Theoretical Study on the Ice Melting with Transient Double Effects of Temperature and Concentration*

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Abstract

The melting of the ice plate with the transient double effects of the temperature and concentration was studied theoretically. In the present study, our main attention was paid to the early stage of melting, when the abrupt changes of the temperature and concentration distributions occur. The results of the present theoretical study were summarized as follows. (1) It was found that the approximate similar solutions can be obtained under the certain assumptions of the velocity component, and we derived the simple formula to evaluate the melting mass of the ice plate. (2) The present formula can evaluate the melting mass of the horizontal ice plate more correctly than the case of the vertical ice plate.

Key words: Natural Convection, Phase Change, Ice, Transport Phenomena, Diffusion

1. Introduction

Melting of ice in mixture induces the abrupt change of the temperature and concentration distributions near the melting front. This abrupt change causes the transient double diffusive natural convection which significantly influences the melting behavior of ice. For this reason, the transient double diffusive natural convection has attracted the interest of many researchers and many studies have been carried out (for example, Carey and Gebhart^{(1),(2)}; Sugawara, Inaba, Nishimura and Mizuno⁽³⁾; Beckermann and Viskanta⁽⁴⁾; Fukusako, Tago, Yamada, Kitayama and Watanabe⁽⁵⁾; Sugawara, Irvin and Tago⁽⁶⁾; Komatsu, Sugawara and Fujita⁽⁷⁾). In these existing studies, the experimental approaches or the numerical approaches were mainly adopted to understand the melting of ice in mixture. Of course, these approaches play an important role to study the ice melting, but it is clear that the theoretical study is effective to understand the physical mechanism of melting. Moreover, the theoretical study plays an important role to establish the simple method to evaluate the melting mass, which has an important meaning from a practical viewpoint.

For this reason, we studied the ice melting with the transient double diffusive natural convection theoretically. When the melting of ice begins, the temperature and concentration distributions near the melting front change rapidly. However, these changes will gradually calm with the progress of time. In the present study, our main attention was paid to the stage when the abrupt changes of the temperature and concentration distributions occur. We call this stage as the early stage of melting. At first, we tried to derive the approximate solutions theoretically for the initial phase of the vertical ice melting through the use of similar solutions. As a result, it was found that we can obtain the similar solutions under the conditions of certain assumptions. Next, we derived the simple formula to evaluate the melting mass of the ice and compared the data obtained from the

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present formula with the existing experimental data. On the basis of these results, we consider the validity of the present approximate solutions. Moreover, we applied the present formula to the melting of the horizontal ice plate and considered the effective range or a limitation of the present formula. Practical use of the ice or snow is an important problem for a long time in the northern countries. The final target of the present study is the contribution for the practical use of the ice or snow.

Nomenclature

a^*	: see Eq.(14)
c	: specific heat
C	: solute concentration
D	: diffusion coefficient
g	: gravitational acceleration
H	: depth of liquid layer
Gc	: concentration Grashof number, $Gc = g\beta_c(C_w - C_0)L^3 / (\mu / \rho)^2$
Gr	: thermal Grashof number, $Gr = g\beta(T_w - T_0)L^3 / (\mu / \rho)^2$
L	: characteristic length
m	: melting mass per unit area and time
M	: melting mass per unit area
Pr	: Prandtl number, $Pr = \mu c / \lambda$
Sc	: Schmidt number, $Sc = \mu / (\rho D)$
t	: time
T	: temperature
u	: x -component velocity
v	: y -component velocity
W	: width of liquid layer
x	: horizontal coordinate component
y	: vertical coordinate component
z^*	: see Eq.(25)

Greek Symbols

β	: temperature expanding coefficient
β_c	: concentration expanding coefficient
ϕ^*	: see Eq.(18)
η^*	: see Eq.(15)
λ	: thermal conductivity
μ	: viscosity
ρ	: density

Subscripts

0	: initial value
w	: value at the melting front
*	: non-dimensional value

2. Vertical Ice Melting

2.1 Model and Analysis

The schematic model of a two-dimensional ice melting in mixture and the coordinate system are shown in Fig.1. As mentioned above, the simple model was used to obtain the

approximate solutions and to clarify the basic characteristics of ice melting. In the present study, the flow is assumed to be incompressible and laminar. In addition, it is assumed that the flow is a Newtonian fluid and the Boussinesq approximation is applicable. Near the melting front ($x=0$), it is clear that the change along the x -direction is much vehement compared to the change along the y -direction (see Fig.1). That is, the following relationship holds true near the melting front ($x=0$).

