Another Possible Origin of Temperature and Pressure Gradients across Vane in the Crookes Radiometer*

Kazuki DENPOH^{*1,a)}

^{*1}Individual Participant, Kai-city, Yamanashi 400-0104, Japan

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In conventional studies on the Crookes radiometer, vane temperature was presumed to be higher at the black side than at the shiny side. In this study, a new hypothesis – vane is isothermal but accommodation coefficients are different at the black side and at the shiny side – has been proposed and examined using heat transfer and Direct Simulation Monte Carlo (DSMC) simulations. The results prove that the vane is indeed isothermal under the sunlight and that gas temperature and pressure at the black side with the perfect accommodation coefficient become higher than those at the shiny side with a small accommodation coefficient. The pressure gradient across the vane acts as an area force to push the vane from the black side. It is also found the glass bulb temperature strongly affects the revolution of the vanes.

1. Introduction

The Crookes radiometer^{1,2)} was invented in the 19th century and even today still attracts many researchers as an interesting subject for study because of complicated physics involved in its simplest machinery. Therefore, significant efforts have been made via numerical simulations³⁻¹⁰⁾ to reveal forces on vanes in the radiometer.

In those studies, the vane temperature was supposed to be much higher at the black side due to larger photon energy absorption than at the shiny side. In addition, accommodation coefficient was assumed to be the same at the both sides of the vane, and in many cases the perfect accommodation coefficient was specified. Their simulation results showed thermal transpiration (or thermal creep force) was produced due to temperature difference ΔT given between both sides of the vane and an area force by pressure difference Δp induced across the vane. Those forces push the vanes from the black side.

However, one might wonder whether such that large temperature difference ΔT can really exist inside the thin vane. Therefore, in this study, a new hypothesis – the thin vane heated by the sunlight is isothermal but accommodation coefficients are different at the black side and at the shiny side – has been proposed and examined.

Accommodation coefficient α ^{11,12)} is a fraction indicating how much gas molecules impinging a wall accommodate to the wall temperature T_w and described as $\alpha = (T_i - T_r)/(T_i - T_w)$, where T_i and T_r are the temperatures of incident and reflected molecules, respectively. The reflection is perfectly specular for $\alpha = 0$ and diffuse one for $\alpha = 1$. It is well known that accommodation coefficient significantly affects gas heat conduction, especially in the transition regime to the free molecular flow regime¹²⁾.

Naturally, the accommodation coefficient at the black

side of the vane must be larger than that at the shiny side since the mica vane in the Crookes radiometer was blackened with soot on one side¹³⁾, and in experiments by Ota, *et al.*^{3,4)} one side of aluminum (Al) vane was coated with carbon-black powder. Porous and rough¹²⁾ surfaces of such soot or carbon-black powder could make the effective accommodation coefficient nearly unity due to frequent interactions between molecules and those surfaces.

This article is organized in the following manner. Time variations of the vane temperature are estimated through heat flux balance equations in the next section. Then, in section 3, effects of accommodation coefficients on rarefied flow fields formed around four vanes in a radiometer are investigated using a DSMC software¹⁴⁾, which runs on the Microsoft Excel. Finally, we give conclusions of the present study.

2. Evaluating Vane Temperature

Figure 1 illustrates a one-dimensional (1D) model of heat fluxes coming into and going out from a vane. The heat flux q_{in} incoming from the sunlight to the vane through the glass bulb is perfectly absorbed only at the black side of the vane. And outgoing fluxes by gas heat conduction and radiation are emitted from the both sides of the vane. The perfect accommodation coefficient is as-

Fig. 1 1D model of heat fluxes coming into and going out from a vane.

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^{a)} E-mail: denpoh@mvi.biglobe.ne.jp

sumed at the both sides of the vane as most conventional studies did. Presuming the Biot number Bi is much less than unity ($Bi \ll 1$), heat flux balance equations to solve the vane temperature T_V at time t can be simply written as lumped ones, i.e.,

$$q_{in} - (q_{g,B}^t + q_{r,B}^t) = -\kappa \frac{\partial T_V^t}{\partial x}, \quad (1)$$

$$\rho L_b C_p \frac{\partial T_V^{t+\Delta t}}{\partial t} = q_{in} - (q_{g,B}^t + q_{r,B}^t) - (q_{g,S}^t + q_{r,S}^t), \quad (2)$$

where, κ is the thermal conductivity, ρ the density, L_b the thickness, and C_p the specific heat at constant pressure of the vane, respectively. The superscriptions t and $t + \Delta t$ indicate time, Δt being the time step. The heat fluxes outgoing from the vane, i.e. q_g by gas heat conduction and q_r by radiation heat transfer are described as,

$$q_{g,B/S}^t = \frac{1}{4} n \bar{v} \Delta E = \frac{1}{2} n k \sqrt{\frac{8kT_g}{\pi m}} (T_{V,B/S}^t - T_g), \quad (3)$$

$$q_{r,B/S}^t = \varepsilon_{B/S} \sigma \{ (T_{V,B/S}^t)^4 - T_g^4 \}, \quad (4)$$

where, n is the gas density, $\bar{v}$ the mean thermal velocity of gas molecules, m the mass of a gas molecule, k the Boltzmann constant, T_g the ambient gas temperature, ε the emissivity, and σ the Stefan-Boltzmann constant, respectively. The subscriptions B and S denote the black side and the shiny side of the vane.

