A Theory of the Cromwell Current (the Equatorial Undercurrent) and of the Equatorial Upwelling —An Interpretation in a Similarity to a Coastal Circulation*

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Abstract: A theory is given to account for the CROMWELL Current—the Equatorial Undercurrent. The equatorial upwelling is closely related to the Current, and a remarkable similarity between the equatorial circulation and the circulation in the coastal upwelling regions. The widths of the equatorial upwelling as well as of the CROMWELL Current the maximum speeds of the upwelling and of the CROMWELL Current, the depth of the core and the thickness of the CROMWELL Current, are all found to be explained quantitatively. The rate of change in the CORIOLIS parameter with latitude and the mean vertical stability of the waters are essential to determine those length scales of the phenomena. The easterly wind stress over the equatorial region is responsible for the processes.

1. Introduction

In 1952, TOWNSEND CROMWELL et al. (CROMWELL et al., 1954) discovered an excitingly strong current flowing eastward just below the west-going surface equatorial current. This was the current which had not been known to us before and has been established by later investigation. Particularly, recent direct measurements of the current by John KNAUSS (1958) have revealed striking features of the current. This subsurface current which had hitherto been entirely unknown and has now become convinced to exist is named the CROMWELL Current or the Equatorial Undercurrent.

We are so curious about this current not only because it is new to us, having entirely been overlooked by the world oceanographers, but also because the discovery seems to provide us with an extremely interesting phenomenon in the ocean. As KNAUSS has reported, the current is narrow, thin, deep and strong, the core of the current sticking to the equator, with a striking symmetry about the equator. All other ocean currents as far as we know do reveal certain variations in their axes, mostly in responses to those in large-scale wind system.

Fig. 1. *Sigma-t*. Northbound section along 140°W longitude, 7°S to 8°N, Stations 17-31, June 3-8, 1952 (Hugh M. SMITH cruise 15).

Sigma-t, in grams per liter, isopleth interval 0.2 gr/l, (according to T. S. AUSTIN, Mid-Pacific oceanography V, U.S. Fish and Wildlife Serv., Spec. Sci. Rep.: Fish. No. 136, Fig. 9)

The temperature and density sections in the meridional plane across the equator reveal the outstanding features of "vertical divergence" of isotherms and isopycnals (Fig. 1), between 50 and 500 meter depths. Most of those sections to the east of about 180° longitude reveal this feature but not usually to the west. This feature looks very similar, at least apparently, to that

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encountered in the regions where pronounced coastal upwelling takes place, such as off California.

The CROMWELL Current appears to be present right in the layer of such characteristic density feature or at least to be closely related to this density structure. In fact, it is apparent from the density distributions that the Undercurrent is nearly in geostrophic balance with the pressure field. The question is then how such pressure field can be produced and maintained. It will be attempted in this paper to give a possible dynamical explanation for the occurrence of the CROMWELL Current as so far observed.

Of course, foregoing observations of the Current have been somewhat limited in the conditions. We have very little or no information as to the pressure of similar equatorial undercurrents in the Atlantic, in the Indian Ocean and even in the western portion of the Pacific. Weak meridional components of flow have not been measured. It is uncertain whether the current might reveal certain fluctuation from season to season. Such a situation may still allow for various different possibilities of theoretical accounts for what have so far been observed. Therefore, the present attempt may only provide one of those possible models.

The model considered in this paper is that within the framework of a linear system. The CORIOLIS change with latitudes is the most important factor, and the density stratification is essential to the process as well. The equatorial upwelling which is confined near the equator is considered to be closely related to the Undercurrent. The easterly winds over the equatorial regions will be responsible for the upwelling and consequently for the Undercurrent. The Undercurrent will be present in the layer below the surface layer within which the wind-induced horizontal *divergence* is assumed to be limited. This lower layer is thus a layer of convergence compensating the surface divergence layer and corre-

sponds to that of characteristic density-divergent features as described before. Two different mechanisms will be considered for maintaining such subsurface convergence: one is through the equatorward flow with longitudinally uniform eastward flow, and the other is through the eastward current decreasing its speed toward eastern boundary. The former model would apply to the regions in the central portion of the equatorial oceans where the undercurrent exists with relatively uniform velocity longitudinally. Although observations seem to show that the core velocity of the Current does not vary appreciably between the Galapagos Island and 140°W and disappears rather abruptly near the Island, the depth of the core seems to decrease toward the east, so that the latter model may be suggested to apply for eastern regions.

It will be found from the following analysis that the principal features as observed may be explained quantitatively by those dynamical models which are highly simplified. The similarity of the equatorial circulation to the coastal circulation will also be considered to be real, and the narrow coastal undercurrent in certain regions of coastal upwelling may be suspected.

