Effect of Tides on the Magnification of the Wave Recorder using a Stand-pipe*

Sanae UNOKI**

At a few places in Japan sea waves have been recorded for a year or so by the mechanical wave gauge with a stand-pipe, which was first designed by Y. Ota, on account of simplicity and cheapness. The gauge of such type, however, has a fault that the magnification is likely to fluctuate corresponding to the rise and fall of water surface due to tides. Hence we shall consider this problem somewhat quantitatively.

A schematic diagram of the installation is shown in Fig. 1. It consists mainly of an iron pipe (C1) constrained to the vertical by any convenient means, a thick rubber-tube (Rt), a capillary (R2), a reservoir (C2) and a copper bellows (C3). They form a hydraulic resistance-capacitance network as shown in the accompanying electrical analog circuit, where we put

\[ E_0 = \rho g H / 2, \quad C_0 = \pi a^2 / \rho g, \quad C_i = V_i / P_i, \quad R_i = 8 \mu l_i / \pi r_i^4 \quad (i = 1, 2, 3) \]

\( H \) is the wave height, \( \rho \) the density of water, \( a \) the inside radius of an iron pipe, \( V_i \) the volume, \( P_i \) the pressure, \( \mu \) the molecular viscosity of air, \( l_i \) and \( r_i \) are the length and radius, respectively, of a tube or capillary.

Since \( R_3 \) is practically negligible, the amplitude ratio becomes

\[ k = \frac{E}{E_0} = \left[ \left( 1 + \frac{R_1}{R_2} \frac{C_1}{C_0} + \frac{C_1 + C_2 + C_3}{C_0} \right)^2 \right]^{-1} \]

\[ + \frac{R_1}{R_2} \left( \frac{C_0 + C_1}{C_2} + \frac{C_2 + C_3}{C_0} \right) \left( \frac{T_0}{T} \right)^2 \]

by the simple network theory, where \( T \) is the period of the incoming wave, \( T_0 \) the period at maximum response defined by

\[ T_0 = 2\pi \left( R_1 R_2 (C_0 + C_1) (C_2 + C_3) \right)^{1/2} \]

Thus, the short-period and long-period noises in the sea can be filtered out if \( C \) and \( R \) are chosen suitably. The magnification of the gauge, or the ratio of the stroke of the recording pen to the height of the sea wave, can be determined by multiplying \( k \) by a constant, since \( k \) refers to the extension of the bellows.

To our inconvenience, \( k \) varies not only with the wave period but also with the wave height and the tidal height. As far as wind waves and swell are taken into consideration and \( R_2 \) is remarkably larger than \( R_1 \), \( k \) is almost independent of the period, because on the right hand side of the Eq. 1 the first term is considerably larger than the second term containing \( T \), and \( k \) is roughly estimated as

\[ k = \left( 1 + \frac{C_1 + C_2 + C_3}{C_0} \right)^{-1} \]

The effect of tides is attributed to the fact that \( C_1 \) varies with the tidal height, that is,

\[ C_1 = \frac{\pi a^2 L_0}{P_0} \left( 1 - \frac{\gamma}{L_0} \right) \]

where \( \gamma \) and \( L_0 \) are the tidal height and the length of the stand-pipe to the top measured above the mean sea-level respectively (Fig. 1). In Fig. 2 \( k \) is plotted against the ratio \( \gamma / L_0 \) for various capacitance ratios \( (C_2 + C_3) / C_0 \).
Fig. 2. Amplitude ratio $k$ as a function of the relative tidal height for various capacitance ratios, calculated from (2) and (3) in the case of $\frac{\pi a^2 L_0}{P_0} = 0.3$.

and we can see that $k$ becomes large at high water and small at low water. A typical example of the circumstances is shown in Fig. 3, where we find that the value itself read off on the recording paper, not yet converted into actual wave height, fluctuates with the same period and phase as those of tides but the actual wave height converted from them does not represent such fluctuation. For practical use, however, that $k$ fluctuates is very troublesome. In order to reduce the fluctuation of $k$, it is most effective to make the ratio $(C_2+3)/C_0$ large, accordingly the volume of the reservoir must be larger several times than that of air column in the stand-pipe. Then, it is inevitable that the value itself of $k$ becomes small, and its reduction must be compensated by suitable means.

Moreover, the effect of wave height is small when the wave is not so high and the volume of the reservoir is large. Discriminating the values in the static state with a dash, the capacitance $\Sigma C_i$ in Eq. 2 corresponding to the wave height $H$ is given by

$$C_1 + C_2 + C_3 = (C_1' + C_2' + C_3') \left(1 - \frac{v_1}{V_1' + V_2' + V_3'}\right)^2,$$

where $v_1$ is the decrease of air column in the stand-pipe and this is a complex function of $H$. (Here it holds naturally that $V_1 = V_1' - v_1$, $V_2 = V_2'$, $V_3 = V_3'$). When the wave height is large, $\Sigma C_i$ becomes small and $k$ becomes large, and vice versa. In our case, since $V_2$ is sufficiently larger than $v_1$, $\Sigma C_i$ is approximately equal to $\Sigma C_i'$ and $k$ is considered to be almost independent of the wave height. On the other hand, the phase-lag is negligibly small both theoretically and experimentally.

The above discussions, however, should be recognized as a first approximation and are not valid strictly at some points. First, in spite of $k$ having been obtained on the assumption that $C_i$ is constant, $k$ was applied for the case when $C_i$ varies with time. But it may be recognizable in considering the effect of tides, where the time-variation of $C_i$ is small. Second, in the range of long waves the isothermal change may hold, but in the range of short waves with several seconds period the adiabatic change may rather hold and $C_i$ becomes $V_i/P r$. Third, in the case of short waves, the dynamic pressure change with depth of water must be taken into consideration. However, it is extremely difficult to discuss the problem completely considering all effects mentioned above.

In conclusion, the wave recorder in question is convenient at places where waves and tides are not so high for its simplicity and cheapness but not suitable at other places.

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