$$|\partial/\partial x| \gg |\partial/\partial y|.$$

For these reasons, we can obtain the following governing equations.

$$\partial u / \partial x = 0, \quad (1)$$

$$\rho(\partial v / \partial t + u \partial v / \partial x) = \mu \partial^2 u / \partial x^2 + \rho g \beta (T - T_0) + \rho g \beta_c (C - C_0), \quad (2)$$

$$\rho c (\partial T / \partial t + u \partial T / \partial x) = \lambda \partial^2 T / \partial x^2, \quad (3)$$

$$\partial C / \partial t + u \partial C / \partial x = D \partial^2 C / \partial x^2. \quad (4)$$

Some additional explanations are presented here. In the case of the boundary layer flow, the velocity component parallel to the wall is much greater than the velocity component perpendicular to the wall. However, our main attention was paid to the early stage of melting as mentioned above. So, it is easily expected that the velocity component parallel to the wall does not develop sufficiently and the order of the velocity component parallel to the wall is the same to the order of the velocity component perpendicular to the wall. We can understand the adequacy of these equations from these considerations.

Subsequently, let us consider the initial and boundary conditions. At $t=0$, the fluid velocity is assumed to be zero everywhere. And the concentration and temperature in mixture are assumed to be constants everywhere. So, the initial conditions are written as follows.

$$u = v = 0, \quad C = C_0, \quad T = T_0 \quad \text{for all } x > 0. \quad (5)$$

As for the boundary conditions, the following conditions are applied at the melting front ($x=0$).

$$u = v = 0, \quad C = C_w, \quad T = T_w, \quad C_w m = \rho D \partial C / \partial x \quad \text{at } x = 0. \quad (6)$$

In this equation, we can obtain the value of C_w from the thermodynamic equilibrium condition and the temperature at the melting front⁽⁸⁾. Additionally, the following conditions are applied in the limit of $x \rightarrow \infty$.

$$u = v = 0, \quad C = C_0, \quad T = T_0 \quad \text{as } x \rightarrow \infty. \quad (7)$$

Usually, the heat balance equation including the phase change is added as the boundary condition at the melting front. However, we assumed that the temperature at the melting front is kept constant ($= T_w$). So, we can omit this condition. Of course, the temperature at the melting front changes with time, but the change of the melting front temperature is not so violently⁽⁶⁾. For this reason, we can understand the reliability of this assumption

Fig.1 Schematic model of vertical ice melting and coordinate system

easily.

Here, we introduce the following non-dimensional quantities.

$$t_* = \mu t / (\rho L^2), \quad x_* = x / L, \quad y_* = y / L, \quad u_* = \rho L u / \mu, \quad v_* = \rho L v / \mu. \quad (8)$$

$$T_* = (T - T_0) / (T_w - T_0), \quad C_* = (C - C_0) / (C_w - C_0). \quad (9)$$

By using these non-dimensional quantities, we can obtain the following non-dimensional equations.

$$\partial u_* / \partial x_* = 0, \quad (10)$$

$$\partial v_* / \partial t_* + u_* \partial v_* / \partial x_* = \partial^2 u_* / \partial x_*^2 + Gr T_* + Gc C_*, \quad (11)$$

$$\partial T_* / \partial t_* + u_* \partial T_* / \partial x_* = Pr^{-1} \partial^2 T_* / \partial x_*^2, \quad (12)$$

$$\partial C_* / \partial t_* + u_* \partial C_* / \partial x_* = Sc^{-1} \partial^2 C_* / \partial x_*^2. \quad (13)$$

Next, we try to obtain the analytical solutions of these equations. At first, let us consider the solution of Eq.(10). From this equation, we can find that the function of the x -component velocity u_* is a constant or a function of t_* . In the present study, we assume the function u_* is given by

$$u_* = a_* t_*^{-1/2}, \quad (14)$$

where a_* is a constant. It should be noted that this assumption allows the existence of the similar solutions⁽⁹⁾. This is the most important feature of this assumption. On the other hand, the x -component velocity u_* means the injection flow due to the melting of ice. So, this assumption means that the injection flow due to the melting decreases suddenly with the progress of time. This result is easy to understand. For these reasons, we adopt this assumption. Of course, the comparison with the experimental results must be carried out to clarify the validity of the present study. Here, we introduce the following quantity according to Hashimoto⁽⁹⁾.