2. The observed facts to be accounted for will be those as follows: (1) The CROMWELL Current exists at and very near the equator, between 2°S and 2°N approximately and is symmetrical about the equator; (2) The core of the current is at about 100 meters; (3) The thickness of the current is about 400 meters between the depth of 50 and 500 meters; (4) The maximum velocities of the Current (at the core) are about 100–500 cm-sec⁻¹; (5) The core of the current is likely to be fixed at the equator and to be quite steady; (6) The temperature and density profiles on the meridional section across equator indicate striking features of the vertical stretching of the layer near the thermocline which appear to be characteristic to the CROMWELL Current.

Of course, it would be quite probable that

nonlinear terms are comparable with other terms in the dynamic balances* very near the equator, especially around the core of the undercurrent. The significances of the use of vertical mixing without these nonlinear effects might be doubtful, so that it would not be justified to go much in details of the Undercurrent at the very equator, such as the depth of the core and the thickness of the Current within the framework of the linear system considered.

Highly concentrated activities within the narrow equatorial zone are *not* the direct consequence of the external forces such as winds having similar concentration. Instead, the winds have rather much more uniform distribution over larger areas. The physical factors which may cause such a concentrated pattern in the ocean must therefore be sought for in dynamical mechanism within the interior of the ocean system.

As far as the surface winds over the equatorial regions are relatively uniform, the distributions of the upwelling centered at the equator will be nearly symmetrical about the equator. The width of the upwelling zone may not essentially depend upon wind conditions but only on characteristics of density stratification and on a rate of change in the CORIOLIS parameter. It will be shown that *if* the horizontal divergence associated with the wind-induced upwelling is limited within the surface layer, the convergence to maintain the surface divergence would require the eastward current in the layer just below the surface layer.

3. Equatorial upwelling

Alike the processes along the coast (SVERDRUP et al., 1942), the easterly winds along the equator appear to be essentially responsible for the equatorial processes under consideration. We may note that the easterly component of the average winds along the equator of the Pacific has the maximum

intensity at the longitudes nearly in the half way between the 180th meridian and the eastern coast, that is, around 130° to 135° W in all seasons (HIDAKA, 1958). The surface current charts (such as CROMWELL et al., 1959; the Oceans) seem to show that the westward surface drift has the maximum speed at some longitudes between 110° and 140°W, nearly in consistence with the wind distributions.

Such distributions in the westward surface drifts will tend to give the surface divergence in the east and convergence in the west. However, the equatorial upwelling may seem to be indicated to occur at least in the eastern half of the equator, to the east of 180° meridian. Hence the principal factor which may be responsible for the equatorial upwelling in wide range of the equator will be considered to be the horizontal divergence due to the distribution of the meridional component of flow.

The mechanism may then be similar to for the coastal upwelling and may be close to that described qualitatively by CROMWELL (1952). The following analysis will give the quantitative explanation for the latitudinal width of the equatorial upwelling, the speed of the upwelling, and so forth in terms of the measurable quantities.

Taking x eastward, y northward (from the equator), and z downward (from the sea surface), the equations for the layer above are

$$-fv = -\frac{\partial p}{\partial x} + A \frac{\partial^2 u}{\partial z^2} \quad (3.1)$$

$$fu = -\frac{\partial p}{\partial y} + A \frac{\partial^2 v}{\partial z^2} \quad (3.2)$$

where f is the CORIOLIS parameter, p is the pressure, u and v are the eastward and northward components of the horizontal currents respectively, and A is the eddy viscosity (vertical mixing coefficient).

The term $A \frac{\partial^2 v}{\partial z^2}$ in (3.2) can be ignored as it can be seen unimportant essentially. For this surface layer, we may put approximately

* The order of magnitudes is attempted in the recent preprint manuscript by Dr. R.S. ARTHUR (1959).

$$A \frac{\partial^2 u}{\partial z^2} \simeq \frac{\tau_x + \left(A \frac{\partial u}{\partial z} \right)_H}{H} \quad (3.3)$$

where τ_x is the eastwest component of the surface wind stress.

Then (3.1) becomes

$$-\frac{1}{H} \left(A \frac{\partial u}{\partial z} \right)_H - fv = -\frac{\partial p}{\partial x} + \frac{\tau_x}{H} \quad (3.4)$$

Let us assume for the whole layers,

$$w \frac{\partial \rho}{\partial z} = \frac{A}{\rho} \frac{\partial^2 \rho}{\partial z^2} \quad (3.5)$$

where ρ is the density of sea water and w is the vertical component of velocity.

