A Note on the Compressional Disturbances Accompanying the Tsunami*

By

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Abstract: As the natural case, when a broad bottom deformation superimposed by the secondary unevennesses or by the rather narrow mainly deformed part happens, the check whether the energy of the compressional disturbances may concentrate or may not, is required. Our check proves that no energy concentration may occur.

In Part II of the referred papers, we treated the wave form due to the compressional disturbances when the radii of the deformed areas are much larger (or smaller) than the water depths. In this paper, we will treat the case in which the radius of the deformed area and the water depth are of the same orders.

The ratio of them is, therefore, now

$$\frac{H}{a} = h \approx 1.$$

Then using the ordinary t and the asymptotic expansion of J_0 , (19) (in Part I of the referred papers) can be written as

$$\begin{aligned} & \frac{\pi a^2}{V} \xi_0(t) \\ &= \sqrt{\frac{2}{\pi}} \frac{z_0}{h^3} \int_0^1 s ds \int_0^\pi d\omega \int_0^\infty \sum_{n=1}^\infty \frac{(-1)^n (2n-1) \sin \left\{ \sqrt{m^2 + b_n^2} - \delta \frac{(m^2 - b_n^2)}{\sqrt{m^2 + b_n^2}} \right\} \frac{ht}{z_0} \cos \left(m\Omega - \frac{\pi}{4} \right) \sqrt{m}}{\sqrt{\Omega} (m^2 + b_n^2)^{3/2}} dm \\ &= \frac{z_0}{\sqrt{2\pi} h^3} \Re \frac{e^{-\pi/4 + i}}{i} \int_0^1 s ds \int_0^\pi d\omega [I_1 - I_2] \end{aligned} \quad (54),$$

here
$$I_{1,2} = \int_0^\infty \sum_{n=1}^\infty \frac{\sqrt{m} (-1)^n (2n-1)}{\sqrt{\Omega} (m^2 + b_n^2)^{3/2}} \exp \left(i\Omega \left\{ m \pm K \sqrt{m^2 + b_n^2} \mp K\delta \frac{(m^2 - b_n^2)}{\sqrt{m^2 + b_n^2}} \right\} \right) dm$$

$$K = \frac{Ut}{a\Omega} \quad (55).$$

Let us put
$$f(m) = m \pm K \sqrt{m^2 + b_n^2} \mp K\delta \frac{(m^2 - b_n^2)}{\sqrt{m^2 + b_n^2}} \quad (56),$$

and we will differentiate it as follows,

$$f'(m) = \frac{1}{\sqrt{m^2 + b_n^2}} \left[\sqrt{m^2 + b_n^2} \pm Km \left\{ 1 - 2\delta + \delta \frac{(m^2 - b_n^2)}{(m^2 + b_n^2)} \right\} \right] \quad (57).$$

As $\left| \delta \frac{(m^2 - b_n^2)}{(m^2 + b_n^2)} \right| < \delta$, then $Km \left\{ 1 - 2\delta + \delta \frac{(m^2 - b_n^2)}{(m^2 + b_n^2)} \right\} > 0$.

I_1 has, therefore, no stationary phase, and I_2 has one. We will represent the latter by m_s .

Then
$$m_s = m_{s0} + m_{s1}\delta + \dots, \quad m_{s0} = \frac{b_n}{\sqrt{K^2 - 1}}, \quad m_{s1} = \frac{b_n(3K^2 - 2)}{(K^2 - 1)^{3/2}} \quad (58).$$

If we assume the order of Ω as follows

$$\delta^2 \ll 1/\Omega \ll \delta \quad (59),$$

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$$\text{then } i\Omega f(m_s + \varepsilon) = i\Omega \{f(m_{s0}) + \delta m_{s1} f'(m_{s0})\} + \frac{i\Omega \varepsilon^2}{2} \{f''(m_{s0}) + \delta m_{s1} f'''(m_{s0})\} + \dots \quad (60).$$

Omitting the small modifications on the amplitude, we get

$$\frac{\pi a^2}{V} \xi_0 = \frac{z_0}{h^3} \Re \int_0^1 s ds \int_0^\pi d\omega \sum_{n=1}^\infty \frac{(-1)^n (2n-1) \sqrt{m_{s0}}}{\Omega \sqrt{K(m_{s0}^2 + b_n^2)^{3/4} b_n}} \exp(i\Omega \{f(m_{s0}) + \delta m_{s1} f'(m_{s0})\}) \quad (61).$$

And considering that $f'(m_{s0}) \sim O(\delta)$, the following relation is obtained

$$\frac{\pi a^2}{V} \xi_0 = \frac{2z_0^3 \sqrt{(Ut)^2 - r^2}}{\pi h^4 t^2 a} \int_0^1 s ds \int_0^\pi d\omega \sum_{n=1}^\infty \frac{(-1)^n}{b_n} \cos \Omega f(m_{s0}) \quad (62).$$

Here, since

$$\begin{aligned} \Omega f(m_{s0}) &= - \left\{ \Omega b_n \sqrt{K^2 - 1} - \frac{\Omega b_n \delta (2 - K^2)}{\sqrt{K^2 - 1}} \right\} \\ &\doteq -b_n \left\{ \frac{\sqrt{U^2 t^2 - r^2}}{a} + \frac{\delta (U^2 t^2 - 2r^2)}{a \sqrt{U^2 t^2 - r^2}} + \frac{rs \cos \omega}{\sqrt{U^2 t^2 - r^2}} \right\}, \\ \therefore \frac{\pi a^2}{V} \xi_0 &= \frac{2z_0^2 (1 - \vartheta^2)}{\pi t \vartheta} \int_0^\pi d\omega \left[-\frac{\cos \omega}{h} F \left(\frac{\pi r}{2H \sqrt{\frac{1}{\vartheta^2} - 1}} + \frac{\pi r}{2H} \delta \frac{(1/\vartheta^2 - 2)}{\sqrt{1/\vartheta^2 - 1}} + \frac{\pi \cos \omega}{2h \sqrt{1/\vartheta^2 - 1}} \right) \right. \\ &\quad + \frac{2}{\pi} \sqrt{\frac{1}{\vartheta^2} - 1} \left\{ H \left(\frac{\pi r}{2H \sqrt{\frac{1}{\vartheta^2} - 1}} + \frac{\pi r}{2H} \delta \frac{(1/\vartheta^2 - 2)}{\sqrt{1/\vartheta^2 - 1}} + \frac{\pi \cos \omega}{2h \sqrt{1/\vartheta^2 - 1}} \right) \right. \\ &\quad \left. \left. - H \left(\frac{\pi r}{2H \sqrt{\frac{1}{\vartheta^2} - 1}} + \frac{\pi r}{2H} \delta \frac{(1/\vartheta^2 - 2)}{\sqrt{1/\vartheta^2 - 1}} \right) \right\} \right] / \cos^2 \omega \quad (63). \end{aligned}$$

Now, the elevation of the sea surface is given by

$$\xi_0 = \frac{1}{T} [\xi_0(t) - \xi_0(t-T)].$$

We can know from the above analysis, that the special feature that much energy can concentrate at any areas, does not appear when the radius of the deformed bottom and the water depth are of the same orders. And this may become the foundation upon which we can suppose that when a broad deformation superimposed by the secondary unevennesses or by the rather narrow mainly deformed part occurs, energy concentrations at any areas are not expected.

In this treaty, we adopted the notations used in the referred papers. Finally, the author's sincere thanks are due to the people of Geophysical Institute and Professor R. Takahashi of the Earthquake Research Institute, Tokyo University, who kindly discussed with him.

References

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