$$\eta_* = x_* / (2t_*^{1/2}). \quad (15)$$

Through the use of Eqs.(14) and (15), Equations (12)-(13) can be rewritten as follows.

$$T_*'' + 2Pr(\eta_* - a_*)T_*' = 0, \quad (16)$$

$$C_*'' + 2Sc(\eta_* - a_*)C_*' = 0. \quad (17)$$

In these equations, the prime (') means the differentiation with respect to η_* . As for Eq.(11), some consideration is needed to obtain the similar solution. To obtain the solution, we introduce the quantity ϕ_* which is given by

$$\phi_* = v_* / t_*. \quad (18)$$

By using this quantity, Eq.(11) is rewritten as

$$\phi_*'' + 2(\eta_* - a_*)\phi_*' - 4\phi_* = -4GrT_* - 4GcC_*. \quad (19)$$

Here, we introduce the following function^{(10), (11)}.

$$I_n(z) = \int_z^\infty \frac{1}{n!} (z-s)^n \exp\left(-\frac{s^2}{2}\right) ds, \quad (n = 0, 1, 2, \dots). \quad (20)$$

It should be noted that this function has the following important features.

$$\begin{aligned} \{I_0(A(\eta_* - a_*))\}'' + A^2(\eta_* - a_*)\{I_0(A(\eta_* - a_*))\}' &= 0, \\ \{I_2(A(\eta_* - a_*))\}'' + 2(\eta_* - a_*)\{I_2(A(\eta_* - a_*))\}' - 4I_2(A(\eta_* - a_*)) \\ &= (A^2 - 2)I_0(A(\eta_* - a_*)). \end{aligned}$$

In these equations, A is a constant. By using this function, we can obtain the solutions of Eqs.(16) and (17) which can satisfy the boundary conditions. That is,

$$T_* = I_0(\sqrt{2Pr}(\eta_* - a_*))/I_0(-a_*\sqrt{2Pr}), \quad (21)$$

$$C_* = I_0(\sqrt{2Sc}(\eta_* - a_*))/I_0(-a_*\sqrt{2Sc}). \quad (22)$$

In addition, the solution of Eq.(19) can be written as

$$\phi_* = f_{*1} + f_{*2} + f_{*3}, \quad (23)$$

$$f_{*1} = K_1 I_2(\sqrt{2}(\eta_* - a_*))/I_2(-a_*\sqrt{2}),$$

$$f_{*2} = K_2 I_2(\sqrt{2Pr}(\eta_* - a_*))/I_0(-a_*\sqrt{2Pr}),$$

$$f_{*3} = K_3 I_2(\sqrt{2Sc}(\eta_* - a_*))/I_0(-a_*\sqrt{2Sc}),$$

$$\begin{aligned} K_1 &= 2GrI_2(-a_*\sqrt{2Pr})/((Pr-1)I_0(-a_*\sqrt{2Pr})) \\ &\quad + 2GcI_2(-a_*\sqrt{2Sc})/((Sc-1)I_0(-a_*\sqrt{2Sc})), \end{aligned}$$

$$K_2 = -2Gr/(Pr-1),$$

$$K_3 = -2Gc/(Sc-1).$$

Some additional explanations are presented here. Hossain and Rashid⁽¹¹⁾ studied the Hall effects on the hydromagnetic flow with mass transfer theoretically and they obtained the solutions of velocity, temperature and concentration fields. In their analysis, the natural convection of the hydromagnetic fluid was taken into consideration. For this reason, the present solutions almost agreed with their results. This result means that these results will have a wide adaptation fields. However, they didn't consider the phase change of the test fluid and cannot determine the value of a_* in Eq.(14). In the present study, we determine the value of a_* through the use of the boundary condition.

At first, we rewrite the boundary condition of Eq.(6) with the non-dimensional quantities. The results is given by

$$u_* = \frac{1}{Sc} \left(1 - \frac{C_0}{C_w} \right) \frac{\partial C_*}{\partial x_*} \Big|_{x_*=0}. \quad (24)$$

To derive this equation, we should note the relation of $m = \rho u$. Moreover, we can obtain the following result through the use of Eq.(24).

$$z_* = \left(\frac{C_0}{C_w} - 1 \right) \frac{1}{I_0(-z_*)} \exp\left(-\frac{1}{2}z_*^2\right), \quad a_* = \frac{z_*}{\sqrt{2Sc}}. \quad (25)$$

We can determine the value of a_* from these equations. These equations mean that the value of a_* is in proportion to $\sqrt{Sc}$ and z_* denotes the proportional coefficient

between a_* and $\sqrt{Sc}$. Moreover, these equations show that z_* is a function of the concentration. These results will play an important role to understand the calculated results. From a practical viewpoint, the melting mass per unit area M has an important meaning. This value is given by

$$M = \int_0^t m dt = \int_0^{t_*} \rho a_* L / \sqrt{t_*} dt_* = 2a_* \rho L \sqrt{t_*}. \quad (26)$$

This equation means that the value of M is in proportion to $\sqrt{t}$. Moreover, we can find that the value of M is in proportion to $\sqrt{D}$, too (see the definition of a_* and Sc).