Then from (1.2) and (1.4),

$$A \frac{\partial^3 u}{\partial z^3} \simeq -\frac{gA}{f} \frac{\partial^3 \rho}{\partial y \partial z^2} = -\frac{gs}{f} \frac{\partial w}{\partial y} \quad (3.6)$$

where $s = \frac{\partial \rho}{\partial z}$.

It may be assumed that the vertical velocity having the maximum value at $z=H$ and vanishing at the sea surface would be expressed, to the first approximation, by

$$w \simeq w_H \sin \frac{\pi}{2H} z \quad (3.7)$$

near the depth $z=H$.

In this case, from (3.6)

$$\left(A \frac{\partial u}{\partial z} \right)_H = \frac{gs}{f} \left(\frac{2H}{\pi} \right)^2 \frac{\partial w_H}{\partial y} \quad (3.8)$$

so that (3.4) becomes

$$-\frac{4gsH}{f\pi^2} \frac{\partial w_H}{\partial y} - fv = -\frac{\partial p}{\partial x} + \frac{\tau_x}{H} \quad (3.9)$$

If we follow the CROMWELL's model for the upwelling, we could ignore $\partial u / \partial x$ and $\partial p / \partial x$. On the other hand, we might have somewhat better approximation, if we retain $\partial p / \partial x$ and assume it to be equal to $2/5 \tau_x / H$ according to the REID's density model.

In this case, $\partial v / \partial y \simeq -w_H / H$ and (3.9) become

$$\frac{\partial^2 v}{\partial y^2} - \frac{f^2}{gH_1 \Delta \rho} v = \frac{3}{5} \frac{\tau_x}{H} \frac{f}{gH_1 \Delta \rho} \quad (3.10)$$

$$\text{where } gH_1 \Delta \rho = 4 \frac{gsH^2}{\pi^2} \quad (3.11)$$

The equation (3.10) can be written

$$\frac{d^2 P}{d\xi^2} - \xi^2 P = -\xi \quad (3.12)$$

where P and ξ are dimensionless quantities defined by

$$v = \left(-\frac{3}{5} \frac{\tau_x}{H} \right) \frac{1}{\beta L_\beta} P \quad (3.13)$$

$$y = L_\beta \xi$$

$$\text{and } L_\beta = \frac{(gH_1 \Delta \rho)^{1/4}}{\sqrt{\beta}} \quad (3.15)$$

where $\beta = df / \partial y \simeq 2 \times 10^{-13} \text{ cm}^{-1} \text{ sec}^{-1}$, and quantity L_β is a characteristic length which is essential to the lengths involved in the process.

It would be desirable to obtain the solution of (3.12) which tends to ξ^{-1} for sufficiently large values of ξ . Instead of this, we may assume $P=0$ for $\xi=0$ and $P \sim \xi^{-1}$ for sufficiently large values of ξ .

Although the numerical solutions** under these conditions give some small departures from ξ^{-1} for very large values of ξ , they are sufficiently accurate for the range of ξ which is of interest. Assuming $P=0$ for $\xi=0$ and $P=\xi^{-1}$ for $\xi=4$, the solution is obtained as illustrated in Fig. 2A. consider-

Fig. 2A. Latitudinal distribution of the meridional velocity: expressed by the dimensionless parameters.

* Although A may differ from the eddy viscosity defined in (3.1), we have little knowledge as to the difference and will simply disregard the difference under this crude model.

** The solution of (1.11) is given by

$$P(\xi) = \frac{dP}{d\xi} \bigg|_0 \xi \left(1 + \frac{\xi^4}{5.4} + \frac{\xi^8}{9.8 \cdot 5.4} + \frac{\xi^{12}}{13 \cdot 12.9 \cdot 8.5.4} + \dots \right) - \frac{\xi^3}{3.1} \left(1 + \frac{\xi^4}{7.6} + \frac{\xi^8}{11.10 \cdot 7.6} + \frac{\xi^{12}}{13.14 \cdot 11.10 \cdot 7.6} + \dots \right)$$

ing that (3.10) is rather crude equation with τ_x constant and that obtained values of P approach to ξ^{-1} well for these large values of ξ concerned, this approximation may practically be good enough.

Fig. 2B. Latitudinal distribution of the vertical velocity: expressed by the dimensionless parameters.

