208 Wavelet Reynolds Stress Analysis of Flow Structure in a Bounded Jet

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Abstract

The major purpose of the present study is first to propose the definition of the wavelet Reynolds stress using the wavelet transform, in order to study the characteristics of Reynolds stress in frequency space. Then, the fluctuating velocities at various positions in a bounded turbulent jet are analyzed by the wavelet transform and wavelet Reynolds stress analysis. The wavelet Reynolds stress analysis reveals the contributions of the different frequency to the Reynolds stress, which is directly related to coherent structure of various scale eddies.

1. Introduction

Since wavelet analysis was first proposed by Morlet, it has been applied mostly to various fields because of its ability to decompose arbitrary signals into localized contributions that can be labeled in terms of time and scale.

Recently, wavelet analysis has also been applied mostly in the field of fluid mechanics. Lima and Toh⁽¹⁾ applied wavelet analysis to analyzing the energy transfer processes within and across scales due to spatial structures. Zimin and Hussain⁽²⁾ developed a simple model for the locally isotropic turbulence using a vector wavelet decomposition of the velocity field and applied it to representing the small-scale turbulence in LES schemes. Wavelet auto-correlation and cross-correlation analysis, which was proposed by authors⁽³⁾⁻⁽⁷⁾, have been applied to analyze the similar structure of flow and provide a very clear picture of the eddy coherent structure both in time space and in scale space.

In the present study, the definition of the wavelet Reynolds stress using the wavelet transform is firstly proposed, in order to study the characteristics of Reynolds stress in frequency space. Then, the fluctuating velocities at various positions in a bounded turbulent jet are analyzed by the wavelet transform and wavelet Reynolds stress analysis.

2. Definition of Wavelet Transform

The wavelet transform of a function $f(t)$ is defined as

$$Wf(b, a) = \frac{1}{a} \int_{-\infty}^{\infty} f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt \quad (1)$$

where $Wf(b, a)$ is called the wavelet coefficients of $f(t)$ with respect to the analyzing wavelet $\psi(\xi)$, a and b are the wavelet scale and analyzing position (here $\bar{}$ and $\wedge$ stand for complex conjugate and Fourier transform respectively) respectively. In this paper the

following revised form of the Mexican hat function, which is proposed by authors⁽³⁾, is used as the analyzing wavelet.

$$\psi(\xi) = \left\{1 - (k\xi)^2\right\} \exp\left(-\frac{(k\xi)^2}{2}\right) \quad (2)$$

where $k = \sqrt{2}\pi$. Its Fourier transform is written as

$$\hat{\psi}(\omega) = \frac{2\sqrt{2}\pi}{k} \left(\frac{\omega}{k}\right)^2 e^{-\frac{(\omega/k)^2}{2}}. \quad (3)$$

From Eq.(2) and (3) it is obvious that $\psi(t)$ is localized around $t = 0$, and $\hat{\psi}(\omega)$ is localized around the central frequency $\omega_0 = \sqrt{2}k$. In applications of signal processing it has been found that a particularly useful value for the central frequency ω_0 is the one for which the wavelet scale a presents the period or frequency. Therefore the central frequency is defined as $\omega_0 = 2\pi$ for $k = \sqrt{2}\pi$ in this paper. Therefore, the wavelet coefficient Eq.(1) can describe a signal as localized strength of the signal over a time-period plane.

The wavelet transform can also be expressed in Fourier space via

$$Wf(b, a) = \frac{\sqrt{a}}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) \hat{\psi}(a\omega) e^{ib\omega} d\omega, \quad (4)$$

where $\hat{f}$ and $\hat{\psi}$ are the Fourier transform of f and ψ respectively.

3. Definition of Wavelet Reynolds Stress

The Reynolds stress component $-\overline{uv}$ can be written as the following form

$$-\overline{uv} = -\frac{1}{T} \int_{-\infty}^{\infty} u(t)v(t)dt = -\frac{1}{T} \int_{-\infty}^{\infty} \hat{u}(\omega)\hat{v}(\omega)d\omega. \quad (5)$$

Using Eq.(5) it is impossible to determine the Reynolds stress distribution in frequency space. In the present study, using the wavelet coefficient of fluctuating velocities u and v , the definition of the wavelet Reynolds stress $-\overline{WuWv}(a)$ is proposed by the formula

$$-\overline{WuWv}(a) = -\frac{1}{T} \int_{-\infty}^{\infty} Wu(a, b)Wv(a, b)db. \quad (6)$$

From above equation it is clear that the wavelet Reynolds stress can describe the distribution of the Reynolds stress in scale space, and also be called the spectral distribution of Reynolds stress. Corresponding to Eq.(5), Eq.(6) can be rewritten as

$$-\overline{WuWv}(a) = -\frac{1}{T} \int_{-\infty}^{\infty} \hat{u}(\omega)\hat{v}(\omega)|\hat{\psi}(a\omega)|^2 d\omega. \quad (7)$$

The wavelet Reynolds stress permits the identification of the contributions of the different scales to Reynolds stress, which is a quantity directly related to the structures of the flow.

