for a range of Rayleigh numbers. It is clear in Figure 13 that, for a given $Ra$, the peak velocity increases with time and then approaches a constant value. For a higher $Ra$, the peak velocity increases faster with time and approaches the constant value earlier. The peak velocity at the quasi-steady state is larger for a higher $Ra$. The time history of the location of the peak velocity is more interesting. For $Ra = 2.1 \times 10^3$ and $2.1 \times 10^7$ the location initially moves in the offshore direction, reaching a maximum value, and then retracts towards the tip region. After that the location moves slowly in the offshore direction again until it stops at a final location. For $Ra = 2.1 \times 10^3$, this final location is closer to the tip compared with the maximum value at the end of the initial increasing stage. For $Ra = 2.1 \times 10^3, 2.1 \times 10^6, 2.1 \times 10^7$ the location moves in the offshore direction with time, and then approaches a constant value. The peak velocity continues to increase for a short period of time after its corresponding location has stopped moving after reaching a certain position.

Figure 14 summarizes the flow properties at the quasi-steady state for different $Ra$. Figure 14a shows that the horizontal velocity along the surface initially increases and then decreases with increasing offshore distance. For any location, the velocity at a higher $Ra$ is
greater. The velocity reaches its peak value at a location closer to the tip for a higher $Ra$. For $Ra = 2.1 \times 10^6$ and $2.1 \times 10^7$, the velocity drops from its peak value rapidly until the end of the sloping bottom region and then drops slowly to zero due to the presence of the endwall. The behaviour of the volumetric flow rate shown in Figure 14b is rather similar to that of the horizontal velocity.

The horizontal profiles of the temperature averaged over the local water depth for different $Ra$ are shown in Figure 14c. The average temperature is the highest at the tip region. It decreases rapidly with the increase of the offshore distance, and then decreases slowly with further increase of the offshore distance. A more rapid decrease in temperature means a larger temperature gradient, indicating stronger effect of conduction there. Obviously, for a smaller $Ra$, the region over which the temperature decreases rapidly is larger, indicating that the conduction-dominated region is bigger. This is consistent with the results of Mao et al.\(^{(12)}\). For a smaller $Ra$, the temperature is higher at the tip region and lower near the endwall region compared with that for a higher $Ra$. This indicates a larger horizontal temperature gradient in the reservoir at a smaller $Ra$. This is understandable because the temperature is already normalized by the surface flux ($H_0 h c^3$). It takes a longer time for the flow at a smaller $Ra$ to reach the quasi-steady state, which results in a larger normalized horizontal temperature gradient. On the other hand, at a smaller $Ra$, the conduction-dominated region is bigger and the flow velocity is smaller, indicating that convection is weaker, thus resulting in a larger temperature gradient.

Fig. 14 Horizontal profiles of the (a) Horizontal velocity along the water surface; (b) volumetric flow rate; and (c) temperature averaged over the local water depth for different $Ra$ at the quasi-steady state.
Figure 15 shows the intersecting position of the horizontal conduction and convection curves $X_i$ and the horizontal position of the peak velocity $X_m$ at the quasi-steady state for different $Ra$. For a given $Ra$, $X_i$ is always less than $X_m$. Both $X_i$ and $X_m$ decreases with increasing $Ra$, which is consistent with the previous observation that the conduction-dominated region becomes smaller at a higher $Ra$. Mao et al.\textsuperscript{(12)} has proposed a scaling correlation for $X_i$:

$$X_i \sim Ra^{-1/4} A^{3/2} h$$

(20)

Figure 15 clearly confirms the linear relationship between $X_i$ and $Ra^{-1/4}$.

Although the flow features at the quasi-steady state are consistent with the scaling analysis provided by Mao et al.\textsuperscript{(12)}, the transient behaviour of flow shows some unexpected aspects. Similar to the cases of $Ra < A^{-2}$, it takes much longer time for the flow to reach the quasi-steady state than that predicted by Mao et al.\textsuperscript{(12)}. For $Ra > A^{-2}$, it is expected that the flow reaches the quasi-steady state at $t_c < 1$, where $t_c$ is given in Mao et al.\textsuperscript{(12)} as:

$$t_c \sim X^{2/3} Ra^{-1/3} h^{2/3} K^{-1}$$

(21)

However, Figure 13 shows that the flow is still developing at $t \approx 10$ for $Ra = 2.1 \times 10^3$ and at $t \approx 4$ for $Ra = 2.1 \times 10^4$. Again, this difference may be due to the fact that the horizontal conduction effect was not considered in Mao et al.\textsuperscript{(12)}.