Important numerical values thus determined are of the following quantities:

- (i) Initial slope of P ,

$$\left. \frac{dP}{d\xi} \right|_{\xi=0} \simeq 0.60$$

- (ii) ξ for the maximum value of P ,
 $\xi_M \simeq 1.45$

- (iii) The maximum value of P ,
 $P_{\max} \simeq 0.51$

- (iv) The inflection point of P , and the slope of P at this point,

$$\xi_i \simeq 2.3, \text{ and } \left. \frac{dP}{d\xi} \right|_{\xi_i} \simeq -0.14$$

With these values, the physical quantities are given as follows:

- (i) The half meridional width of the upwelling region,

$$y_M \simeq 1.45 L_\beta$$

- (ii) The maximum value of v ,

$$v_M \simeq -\frac{0.60}{\beta L_\beta} \left(\frac{5}{3} \frac{\tau_x}{H} \right)$$

- (iii) The upwelling velocity at the equator,

$$w_H \Big|_{y=0} \simeq 0.60 \left(\frac{5}{3} \frac{\tau_x}{\beta L_\beta^2} \right)$$

- (iv) The value of y for maximum sinking and the maximum velocity,

$$y_s \simeq 2.3 L_\beta$$

$$\text{and } w_s \simeq -\frac{3}{5} \frac{\tau_x}{\beta L_\beta^2} \times 0.14$$

The actual density data in the equatorial region of the Pacific indicate that the mean vertical stability $\partial \rho / \partial z$ over the upper 500 meters is about 6 to 8×10^{-8} . As the values of those physical quantities are relatively insensitive to the changes in the values of the stability chosen for this range, the values of 6×10^{-8} will be employed.

Putting $H = 10^4$ cm and $\tau_x = -0.5$ dynes cm^{-2} , we obtain $gH_1 \Delta \rho \simeq 2.5 \times 10^3 \text{ cm}^2 \cdot \text{sec}^{-2}$

and $L_\beta \simeq 1.57 \times 10^7$ cm.

With these values,

- (i) $y_M \simeq 2.3 \times 10^7$ cm

(latitudes: 2.3°N and S)

- (ii) $v_M \simeq 4.9 \text{ cm} \cdot \text{sec}^{-1}$

(poleward)

- (iii) $w_H \simeq -3.0 \times 10^{-3} \text{ cm} \cdot \text{sec}^{-1}$ (upwelling)

- (iv) $y_s \simeq 3.6 \times 10^7$ cm

(latitudes: 3.6°N and S)

$w_s \simeq 0.7 \times 10^{-3} \text{ cm} \cdot \text{sec}^{-1}$ (sinking)

All of these values look very reasonable in comparison with the average values so far observed. It should be noted that these values depend only upon the average vertical stability as far τ_x and H are given as observed.

4. Equatorial Undercurrent (CROMWELL Current)

Let us now deal with the layers below $z=H$ with the same equations (3.1), (3.2) and (3.5). The approximation (3.3) and hence (3.4) are not valid for the layers now concerned. From (3.1) and (3.2), eliminating the pressure terms,

$$-f \frac{\partial w}{\partial z} + \beta v \simeq -A \frac{\partial^3 u}{\partial y \partial z^2} \quad (4.1)$$

As (3.6) is valid, (4.1) becomes

$$f \frac{\partial^2 w}{\partial z^2} - \beta \frac{\partial v}{\partial z} \simeq -gsf \frac{\partial}{\partial y} \left(\frac{1}{f} \frac{\partial w}{\partial y} \right) \quad (4.2)$$

Then if we assume $\partial v / \partial y = -\partial w / \partial z$, (4.2) yields

$$\frac{\partial}{\partial y} \left(f^2 \frac{\partial^2 w}{\partial z^2} \right) \simeq -gsf \frac{\partial^2}{\partial y^2} \left(\frac{1}{f} \frac{\partial w}{\partial y} \right) \quad (4.3)$$

If we further assume,

$$w(y, z) = w_H(y) F(z)$$

where $w_H(y)$ satisfies (3.10) with

$$w_H = -H \partial v / \partial y$$

we can readily see from (4.3) and (3.10)

$$\frac{d^2 F}{dz^2} + \frac{s}{H_1 \Delta \rho} F = 0 \quad (4.4)$$

Thus $F(z) = F_0 \cos \sqrt{\frac{s}{H_1 \Delta \rho}} (z-H)$,

so that

$$w(y, z) = w_H(y) \cos \frac{2\pi}{L_z} (z-H) \quad (4.5)$$

where $L_z = 2\pi \left(\frac{H_1 \Delta \rho}{s} \right)^{\frac{1}{2}} = 4H$ (4.6)

in virtue of (1.10)

