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Particle-Based Flow Visualization

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Abstract. Several methods of flow measurement and flow visualization rely on the use of particulates to mark the fluid and make its motion visible, thereby rendering it amenable to optical study. In comparison to molecular markers like dyes, particles offer the unique capability of marking very small points in space which remain small and do not diffuse. In general, particle-based methods such as laser Doppler velocimetry (LDV) and particle image velocimetry (PIV) have been developed to a high degree, and they are used widely, especially for quantitative measurements. The properties of the particles must be managed correctly if the particle-based measurements are to be of the highest quality. The particles must follow the fluid motion accurately, they must scatter adequate light to produce high signal-to-noise ratio, and they must produce uniform, repeatable .

Introduction

Particles are a prevalent choice for marking fluids because they can be made small enough to follow the fluid with high fidelity and still scatter enough light to produce sensible images when illuminated by light of practical intensity. Particles also occur naturally in all fluids, unless the fluid has been cleaned with extreme care, but the best PIV results are obtained by using artificially seeded particles, thereby permitting careful control of the uniformity of the particle concentration and the optical and fluid mechanical properties of the particles. The particle properties of primary interest are those related to the light scattering efficiency and the particle dynamics. The former is characterized by the scattering as a function of the scattering angle and particle size, and the latter is characterized by a time constant of the particle which represents its ability to follow oscillations of the velocity of the fluid that surrounds a particle.

Particulate Markers

Particles are used to mark points in space. They are extremely well suited for this purpose because they are small, and large numbers of them can be dispersed throughout the fluid, so that the density of information, i.e. the number of measurements per unit volume, can be very large. With the exception of strongly magnified images, the particles used in PIV are typically smaller than the diffraction limited spot size of the recording optics, i.e. they are effectively point sources that sample fluid position with a spatial uncertainty that is negligible relative to the inherent optical uncertainty. In this sense, particles make it possible to extract the maximum velocity information from the flow. In comparison, patches of fluid marked by dyes are often considerably larger, of order of one hundred microns or more.

Dyes and other similar molecular markers disperse continually by diffusion, both molecular and turbulent. Because of these phenomena, they must be injected close to the site of measurement to achieve confined marking. Unlike dyes, particles retain their identities for long times, so that they continue to mark the fluid long after injection. This makes it possible to inject the particles before an experiment, or far upstream of the site of measurement.

When marking fluid for the purposes of flow visualization, one must inject the marker non-uniformly to optically differentiate different patches of fluid, i.e. by injecting dye locally with one or more narrow streams, or by creating narrow lines of hydrogen bubbles with pulses supplied to the bubble generating wire. In PIV it is important to strive instead to achieve a *uniform* distribution of scattering particles throughout all regions of the flow that pass through the measurement domain, so as to avoid preferentially sampling selected regions of the velocity field, and thereby biasing measurements toward more highly seeded regions. Thus, a typical PIV recording of a flow field appears to be a uniform cloud of particle images. In some experiments where there is an interface between different types of flow, as between the turbulent boundary layer and the non-turbulent free stream, or between burned and unburned gases in combustion system, it is desirable (or perhaps unavoidable) to have one region of the flow seeded differently from the other. In such cases, it is still best to have each region seeded uniformly. One frequently relies on turbulent mixing to achieve uniformity. However, it has been shown that turbulence can also create non-uniform concentration fluctuations in space if the particles do not follow the fluid streamlines. The mechanisms are simple: Vortices centrifuge particles from their centers, and stagnation point flows in the pattern of turbulent eddies concentrate particles around the stagnation point. Thus, in addition to errors caused by particles traveling at velocities different from the fluid, particle lag can also lead to errors by biasing the statistical sampling for or against certain regions of the flow. This error vanishes as the particle time constant becomes small.

Particles are easy and inexpensive to use. Unless special care is taken to clean a fluid carefully, most untreated fluids, especially room air and tap water, contain an abundance of particles which produce sensible images. The natural particles do not, however, lead to the best fluid mechanical measurements because they may have irregular geometry, the size, mass and intensity of scattered light varies from particle to particle, and the concentration of particles may not be optimum.

Since particles are the source of the images and signals, it is better to make the effort to remove natural particles from the fluid, and introduce well-chosen artificial seeding. This approach is feasible because particles are relatively easy to generate in large numbers. Solid particles in the form of powders are available in abundance, and liquid particles, i.e., droplets, can be generated in vast quantities by various aerosol generators. It is desirable to generate the particles with uniform properties, but less easy.