2.2 Results and Comparison with the Experimental Data

As mentioned above, the following important relations were obtained from the present formulas.

- 1) The melting mass per unit area is in proportion to a square root of the time.
- 2) The melting mass per unit area is in proportion to a square root of the mass diffusion coefficient.

We must consider the propriety of these results at first. For this reason, we compare these results with the existing data. As the detailed characteristics of the melting mass per unit area were reported by Sugawara, Irvine and Tago⁽⁶⁾, the present results are compared with

Fig.2 Schematic model of the experimental apparatus⁽⁶⁾

Table 1 Physical properties used in the comparison with the experimental data

c [kJ/(kg·K)]	3.044
D [m ² /s]	0.3×10^{-9}
Pr	22
β [1/K]	3.3×10^{-4}
β_c [1/(kg/kg)]	-0.916
λ [kJ/(m·s·K)]	5.35×10^{-4}
μ [kg/(m·s)]	3.86×10^{-2}
ρ [kg/m ³]	1.1939×10^3

Fig.3 Comparison of the present results with the experiments⁽⁶⁾

Fig.4 Influence of the diffusion coefficient on the melting mass per unit area

their experimental data. Figure 2 shows the schematic model of their experiments. In this figure, we show the half model because of the symmetry. They used the CaCl_2 aqueous solution and an initial temperature of the liquid was set at -5°C , which equals to the initial temperature of the ice. In their experiments, the liquid height H was set at 100 mm and three different liquid widths $W = 8, 15, 50$ mm were adopted. The liquid height H was used for the definition of Gr and Gc in this chapter. In addition, the physical properties used in the present calculations are shown in Table 1. Figure 3 shows the comparison of the results by the present formulas with the experiments by Sugawara, Irvin and Tago⁽⁶⁾ for the melting mass per unit area. Their experiments show that the liquid width has a weak influence on the melting mass per unit area. On the other hand, our main attention was paid to the early stage of melting near the front and the influence of the side wall was not taken into consideration. So, we can easily expect the limitation of the present study. Indeed, this figure shows that the difference between the present results and the experimental results increases with the time. However, this figure shows that the predictions by the present formulas are effective within the early stage of melting. These results suggest that the present formula is effective under some limitation. More detailed discussion will be performed later.

Sugawara, Irvin and Tago⁽⁶⁾ were studied the relation between the mass diffusion coefficient and the melting mass per unit area numerically. It is well known that the studies on the mass diffusion coefficient of CaCl_2 aqueous solutions are not enough to determine the exact values⁽⁶⁾. In other word, the value of the mass diffusion coefficient in Table 1 is expected to possess some uncertainty. For this reason, the relation between the mass diffusion coefficient and the melting mass per unit area will have an important meaning to evaluate the influence of the uncertainty related to the mass diffusion coefficient. It was shown that the diffusion coefficient $D_{0.3} = 0.3 \times 10^{-9} \text{ m}^2/\text{s}$ is thought to be a plausible value within the present conditions⁽⁶⁾. So, this value was used as a reference one after Sugawara, Irvin and Tago⁽⁶⁾. Figure 4 shows the relation between the value of $D/D_{0.3}$ and $M/M_{0.3}$. In this figure, $M_{0.3}$ is the melting mass per unit area under the condition of $D = D_{0.3}$. From this figure, we can find the close agreement between the results obtained from the present formulas and the numerical results by Sugawara, Irvin and Tago⁽⁶⁾. This result means the adequacy of the present formulas.

Equation (25) shows that the value of z_* has a strong relation with the melting mass. So, this value will play an important role from a practical viewpoint. For this reason, we show the relation between the value of z_* and $C_0/C_w - 1$ in Fig.5. This figure shows that the value of z_* increases with the increase in $C_0/C_w - 1$. This result means that the increase in the concentration enhances the melting of ice. This result is easy to understand. Moreover, this figure shows that the slope of this curve decreases with the increase in $C_0/C_w - 1$. This result means that the sensitivity of the concentration to the melting mass becomes remarkable in a low concentration region.

