A Theory of the Under-water Unit with a Slow-leak for Wave Recorders*

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Abstract: Although the structure of the under-water unit with a slow-leak has been described in many papers, it appears that the theoretical analysis for the characteristics of the slow-leak itself has not yet been mentioned. The present paper is an attempt to give a method designing under-water unit with a slow-leak based upon a theoretical consideration. Here are illustrated some examples of the response curves drawn by introducing the slow-leak constant.

1. Introduction

Numerous wave measurements are made by means of pressure recorders placed at some depths below ocean surface. The recorder of this type generally consists of an under-water unit installed at bottom or some depth and electrically recording device on land or vessel. The pressure fluctuations at bottom caused by the passage of waves overhead compress or expand the bellows of the under-water unit, and the movements of bellows are converted into electrical signals with adequate device. The measurements of wind waves and swells proper are effected by an under-water unit usually incorporated with a slow-leak mechanism filtering long period fluctuations of pressure other than wind waves and swells, which might scale out and destroy. The structure of the under-water unit with a slow-leak has hitherto been discussed by numerous authors (FOLSOM, 1949; KLEBBA, 1954; MUNK, 1947), but it seems that the characteristics of the slow-leak itself have not yet been theoretically analysed. The present paper aimed at to treat the problem giving a method of design.

2. Theory

A typical construction of the under-water unit is schematically shown in Fig. 1. R is an air reservoir which communicates through a capillary tube L with the outside of the reservoir. Sometimes other devices have been used instead in many prac-

tical recorders, but the capillary tube is a representative in the principle. B_i and B_o are sensing bellows placed inside and outside the reservoir respectively, which are tightly connected with each other by a rod. Therefore, this rod moves upwards or downwards according to the difference of pressures inside and outside. And E is an electrical converter which converts the movements of the rod into electrical signals.

Fig. 1 Schematic diagram of the under-water unit with a slow-leak

Let P_R and V_R be the pressure and volume of the air of the reservoir respectively, l and d the length and diameter of the capillary tube respectively. Further let B be the effective area of bellows, B_i and B_o having equal dimensions. If the ambient

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water pressure of the under-water unit is P , and the travel of the rod is Z , the relation between P and Z is

$$WZ = B(P - P_R), \quad (1)$$

where W is the additive elastic constant of the bellows system. The water flowing through the capillary into the reservoir during the time interval dt is

$$dQ = m(P - P_R) dt, \quad (2)$$

where m is POISEUILLE'S constant for the capillary. Therefore, μ standing for the coefficient of viscosity of sea water,

$$m = \frac{\pi d^4}{8\pi l}. \quad (3)$$

At the same time, the volume dQ can be written as

$$dQ = -dV_R - BdZ. \quad (4)$$

As the change can be assumed isothermal, we have

$$P_R V_R = P' V',$$

$$\therefore \frac{dV_R}{dt} = -\frac{V'}{P'} \frac{dP_R}{dt}, \quad (5)$$

where P' is the mean air pressure inside the reservoir placed at the given depth, V' is the air volume inside the reservoir at the pressure P' . Neglecting the inertia effect of the bellows and rod, we get as the first approximation of the above discussion

$$\dot{Z} + aZ - b\dot{P} = 0, \quad (6)$$

where

$$\left. \begin{aligned} a &= \frac{mWP'}{V'W + P'B^2}, \\ b &= \frac{V'B}{V'W + P'B^2}. \end{aligned} \right\} \quad (7)$$

If $Z=0$ initially, the solution of (6) is

$$Z = be^{-at} \int_0^t e^{at} \frac{dP}{dt} dt. \quad (8)$$

Considering the pressure fluctuation to be sinusoidal, we put

$$P = p_0 + p \sin \omega t,$$

$$\therefore \dot{P} = \omega p \cos \omega t.$$

Thence, we have

$$Z = \frac{\pm bp}{\sqrt{1 + \tan^2 \delta}} \sin(\omega t + \delta)$$

$$- \frac{\tan \delta}{1 + \tan^2 \delta} bpe^{-at}, \quad (9)$$

where δ is the phase difference defined as

$$a/\omega \equiv \tan \delta.$$

After a sufficient time lapse, the second term of (9) vanishes entirely, and the solution becomes

$$Z \approx \frac{\pm bp}{\sqrt{1 + \tan^2 \delta}} \sin(\omega t + \delta). \quad (10)$$