Figure 2 shows the vane temperature T_V at the black side and at the shiny side as a function of time, obtained by solving equations (1) and (2) alternately. In this calculation, Al and mica are chosen as a base material of the vane, and soot as its black side. Their thermal properties^{15–18)} are tabulated in **Table 1**. The thickness is

Fig. 2 Time variations of vane temperature for two different base materials, Al and mica. Each vane has a soot layer as the black side.

Table 1 Thermal properties of vane materials^{15–18)}.

	ρ (kg/m ³)	C_p (J/kg-K)	κ (W/m-K)	ε
Al	2688	905	237	0.17
Mica	2100	880	0.5	0.72
Soot	100	1000	0.05	0.95

0.1 mm for the base material and 0.01 mm for the soot layer as shown in Fig. 1. The sunlight heat flux^{19,20)} is assumed to be 700 W/m², and T_g is fixed at 298 K (25 deg. C).

As plotted in Fig. 2, T_V rises with time from T_g to 363 K (90 deg. C) for Al and 347 K (74 deg. C) for mica, respectively. The temperature difference between the black side (soot) and the shiny side is quite small and at most 0.12 K. The actual Bi calculated is less than 0.01 for each Al and mica vane, and therefore solving the lumped equations (1) and (2) is reasonable. Consequently, the results suggest that thin vane should be almost isothermal under the sunlight.

3. Simulation of rarefied flows in radiometer

Next, rarefied flows induced under the new hypothesis have been investigated using the DSMC software¹⁴⁾. **Figure 3** depicts thermal boundary conditions of a two-dimensional (2D) radiometer. The temperatures of the vane and the glass bulb are kept at T_V and T_G , respectively. Each accommodation coefficient is α_B at the black side and α_S at the shiny side of the vane. The perfect accommodation is assumed at the glass bulb surface. The length L_a of the vane is 13 mm. The thickness L_b is 2 mm, which is thicker than actual one (~ 0.1 mm) because of a limitation of the software. The inner radius of the glass bulb is 25 mm.

The gas in the radiometer is air with its molecular weight of 28.97 g/mol and two rotational degrees of freedom. The inter-molecular collision diameter is calculated from the viscosity at the reference temperature $T_{ref} = \sqrt{T_V T_G}$. The Maxwell molecule²¹⁾ is specified as a molecular model, and collisions are treated using the variable hard sphere model²²⁾ and the Larsen-Borgnakke model²³⁾. As initial conditions, the radiometer is filled with the stationary air at a reference pressure p_{ref} and T_{ref} .

Figure 4 shows flow fields obtained for $p_{ref} = 1$ Pa, $T_V = 348$ K (75 deg. C), $T_G = 298$ K (25 deg. C), $\alpha_B = 1$ (perfect accommodation), and $\alpha_S = 0.01$. The gas temperature and thus the pressure which corresponds to the total kinetic energy of gas molecules²⁴⁾ become higher at the black side of the vane than at the shiny side. As a conse-

Fig. 3 Thermal boundary conditions in 2D radiometer model.

Fig. 4 Air flow fields simulated for $p_{\text{ref}} = 1$ Pa, $T_v = 348$ K, $T_G = 298$ K, $\alpha_B = 1$, and $\alpha_S = 0.01$.

Fig. 5 Torque as a function of α_S , calculated for $p_{\text{ref}} = 1$ Pa, $T_v = 348$ K, $T_G = 298$ K, and $\alpha_B = 1$.

quence, it is proved that gas temperature and pressure gradients are produced across the vane under the new hypothesis.

Thermal creep flows are also induced around the vanes and form a large stream in the radiometer as already reported and discussed in many papers.^{4,6-9)}

In **Fig. 5**, the torque calculated from the gas pressure difference Δp across the vanes is plotted as a function of α_S . The torque increases with decreasing α_S for $\alpha_S > 0.1$, then becomes a constant ($= 4.3 \times 10^{-8} \text{ N}\cdot\text{m}$) for $\alpha_S < 0.1$. The rotation speed estimated from the torque is about 4 rps at 10 s after starting revolution. Although the estimation is valid only at the early state of starting rotation⁸⁾ since the vanes are stationary in this study, the time-scale of the rotation is coincident with common ob-

Fig. 6 Air flow fields simulated for $p_{\text{ref}} = 1$ Pa, $T_v = 348$ K, $T_G = 348$ K, $\alpha_B = 1$, and $\alpha_S = 0.01$.

servations.

It is said that the revolution slows down and sometimes stops when the radiometer is exposed to the sunlight for a long time. **Figure 6** shows flow fields for the glass bulb heated up to the same temperature as the vanes, i.e., $T_G = T_v = 348$ K. Despite the large difference in accommodation coefficients ($\alpha_B = 1$, $\alpha_S = 0.01$), spatial distributions of the gas temperature and the pressure are almost uniform. And any apparent thermal creep flow is not induced in the radiometer. That indicates the vanes will stop as the glass bulb is heated up to T_v .

4. Conclusion

Unlike conventional simulation studies on the Crookes radiometer, the new hypothesis – vane is isothermal but accommodation coefficients are different at the black side and at the shiny side – has been proposed and investigated in this study. The solutions of the heat flux balance equations suggest that the thin vane should be isothermal under the sunlight. Then, the DSMC simulations proved that the contrast of the accommodation coefficients between both sides of the vane can be an origin of gas temperature and pressure gradients across the vane. The torque calculated from the pressure difference across the vanes increases as the contrast of the accommodation coefficients becomes large. Moreover, it is found that the gas temperature and the pressure gradients are also affected by the glass bulb temperature. The revolution of the vanes will stop when the glass bulb is heated up to the same temperature as the vanes.

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