Subsequently, we consider the velocity distributions near the melting front. Usually, the diffusion coefficient D is much smaller than the kinetic viscosity coefficient μ/ρ . That is, the value of the Schmidt number Sc becomes much greater than 1.0. Moreover, the order of Gc is much greater than the order of Gr . It is clear that the differences of these non-dimensional parameters make the numerical calculations difficult. However, these values of non-dimensional parameters are not so important problem in the present study because of the analytical study. The y -component velocity distributions are shown in Figs.6 - 9. In these figures, ϕ_* is given in Eq.(18) and the values of the non-dimensional parameters were determined after the study of Sugawara, Irvin and Tago⁽⁶⁾. Some additional explanations are presented here. Of course, we can fix the value of z_* from Eq.(25). However, in these figures, we changed the value of z_* within the range of

$0 \leq z_* \leq 1$. These calculations will be effective to understand the influence of the injection flow due to the melting. It should be noted that the influence of the injection flow due to the melting on the velocity distributions is not studied yet. So, these calculations will have an important meaning. Figure 6 shows the results for $Gr = -1 \times 10^5$, $Gc = 2 \times 10^7$, $Pr = 20$ and $Sc = 1.0 \times 10^4$. Some additional explanations are presented here. In the case of the ice melting, the temperature at the melting front T_w is lower than the initial liquid temperature T_0 . This relation leads to the condition of $Gr < 0$. In addition, the concentration at the

Fig.5 Relation between z_* and $C_0 / C_w - 1$

Fig.6 Distribution of the vertical velocity component
($Gr = -1.0 \times 10^5$, $Gc = 2.0 \times 10^7$,
 $Pr = 20$, $Sc = 1.0 \times 10^4$)

Fig.7 Distribution of the vertical velocity component
($Gr = -1.0 \times 10^5$, $Gc = 6.0 \times 10^7$,
 $Pr = 20$, $Sc = 1.0 \times 10^4$)

Fig.8 Distribution of the vertical velocity component
($Gr = -1.0 \times 10^6$, $Gc = 2.0 \times 10^7$,
 $Pr = 20$, $Sc = 1.0 \times 10^4$)

Fig.9 Distribution of the vertical velocity component
($Gr = -1.0 \times 10^6$, $Gc = 6.0 \times 10^7$,
 $Pr = 20$, $Sc = 1.0 \times 10^4$)

melting front C_w is lower than the initial concentration C_0 . This relation and the condition of $\beta_c < 0$ lead to the result of $Gc > 0$. We can understand the sign of the non-dimensional parameters easily from these considerations. From this figure, we can find that the y -component velocity changes suddenly near the melting front and the positive velocity region appears. Moreover, we can find the injection flow due to the ice melting has an effect to increase the y -component velocity. The low temperature region near the melting front brings on the descending flow. And the low concentration region near the melting front brings on the ascending flow. Moreover, the thickness of the temperature boundary layer is much thinner than the thickness of the concentration boundary layer. The result of this figure will be explained by these considerations easily. In addition, it is clear that the injection flow due to the ice melting has an effect to decrease the concentration near the melting front. We can understand the reason that the injection flow has an effect to increase the ascending flow easily from this consideration. This discussion does not conflict with the result of Fig.6. Figure 7 shows the results for $Gr = -1 \times 10^5$, $Gc = 6 \times 10^7$, $Pr = 20$ and $Sc = 1.0 \times 10^4$. It is clear that the increase in Gc has an effect to enhance the ascending flow due to the difference of the concentration. The remarkable ascending flow of this figure will be explained by this consideration. The results for large thermal Grashof number are shown in Figs.8 and 9. The large thermal Grashof number enhances the descending flow near the melting front because of the density difference due to the temperature difference. The results shown in these figures are not conflict with this consideration. Moreover, these figures show that the large thermal Grashof number has an effect to weaken the influence of the injection flow (influence of z_*). This result is easy to understand. These velocity distributions are reasonable and agree with the results of Sugawara, Irvin and Tago⁽⁶⁾ qualitatively. For this reason, we can understand the validity of the present results.

