ABSTRACT
In this study, the wavelets-based PIV technique is developed for improving spatial resolution and reducing physical storage. For evaluating the effect of image parameters, this technique is applied to standard experimental PIV images. It is found that the higher accuracy of PIV measurement can be achieved by using wavelet multi-resolution technique without losing information of flow structure.

Keywords: Visualization, Particle image velocimetry, Wavelet Multi-resolution Analysis

1. Introduction
Recently, the application of wavelets has been rapidly developed to PIV technique. As far as we know, the wavelet technique has not only been used to compressing PIV images\(^1\), but also been performed as a post-processing of velocity vector field, such as the multi-scale analysis of PIV measurement results\(^2\) and compression of PIV velocity vector for efficiently eliminating erroneous vectors and reducing physical storage\(^3\). It is well-known fact that the wavelet multi-resolution technique may decompose image into several image components depended on level, where low- and high-level represent the low- and high-spatial frequency components, respectively. Although a wavelet-based PIV algorithm\(^4\), which derives a windowed cross-correlation function for wavelet-based representations of PIV images, improved performance in terms of spatial resolution and reliability, it only removed the lowest-level image component from original PIV image. In this study, a wavelets-based PIV technique is developed and applied to experimental PIV images, in order to improve spatial resolution and reducing physical storage.

2. Wavelet Multi-resolution Analysis
For a two-dimensional scalar field \(f(x_1, x_2)\), the two-dimensional discrete wavelet transform is defined by

\[ W_{f_{m,n_1, n_2}} = \sum \sum f(x_1', x_2') \psi_{m,n_1, n_2}(x_1', x_2') \]

The reconstruction of the original scalar field can be achieved by using

\[ f(x_1, x_2) = \sum \sum \sum W_{f_{m,n_1, n_2}} \psi_{m,n_1, n_2}(x_1, x_2) \]

where the two-dimensional wavelet basis, \(\psi_{m,n_1, n_2}(x_1, x_2)\), is simply to take the tensor product functions generated by two one-dimensional bases as

\[ \psi_{m,n_1, n_2}(x_1, x_2) = 2^{-m} \varphi(2^{-m} x_1 - n_1) \varphi(2^{-m} x_2 - n_2) \]

The oldest example of a function \(\varphi(x)\) for which the \(\psi_{m,n}(x)\) constitutes an orthogonal basis is Haar function, constructed long before the term “wavelet” was coined. In the last ten years, various orthogonal wavelet bases, such as Meyer basis, Daubechies basis, Coifman basis, Battle-Lemarie basis, Baylkin basis, spline basis, etc., have been constructed. They provide excellent localization properties both in physical space and frequency space. In this study we use the Daubechies basis with index \(N=20\), which is not only orthonormal, but also have smoothness and
Fig. 1 Velocity field of self-induced sloshing flow

The procedure of the wavelet analysis, i.e. wavelet multi-resolution analysis, can be summarized in two steps:

1. Wavelet coefficients or wavelet spectrum of an image is computed based on the discrete wavelet transform of Eq. (1).

2. Inverse wavelet transform of Eq. (2) is applied to wavelet coefficients at each wavelet level, and image components are obtained at each level or scale in the wavelet spaces.

The detail regarding the wavelet multi-resolution analysis can be found in Li et al. 3.

3. Results

Figure 1(a) shows the velocity field computed from original experimental PIV images of a self-induced sloshing flow. From the Fourier and wavelet multi-resolution analysis, the dominant spatial frequency of this PIV image distributes in the range of wavelet levels 4 and 5. Therefore, we use the image component of levels 4 and 5 to compute the velocity field and its result is showed in Fig.1 (b). A reduced number of erroneous vectors can be realized in comparison with Fig.1 (a). These erroneous vectors can be easily removed by average neighboring reasonable vectors (no shown). This result indicates that the higher accuracy of PIV measurement can be achieved by using wavelet multi-resolution technique without losing information of flow structure. On the other hand, since only PIV image component of levels 4 and 5 is used as PIV analysis, the image storage may be also reduced.

References