It is a fortunate coincidence that if the particles are of a size that scatters light adequately, i.e. about one-micron and larger, the Brownian motion of the particle is small, and contributes very little noise to the velocity measurement unless the fluid velocity is less than a fraction of a millimeter per second. In contrast, the thermal motion of molecular markers is several hundred meters per second. If a patch of molecules is used as a marker, the primary effect of thermal motion is the diffusion of the molecules, as mentioned above. However, if the velocities of the molecules are sensed directly, as in Doppler global velocimetry (DGV), then the fluid velocity needs to be large. Hence, even in the practice of DGV one finds that the instrument is operated in a mode that senses the motion of particles rather than molecules. Often this is unavoidable because the intensity of the light scattered from natural particles far exceeds the intensity of the molecular scattering.

Particulate markers are not without their disadvantages. Because of their large mass relative to the mass of the fluid molecules, particles may not follow the motion of the fluid. The lag results in erroneous velocity measurements. With proper seeding, errors due to particle lag can be reduced to acceptable levels, i.e. a fraction of one per cent in flows having velocities up to several hundred meters per second. This topic will be discussed at some length because of its importance. It is also the nature of small particles to be randomly located in the fluid. This either results in randomly located sites of velocity measurement, or the need for special methods of analysis such as correlation analysis which account for this random aspect of the images. Lastly, particles may cause chemical contamination, or otherwise affect the thermophysical properties of the fluid. Ordinarily, the concentration of particles is so low, that one does not expect to see effects due to two-phase phenomena. For example, in PIV one typically requires 10-20 particles per resolution cell, and a typical dimension for a resolution cell is 0.2-2 mm (Adrian 1991). The maximum concentration is therefore of order 2.5×10^{-6} particles mm^{-3} . If the particles are one micron diameter droplets in gas (a typical value) the volume concentration is 2.5×10^{-6} and the mass concentration is of order 10^{-3} . If the particles have 10 micron diameter, as is often required to achieve adequate scattering from particles in liquids, the volume concentration and the mass fraction are both of order 2.5×10^{-3} . In either case neither the mass concentration nor the volume concentration is large enough to have a significant effect on the mean density or viscosity of the fluid. That is, the fluid may contain millions of particle scattering sites without causing any two-phase flow phenomena that might influence the behavior of the flow by more than a fraction of one-percent. (Note, however, that at the maximum concentration cited in the foregoing example, the use of particles larger than 10 microns, i.e., 100 microns, would certainly introduce two-phase behavior. Particles larger than about 10 microns should be avoided, or if that is not possible, the concentration must be reduced.

The effects of small concentrations of particles can be important and must be evaluated to ascertain their importance when treating certain flow phenomena that involve an element of chemistry or fine scale behavior. For example, turbulent flows exhibit polymer drag reduction at only a few parts per million of polymers, and at these low levels, the

concentration of particles seems large, and perhaps significant. Similarly, in surface tension-driven phenomena the volume of the relevant surface layer is very small and particles may tend to collect in it, thereby raising their concentration to a high level in the region of most importance.

Particle velocity, Eulerian velocity and Flow Tracking Fidelity

Since measurements are taken wherever a particle happens to be at the time of measurement, the full trajectory of the particle is not of interest in PIV measurements of the fluid phase. Rather, the important quantity is slip if the particle relative to the fluid. Some slip must always occur if the flow is accelerating in order to create the drag force needed to accelerate the particle.

The equation governing a particle moving in a fluid is complicated in general, and will be discussed later. For the moment, the following simple equation provides a reasonable description:

$$\dot{\mathbf{v}}_p = \frac{\mathbf{u}(t) - \mathbf{v}_p(t)}{\tau_p} + \mathbf{b}. \quad (1)$$

Here $\mathbf{v}_p(t)$ is the velocity of the p th particle, $\mathbf{u}(\mathbf{x}, t)$ is the Eulerian velocity of the fluid, and

$$\mathbf{u}(t) = \mathbf{u}(\mathbf{x}_p(t), t) \quad (2)$$

is the fluid velocity at the location of the p th particle, $\mathbf{x}_p(t)$. The body force per unit mass of the particle is $\mathbf{b}$, (i.e. $\mathbf{b}$ equals the acceleration of gravity if the force is gravitational), and the conventional viscous time constant is given by

$$\tau_p = \frac{2\rho_p d_p^2}{9\rho_f \nu_f} \quad (3)$$

for the case of low Reynolds number Stokesian drag on the particle. In (3), d_p is the particle diameter, ρ_p is the particle density, ρ_f is the fluid density, and ν_f is the fluid kinematic viscosity. Equation (1) is obtained a simple balance of the Stokesian drag force and the particle inertia.