From the definition of the wavelet Reynolds stress in Eq.(6) or (7), it is evident that $-\overline{WuWv}(a)$ is complex-valued, and consists of a real part $-\overline{RWuWv}(a)$ and a imaginary part $-\overline{IWuWv}(a)$. Because $-\overline{RWuWv}(a)$ can express the strength of the wavelet Reynolds stress, it is used to analyze the structure of the Reynolds stress in the present study.

4. Application to Eddy Structure in a Bounded Jet

The experimental apparatus of the bounded jet is shown in Fig.1, where the origin O is taken at the center of the nozzle exit, the x-axis is the streamwise coordinate, the z-axis is

Fig.1 Experimental apparatus

perpendicular to the wall, and the y-axis is normal to x-axis and z-axis. The jet is generated by a blower-type wind tunnel having flow-straightening elements, screens, and settling length, a 24:1 contraction leading to a 200x100 mm nozzle ($2H=200\text{mm}$, $2b_0=100\text{mm}$). For all measurements, the jet is operated at a constant Reynolds number $Re=10000$ (exit velocity is 0.7m/s) based upon nozzle width $2b_0$. The velocity components of u , v are

measured simultaneously using a X type hot wire probe located in the (x, y) -plane. The recording frequency is 1.25 kHz .

The wavelet coefficients $Wf(t, a)$ of u -component near the wall ($x/b_0=2$, $y/b_0=0.96$ and $z/b_0=1.97$) is shown on the (t, a) B&W plane representation in Fig.2. Making a comparison between $Wf(t, a)$ and $u(t)$, we find that the positive and negative peaks of $Wf(t, a)$ in the large-period region correspond to large peaks of $u(t)$. From Fig.2 as the time t increases, the succession of positive and negative peaks for some periods are observed. The dominant feature of this figure is the periodic oscillation in the range of $a=170\text{-}200\text{ms}$ approximately. That indicates that the periodic large-scale motions pass through the mixing region near the wall.

Figure 3 shows the distribution of the wavelet Reynolds stress along z -direction at $x/b_0=2$ and $y/b_0=0.96$. At $z/b_0=1$ $-R\overline{WuWv}(f)$ reaches the peak of $f=5\text{Hz}$ can be observed. That indicates that the large-scale motion of $f=5\text{Hz}$ are much more active in this region. It corresponds to the results of the wavelet transform in Fig.2. The magnitude and frequency of the wavelet Reynolds stress associate with information of the ambient flow and the spots.

At the downstream distance $x/b_0=8$, the variation of wavelet Reynolds stress from the center line to the wall on the center plane ($y/b_0=0.0$) is shown in Fig.4. In this figure, it is obvious that the peak of $-R\overline{WuWv}(f)$ happens at $z/b_0=1$ for $f=4.5\text{Hz}$. When closing to the wall at $z/b_0=1.92$, $-R\overline{WuWv}(f)$ gives again a small peak for $f=4\text{Hz}$. They appear that the large-scale motion are much more active at $z/b_0=1$ and near the wall.

5. Conclusions

The main conclusions can be summarized as follows:

- (1) From the wavelet transform, the periodic large-scale motions of $a=170\text{-}200\text{ms}$ pass through in the mixing region near the wall at $x/b_0=2$.
- (2) By analyzing the wavelet Reynolds stress, the large-scale motion of $f=5\text{Hz}$ are much more active at $x/b_0=2$ and $z/b_0=1$.
- (3) At the downstream the active large-scale motions appear at $z/b_0=1$ and near the wall.

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Fig.2 Wavelet transform of the fluctuating velocity near the wall
($x/b_0=2$, $y/b_0=0.96$ and $z/b_0=1.97$)

Fig.3 Wavelet Reynolds stress
at $x/b_0=2$ and $y/b_0=0.96$

Fig.4 Wavelet Reynolds stress
at $x/b_0=8$ and $y/b_0=0.0$