Efficiency of the Tsunami

By

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Abstract: The destructive energy of the larger tsunami which accompanies the earthquake is about 10^{23} ergs. But the exchanged energy between the sea water and the crust is far beyond this value, and is perhaps comparable to or larger than the whole energy of the seismic waves which is sometimes 10^{25} — 10^{26} ergs. Concerning this pointing out, the author should like to refer to the gravitational wave energy which has recently been described rather ambiguously.

§ 1. Introduction

One is apt to say that in the sinusoidal wave the elevation of which is given by $\eta = a \sin k(x - Ct)$, the potential energy at x is given by

$$E_p = g\rho \int_{-\infty}^{\eta} z dz = \frac{a^2}{2} g\rho \sin^2 k(x - Ct) \quad (\text{Fig. 1}),$$

and further, as

$$VE = \int_{-\infty}^{\infty} p u dz = \frac{a}{2} \rho g C \sin^2 k(x - Ct),$$

he can derive the relation $VE = CE_p$ which indicates the potential energy is transmitted with wave velocity.

Or one is tempted to say that in a solitary wave, the energy per unit crest width between $-x$ and $+x$ equals

Fig. 1. Sinusoidal wave.

Fig. 2. Solitary wave.

$E = \frac{4}{3} \rho g H^2 h \left(2 + \frac{\eta}{h} \right) \sqrt{\frac{h}{3H}} \sqrt{1 - \frac{\eta}{h}}$, so that 98% of the energy is contained between $x = \pm 2.1H$ for $h/H = \frac{1}{3}$ (Fig. 2).

But these are serious mistakes because the quantity named "energy" only expresses the change which occurs in a specific system. When the new system takes the place of the original one, the usual conception of energy cannot be used.

For example, there is a table in Fig. 3, whose height is H . At first, there is nothing on it. Let a mass m be on the floor. If one lifts the mass up to a position just ε above the table surface, one cannot say that there is an increase in energy by $mg\varepsilon$ on the table surface. One should say that the energy increase is $mgH + mg\varepsilon$ for the system consisting of both the table and the floor.

Fig. 3

If we use a symbolic description, taking z -axis vertically upwards and its origin at the table surface, we may say that the increase of energy on the table *and* on the floor are $mg\varepsilon$ and $(-m)g(-H) = mgH$, respectively. We should here remember to couple the mass excess on the table and the mass deficit on the floor.

In the case of the sinusoidal waves, one should call the wave energy, what is contained in one wave length, in which no excess or no deficit of mass exist comparing with the still water state.

In the solitary wave, the energy should be integrated from $-\infty$ to ∞ . In this case, this quantity means the work to make a solitary wave of the virtual excess mass beaten out to a thin skin over the still surface.

The mechanism of the generation of the tsunami resembles the process shown in Fig. 3. The energy which plays the destruction along the coast is $mg\varepsilon$. This is equal to the work which would make a tsunami of the virtual thin skin over the sea surface. But the whole work which the crust have acted on the sea water system, corresponding to $mgH + mg\varepsilon$ in Fig. 3, is far beyond the so-called destructive tsunami's energy.

§ 2. Efficiency of the Tsunami

The so-called energy of the larger tsunami, caught along the coast is about 10^{23} ergs.

In the generation of the tsunami (Fig. 4), if we assume the bottom deformation happens suddenly, (the water is assumed to be incompressible,) the water pile of the same shape as the deformed part of the bottom appears upon the sea surface, which is the new-born form of the tsunami. The increases of the energy on the sea surface *and* at the bottom are $\frac{\rho g}{2} \int_s \eta^2 dS$ and $(-\rho V)g(-H) = \rho g H \int_s \eta dS$,

respectively. Here V is the volume of the deformation and S means the deformed area. The former is the so-called destructive energy, and is quite smaller than the latter. So the efficiency of the tsunami,

$$e = \frac{\int_s \eta^2 dS}{2H \int_s \eta dS + \int_s \eta^2 dS} \doteq \frac{\int_s \eta^2 dS}{2H \int_s \eta dS} \quad \text{is a very small fraction } (\eta \ll H).$$

The whole energy which the sea water received from the crust is, therefore, far beyond the destructive energy, and is perhaps comparable to or larger than the so-called energy of the earthquake, which is sometimes 10^{25} — 10^{26} ergs.

Briefly speaking, when the bottom elevates by a quasi-static process or by a sudden process, the work done to lift the water column with the upper edge which is being rubbed off triflingly, differs slightly from that to lift the column having a fixed vertical length. This slight difference is the true character of the destructive energy of the tsunami. In this case, the efficiency of the tsunami is a small fraction of a positive value.

When the bottom depresses, on the contrary, by a sudden process, the sea water cannot give to the crust the whole energy which would be given only by the quasi-static depression. In this case, the slight remainder becomes the tsunami's destructive energy. The efficiency is, therefore, also a small fraction but of a negative value.

If we use a more quantitative description, the increases of energy in the system of the sea water are E_k , E_p and E_b . E_k is the kinetic energy of the water particle. $E_p = \frac{\rho g}{2} \int_{S'} \eta^2 dS'$ is the so-called potential energy of the tsunami. Here S' means the whole sea surface. We know that $E_k = E_p$ ($= \frac{mg\epsilon}{2}$ in Fig. 3). Finally, E_b corresponds to " mgH ". On the other hand, the energy ex-

Fig. 4. Generation of the Tsunami.

changed at the bottom boundary is $\int_V p dv$, where p is the hydrostatic pressure at the bottom.

$$\therefore \int_V p dv = E_k + E_p + E_b \quad (1).$$

If the hydrostatic pressure during the quasi-static process of the same deformation, is represented as P_q , we can get the following relation

$$\int_V P_q dv = E_b \quad (2),$$

for, in this case, E_k and E_p vanish.*

$$\therefore \int_V (P - P_q) dv = E_k + E_p \quad (3).$$

This has, exclusively, a positive value. And the efficiency is given as

$$e = \frac{\int_V (P - P_q) dv}{\int_V P_q dv} \quad (4).$$

This value takes its maximum value, when the deformation is a sudden one, and tends to zero, when the deformation is a quasi-static one. In the intermediate case, the relative efficiency (the ratio of this efficiency to its maximum value), will be a function of the horizontal dimension and the time duration of the deformation, and the water depth.

The author should like to add that he infers that this rather important commonsense has been become aware of, by the several investigators of the tsunami, earlier than he.

§ 3. Acknowledgement.

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* For example, the deformation of $1/N$ height occur N times, the energy of $1/N^2$ comparing with the original tsunami will be radiated every time. So, the total energy is $1/N^2 \times N = 1/N \rightarrow 0$ ($N \rightarrow \infty$).