The equation that is normally used to infer the particle velocity from measurement of the particle displacement is

$$\mathbf{v}_p \approx \frac{M_o \Delta \mathbf{x}_p}{\Delta t} \quad (4)$$

here Δt is the time interval and $\Delta \mathbf{x}_p$ is the displacement of the particle during the time interval. This equation is based on a first order forward difference approximation which is only valid if the particle does not accelerate during the time increment. A more accurate formula is obtained from the exact equation for the particle displacement,

$$\Delta \mathbf{x}_p = \mathbf{x}_p(t + \Delta t) - \mathbf{x}_p(t) = \int_t^{t + \Delta t} \mathbf{v}_p(t') dt' \quad (5)$$

by expanding the particle velocity in a Taylor series about t and integrating (5). The result

$$\frac{\Delta \mathbf{x}_p}{\Delta t} = \mathbf{v}_p(t) + \dot{\mathbf{v}}_p(t) \frac{\Delta t}{2} + O(\Delta t^2) \quad (6)$$

states that to second-order the particle velocity is the velocity at t plus one-half of the change that occurs during Δt . This estimate for the particle velocity can be transformed into an estimate for the Eulerian fluid velocity by using the particle dynamic equation (1) to eliminate $\dot{\mathbf{v}}_p$, and approximating the particle acceleration by the local acceleration of the fluid:

$$\mathbf{u}(\mathbf{x}, t) \approx \frac{\Delta \mathbf{x}_p}{\Delta t} - \frac{D\mathbf{u}}{Dt} \left[\frac{\Delta t}{2} - \tau_p \right] - \mathbf{b} \tau_p + O(\Delta t^2), \quad (7)$$

where

$$\dot{\mathbf{v}}_p(t) \approx \left. \frac{D\mathbf{u}}{Dt} \right|_{\mathbf{x}=\mathbf{x}_p(t)} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}. \quad (8)$$

The second and third terms on the right hand side of (7) represent the errors due to flow acceleration, particle lag and the effect of particle drift caused by a body force such as gravity, or centrifugal acceleration. Interestingly, the finite difference error opposes the particle lag error to first order, suggesting that they could be canceled if Δt were conveniently close to τ_p . With respect to Figure 1 this occurs because the finite difference estimate tends to lie below $\mathbf{u}$ when the fluid pathline curve downwards, while the effect of particle inertia is to make the particle velocity lie above $\mathbf{u}$.

Since (7) relies upon a simple model of the particle dynamics, it is not in general accurate enough to be used to evaluate large errors, but it can be used for small corrections, and it is certainly useful to estimate the magnitude and direction of error producing effects. It should be noted that each of the errors in (7) is a systematic error in the presence of steady accelerations. The effect of fluctuating turbulent fields is to cause local changes in the Eulerian velocity that a particle would see as a fluctuating acceleration as it moves. If mean flow accelerations and settling velocities due to body forces are ignored, the root mean square error velocity of the first order forward difference algorithm due to turbulence is approximately

$$\left\langle \left| \frac{D\mathbf{u}}{Dt} \right|^2 \right\rangle^{1/2} \left| \frac{\Delta t}{2} - \langle \tau_p \rangle \right| \approx \text{const} \frac{u^2}{\lambda_T} \left| \frac{\Delta t}{2} - \langle \tau_p \rangle \right|, \quad (9)$$

where conventional approximations have been made to express the rms turbulent acceleration in terms of the mean square velocity u^2 and the Taylor microscale λ_T . A substantial reduction of this error can be achieved by interpreting (4) as a central difference and assigning the value to the midpoint between $\mathbf{x}_p(t)$ and $\mathbf{x}_p(t+\Delta t)$. The rms error for this approach is given by (9) with Δt set equal to zero. Since τ_p is usually much less than Δt the improvement is significant.