The time scale $t_c$ in Mao et al.\textsuperscript{(12)} indicates that it takes a longer time to reach the quasi-steady state at a location further away from the tip. To validate this time scale, the surface velocities at different locations for $Ra = 2.1 \times 10^7$ are shown in Figure 16. Note that the velocity is normalized by its corresponding local velocity at the quasi-steady state. All the locations selected are within the convection-dominated region and the sloping bottom region to ensure the validity of the scale. It is seen in Figure 16 that all the velocity curves collapse together, indicating that the flow in the whole convection-dominated region reaches the quasi-steady state at the same time. Again, this deviation may be due to the different configurations used in the scaling analysis of Mao et al.\textsuperscript{(12)} and the present simulation.

Mao et al.\textsuperscript{(12)} also proposed a
velocity scale for the convection-dominated region at the quasi-steady state,
\[ u \sim Ra^{1/3} h^{-4/3} x^{1/3} \kappa \]  
(22)

This equation indicates that the velocity increases with the offshore distance. Without the presence of the endwall and the Rayleigh-Bénard type instability, it is expected from their scaling that the velocity keeps increasing in the offshore direction. However, the present simulations show that the peak velocity along the surface always occurs at a location before the end of the sloping bottom region. The location of the peak velocity moves towards the tip region with increasing \( Ra \), which is consistent with the trend of the intersection of the horizontal conduction and convection curves. This suggests that the location of the peak velocity may also approximately indicate the dividing point between the conduction- and convection-dominated regions. In this sense, the present study suggests that the velocity in the convection-dominated region decreases with the increasing offshore distance, which seems contradictory to the velocity scale provided by Mao et al.\(^{12}\). However, as already noted, Mao et al.\(^{12}\) considered an infinite triangular domain without an endwall while the present study considers a finite sloping bottom region followed by a flat bottom region with an endwall. As the unequal heat gain only occurs in the sloping bottom region, there is no driving force in the flat bottom region. The horizontal velocity in this region is reduced by the viscous effect. Moreover, although the endwall is far away from the sloping bottom region, it may still significantly contribute to the velocity decrease, especially for the high \( Ra \) case with a large horizontal velocity as shown in Figure 14a. Thus, this deviation may also be caused by the geometric variation.

4. Conclusion

In the present study, we numerically investigate transient natural convection in a reservoir model induced by surface heating. Although this type of flow has been carefully studied by Lei & Patterson\(^{11}\) and Mao et al.\(^{12}\), using both numerical simulations and scaling analysis, some new features of the transient behaviour of the flow are revealed from the present simulations. It is found that it takes much longer time for the flow in the conduction-dominated region to reach the quasi-steady state than the time scale predicted by Mao et al.\(^{12}\). At an instantaneous time, within the sloping bottom region, the surface velocity initially increases and then decreases with an increase of the offshore distance. This indicates that the velocity in the convection-dominated region decreases with the offshore distance instead of increasing with the offshore distance as predicted by Mao et al.\(^{12}\). The present study also indicates the flow in the whole convection-dominated region reaches the quasi-steady state at the same time instead of sequentially in the offshore direction as predicted by Mao et al.\(^{12}\). Over a certain range of Rayleigh numbers, the location of the peak surface velocity initially moves in the offshore direction and then retracts with the elapse of time. The intersection of the horizontal conduction and convection curves also retracts towards the tip region with the elapse of time. These two phenomena imply a retraction of the conduction-dominated region.

One of the possible reasons for the above mentioned deviations between the present study and Mao et al.\(^{12}\) is that different reservoir configurations are considered. The scaling analysis of Mao et al.\(^{12}\) considered an infinite sloping bottom region with no bounding boundary at the deep end, which is quite different from the present configuration. In the present simulation, the driving force is absent in the flat bottom part. The viscous effect reduces the horizontal velocity in this region. With the presence of the endwall, the velocity and convection are further reduced there. On the other hand, the presence of the endwall forces the flow to turn over near the endwall. This returning flow may also greatly affect the transient behaviour of the flow. Moreover, Mao et al.\(^{12}\) did not consider the effect of horizontal diffusion. The present study suggests that in the conduction-dominated region, the horizontal diffusion may play an important role as the time for flow to reach
quasi-steady state is $O(100)$, which is corresponding to the diffusion time in the horizontal direction. Finally, the model used to represent the reservoir in the present study is an enclosed chamber with all boundaries insulated except for the surface. Thus, the energy gain/loss from the surface will cause an increase/decrease in the average temperature within the reservoir. However, this is irrelevant to the investigation of Mao et al.\(^{(12)}\) as they considered an infinitely long open triangular chamber. This difference in the energy balance may also contribute to the deviations of the transient flow behaviour between the present study and theirs. The present results suggest that a new set of scales beyond those reported in Mao et al.\(^{(12)}\) is required to capture the transient behaviour of the flow in the present configuration.

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