3. Horizontal Ice Melting

3.1 Model and Analysis

Subsequently, the horizontal ice melting is analyzed theoretically. The schematic model of a two-dimensional horizontal ice melting in mixture and the coordinate system are shown in Fig.10. In the analysis of the horizontal ice melting, we adopted the same assumptions and same non-dimensional quantities mentioned above. Moreover, the following relationship holds true near the melting front ($y = 0$).

$$|\partial / \partial y| \gg |\partial / \partial x|.$$

Therefore, we can obtain the following governing equations.

$$\partial v / \partial y = 0, \quad (27)$$

$$\rho c (\partial T / \partial t + v \partial T / \partial y) = \lambda \partial^2 T / \partial y^2, \quad (28)$$

$$\partial C / \partial t + v \partial C / \partial y = D \partial^2 C / \partial y^2. \quad (29)$$

From Eq.(27), we can find that the function of the y -component velocity is a constant or a function of time. The y -component velocity means the injection flow due to the melting of ice and it is clear that the y -component velocity decreases suddenly with the progress of time. For this reason, we assume the non-dimensional y -component velocity v_* is given by

$$v_* = -a_* t_*^{-1/2}. \quad (30)$$

Some additional explanations are presented here. In case of the vertical ice melting, we used the Navier-Stokes equation to obtain the y -component velocity. However, in this case,

we assumed the y -component velocity as shown in Eq.(30). In addition, the x -component velocity does not play the important role. So, the Navier-Stokes equation was not required here. We can estimate the boundary conditions of the horizontal ice melting easily, and the method to obtain the analytical solution is the same to the case of the vertical ice melting. For this reason, these explanations are omitted here. As the results of the theoretical consideration, it was found that the melting mass per unit area M , which has an important meaning from a practical viewpoint, is given by the same equation to the vertical ice melting. This result is worthy to attention, and means the wide applicable range of Eq.(26). Of course, the comparison with the experimental data is required.

Fig.10 Schematic model of horizontal ice melting and coordinate system

3.2 Results and Comparison with the Experimental Data

The experimental study on the horizontal ice melting was carried out by Sugawara, Hayata, Tago and Fujita⁽¹²⁾. Figure 11 shows the schematic model of their experimental apparatus. They used the CaCl_2 aqueous solution and the initial temperature of the ice and solution was set at -5°C . Additionally, the value of liquid width W was set at 100 mm and two different liquid depths $H = 25, 50\text{ mm}$ were adopted. In addition, the physical properties used in the present calculations are shown in Table 1. Figure 12 shows the comparison of the present results with their experimental results. From this figure, we can find that the present model can evaluate the melting mass of the horizontal ice more exactly than the melting mass of the vertical ice. This result has an important meaning. The reason of this result will be discussed in the next chapter.

Fig.11 Schematic model of the experimental apparatus on the horizontal ice melting⁽¹²⁾

4. Discussion

At first, let us consider the results of the vertical ice melting. In this case, we omitted the term of $v\partial T/\partial y$ in the energy equation (Eq.(3)) because of the following inequality. That is,

$$|\partial/\partial x| \gg |\partial/\partial y|.$$

At the initial stage of melting, the y -component velocity v does not developed sufficiently. So, this simplification is thought to be correct. However, the y -component velocity v develops with a lapse of time. For this reason, it is easily expected that the validity of this

Fig.12 Comparison of the present results with the experiments on the horizontal ice melting⁽¹²⁾

simplification becomes poor with a lapse of time. The result of Fig.3 will be explained by this consideration. Subsequently, let us consider the results of the horizontal ice melting. In this case, we omitted the term of $u\partial T/\partial x$ in the energy equation (Eq.(28)) because of the following inequality. That is,

$$|\partial/\partial y| \gg |\partial/\partial x|.$$

In this case, we can easily expect that the x -component velocity u does not develop even after a sufficient time. So, it is expected that this simplification is effective for a long time. The result of Fig.12 will be explained by this discussion.

The formula to estimate the melting mass with the transient double effects of the temperature and the concentration has not been proposed until now. So, we can easily understand the important meaning of the present study, but the present formula has some limitation. For this reason, the study to clarify the limitation of the present formula will be required to enhance the validity of the present formula. Future studies are anticipated.

5. Concluding Remarks

The approximate solution for the vertical ice melting with the transient double effects of temperature and concentration was derived theoretically. In addition, we proposed the simple formula to evaluate the melting mass. At first, the present formula was applied to the comparison with the experimental data of the vertical ice melting and it was found that the present result is effective within the early stage of the vertical ice melting. Subsequently, we applied the present formula to the melting of the horizontal ice and it was found that the present formula is effective for a long time. Lastly, we discussed the reason of these results.

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