Modern Particle Dynamics

The dynamics of particle motion in an unsteady flows field is clearly important to the successful application of Particle-based velocity measurement methods such as LDV and PIV. The dynamics used in equation (1) above are, unfortunately, oversimplified. For decades, the classic source of a more complete description has been Tchen's equation, which includes the effects of Basset's history term, added mass and flow acceleration in a non-inertial frame. This equation was derived by patching together the results of several separate analyses, and, as a consequence, it is not entirely correct. More recently, Maxey and Riley (1983) derived the complete dynamics particle motion in high frequency unsteadiness from first principles. The equation assumes that the particle Reynolds number, defined as the velocity difference between the particle and the surrounding fluid times the particle diameter over the kinematic viscosity, is small compared to unity. The Maxey-Riley equation is very similar to Tchen's equation, with small, but sometimes important differences. The limitation to small particle Reynolds number is not a strong restriction in the context of particle-based methods because one strives to use particles whose diameters are small enough to ensure relatively small slip velocity. Thus, for many, if not most applications of particle-based methods, the slip velocity can be estimated using the equations for low particle Reynolds number.

However, in some cases the particle Reynolds number is not small, or one needs to understand exactly when large particle Reynolds number effects become important. For these circumstances the reader is referred to an interesting extension of the particle dynamics equations to finite particle Reynolds numbers. (Mei 1997). This work is particularly useful because Mei manages to find a very simple and easily implemented formula for the frequency response of particles having any density relative to the fluid in an oscillating flow. He also discusses the response of particle to isotropic turbulent fields in terms of the particle time constant the intrinsic scales of the turbulence.

Particle Population Distributions

The characteristics of particles are not normally identical from one particle to the next, and the difference have significant implications for the quality of the measurements. Specifically, in any PIV measurement that uses correlation of the image at one time with the image at another, later time, the key factor determining the fraction of successful measurements and their reliability and accuracy is the extent to which the images correlate. Anything that reduces the correlation, reduces the signal strength and introduces uncertainties. De-correlating factors include any agency that causes the first image of a particle to look different from the second image. For example, non-spherical particles may rotate between exposures, and as a consequence present different images. Well-known particles of this type are the platelet-shaped Kalariscope particles whose scattered light appears to blink on and off as they rotate with the fluid. Particles that are spherical but polycrystalline also suffer from de-correlation of their images. Lastly, some particles may evolve from one exposure to the next. This occurs if the particles are caused to evaporate rapidly or be shattered by the pulse of light.

Polydisperse particle populations also lead to degradation of the measurements. Interrogation cells containing one large particle and many small particles will yield correlations that are dominated by the image of the large particle because the scattered light intensity increases rapidly with particle size. It is better to sample the velocity field within the resolution cell by averaging over many particles of similar size and image characteristics.

In summary, the ideal particles are monodisperse, spherical, scatter light intensely and have density equal to that of the fluid.

Seeding Liquids

In liquids the increased refractive index of the fluid substantially reduces the intensity of scattered light for the same index of refraction of the particle material. Higher refractive index materials such as aluminum oxide and titanium dioxide can be used to increase the refractive index ratio, or hollow spheres which scatter efficiently by total internal reflection at the solid wall-trapped gas interface can also be used, but to fully compensate, it is necessary to use significantly larger particles, in the range of 5-15 microns diameter. If the density of the particle is matched closely with that of the liquid, the tendency of the inertia of such particles to cause particle lag is minimized. If the densities differ, particle lag or settling under the influence of gravitational attraction may be important. In liquids, bubbles scatter light well, but the density difference is significant and the rise velocity causes problems keeping seed in the flow.

Seeding Gases

In gases it is possible to form good images on photographic, video or holographic recording media using pulsed lasers with energies in the range of 1-100 mJoules per pulse and one micron diameter oil droplets whose refractive indices are in the range 1.5-1.6. Natural, non-toxic oils such as vegetable oils and olive oil are safe, easy to use and easy to remove if they should coat laboratory surfaces. The ratio of the refractive index of the particle to that of the gas determines the scattered light intensity, and ratios in the range cited above provide adequate scattering if the illumination is strong. Such droplets also follow very large accelerations, and they have been used in high speed flows at velocities up to 500 m s⁻¹ with good results. The ability of a particle to follow accelerations of the flow increases in inverse proportion to the particle time constant, which in turn increases as the square of the particle diameter. Thus, ten micron particles are roughly one-hundred times more sluggish than one micron particles of the same density, and particles that are more than a few microns in diameter are not likely to be satisfactory for gaseous flows of a few meters per second, or more. Consequently, accurate PIV measurements in all but very slow gas flows are characterized by small particles and high powered pulsed laser illumination.

Acknowledgment

This research was supported by the U S Office of Naval Research and the U S National Science Foundation.

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FIGURE 1. The Lagrangian trajectory of a particle (light line) compared to the Lagrangian trajectory of the fluid particle that coincides with it at time t . (heavier line). The marker particle departs from the fluid trajectory due to inertia.