Here the bellows and rod can be seen to displace sinusoidally with time. We put hereon for convenience sake

$$z = \frac{bp}{\sqrt{1 + \tan^2 \delta}}.$$

Substituting the frequency ω by the period T , the relation between the amplitude Z of the bellows system and T can be written as

$$\frac{z/p}{b} = \frac{1}{\sqrt{1 + T^2/\lambda^2}}, \quad (11)$$

where λ is defined as

$$\lambda = 2\pi/a, \quad (12)$$

which is to be called the 'slow-leak constant'. And the phase difference is given by

$$\delta = \tan^{-1} \frac{T}{\lambda}. \quad (13)$$

3. Response Curves

The equation (11) shows that the response z/p or the ratio of the amplitude z of the bellows system to the pressure amplitude p , changes with the period T of the water pressure and the slow-leak constant λ . Figs. 2 and 3 represent the relations (11) and (13) respectively.

In Fig. 2, we can understand the response z/p is large for shorter periods and small for longer periods for a given value of λ . This fact shows that the slow-leak behaves like a filter to the pressure fluctuations due to ocean waves. And in the same figure we see that the response is also a function of the slow-leak constant λ . From curves illustrated therein the most suitable value of λ can be selected for the periods of waves to be measured.

For example, if such wave recorder is to measure only winds waves and swells of periods less than 20 seconds, and be possibly indifferent to longer-period fluctuations, we

Fig. 2. Response curves

Fig. 3 Phase difference

can find from Fig. 1 that the appropriate slow-leak constant λ should be in the vicinity of 60 sec. And further it is obvious that the value of the response will be 0.9 or more for period T less than 20 sec. and 0.3 or less for T larger than 7 min. Then the recorder will register only ordinary wind waves and swells with its full sensitivity, and the scaling out and destruction due to the large fluctuations of sea level caused by tides and other effects, can safely be avoided.

Once the value of λ be selected, the value of a can be determined by the equation (12), and POISEUILLE's constant m in (3) can be calculated in the following way. For this purpose we must first give the respective value of P' , V' and B , W , which in turn determine the value of b according to the equation (7).

In the range of period T to be measured the denominator of the right-hand side of (11) is nearly unity, and z/b , viz., the maximum sensitivity attainable for every variation of T , is seen to be here almost equal to b . In (7), P' and V' are respectively the mean pressure and mean volume of the air inside the reservoir placed at the given depth, denoted h . If we put P_0 and V_0 for the values P' and V' respectively at the surface, the equation connecting P_0 and V_0 with P' and V' is clearly

$$P'V' = P_0V_0.$$

Therefore, using V_0 for V' in (7), we get

$$b = \frac{BV_0}{V_0W + B^2P^2/P_0}.$$

Measuring h in metres we can express $P \approx (1 + 0.1h)P_0$, and we get

$$b = \frac{BV_0}{V_0W + (1 + 0.1h)^2 P_0 B^2}, \quad (14)$$

where P_0 naturally equals to the standard atmospheric pressure at the surface and V_0 should be taken to be nearly equal to the capacity of the air reservoir. Thus B and W of the bellows being assumed for the design, the maximum sensitivity b for a given bepth h (metres), can be calculated or inversely the volume V_0 of the air reservoir for a desired sensitivity b . In this way the value of POISEVILLE's constant m will accordingly be determined in the equation (7) and, if necessary, the dimension of the capillary tube will be designed in equation (3), or as its counterpart, other slow-leak devices may similarly be effected.

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