Having now (4.5) for $w(y, z)$, we can now put

$$u(y, z) = u_M(y) \sin \frac{2\pi}{L_z} (z-H) \quad (4.7)$$

in which $u_M(y)$ can be determined from (3.6) and (4.5), to be

$$\begin{aligned} u_M(y) &= \frac{1}{A} \left(\frac{L_z}{2\pi} \right)^3 \frac{gs}{f} \frac{\partial w_H}{\partial y} \\ &= -\frac{3}{5} \frac{\tau_x}{A} \left(\frac{L_z}{2\pi} \right) (1-\xi P) \end{aligned} \quad (4.8)$$

For $f=0$,

$$u_M|_0 = -\frac{3}{5} \frac{\tau_x}{A} \left(\frac{L_z}{2\pi} \right) \quad (4.9)$$

Using (4.7), and (4.8), the total transport is

$$\begin{aligned} M_x &= 2 \int_0^{y_s} dy \int_H^{H+L_z/2} u(y, z) dz \\ &\simeq -4 \left(\frac{L_z}{2\pi} \right)^2 \left(\frac{3}{5} \frac{\tau_x}{A} \right) L_\beta \int_0^{\xi_t} (1-\xi P) d\xi \\ &\simeq -4 \left(\frac{L_z}{2\pi} \right)^2 \left(\frac{3}{5} \frac{\tau_x}{A} \right) \times 1.09 \end{aligned} \quad (4.9)$$

since $\xi_t = 2.3$.

The ratio of M_x to $u_M|_0$ does not depend on A , but only on H and L_β , as given by

$$\begin{aligned} \frac{M_x}{u_M|_0} &\simeq 4 \left(\frac{L_z}{2\pi} \right) \times 1.09 L_\beta \simeq \frac{8.72}{\pi} H L_\beta \\ &\simeq 4.4 \times 10^{11} \text{ cm}^2 \end{aligned}$$

The observed values are $M_x \simeq 40 \times 10^{12} \text{ cm}^3 \cdot \text{sec}^{-1}$ and $u_M|_0 \simeq 10^2 \text{ cm} \cdot \text{sec}^{-1}$, so that

$$\frac{M_x}{u_M|_0} \simeq 4.0 \times 10^{11} \text{ cm}^2.$$

Hence the computed value is fairly close to the observed one.

The values of $u_M|_0$ and M_x depend upon the values of A which is less certain than any other observable quantities involved. However, if we assume $A = 20 \text{ cm}^2 \cdot \text{sec}^{-1}$,

$$u_M|_0 \simeq 96 \text{ cm} \cdot \text{sec}^{-1}$$

and

$M_x \simeq 42 \times 10^{12} \text{ cm}^3 \cdot \text{sec}^{-1} = 42 \times 10^6 \text{ m}^3 \cdot \text{sec}^{-1}$ which are very close to the observed values. Although we assume in the above calculations that the eddy viscosity is equal to the eddy diffusivity, the value $A \simeq 20 \text{ cm}^2 \cdot \text{sec}^{-1}$ may be considered quite acceptable.

The thickness of the Undercurrent is given by

$$L_z/2 = 200 \text{ meters} \quad (4.10)$$

and the depth of the core of the Undercurrent is, measuring from the bottom of the upper layer $z=H$,

$$L_z/4 = 100 \text{ meters} \quad (4.11)$$

As far as these of the depth and thickness of the Undercurrent are concerned, the obtained values, (4.10) and (4.11), may only give the rough order of magnitudes. The core of the Undercurrent as observed is shallower and the thickness is somewhat greater. Although it may be difficult to find the source of such discrepancies, the greater vertical stability in the layer between 100 and 200 meter depths as seen from actual density data might possibly make the core much shallower and the thickness greater.

To derive (3.5), the assumption was made that the magnitude of the vertical velocity has the maximum at $z=H$ for all latitudes concerned. Even though this assumption is accepted, the distribution of $w_H(y)$ derived above may not be valid at the latitudes higher than 4 or 5 degrees, because the wind stress curl due to the northward increase in the easterly wind velocities may be as much effective to the distribution of w_H as the vorticity due to the latitudinal change in the CORIOLIS parameter.

Thus, we may have upwelling at latitudes as high as 7° , if we consider τ_x in (3.10) as a function of y as observed. In such more realistic cases, the solution $w_H(y)$ may resemble something like $w_H(0) \cos ly$, where $l = 2\pi/L_y \simeq 2\pi/8 \times 10^7 \simeq 0.8 \times 10^{-7} \text{ cm}^{-1}$.

In this case,** for larger values of y ,

** The equation (4.3) can also be written

$$gs \frac{\partial^4 w}{\partial y^4} + f^2 \frac{\partial^4 w}{\partial y^2 \partial z^2} + 6f\beta \frac{\partial^3 w}{\partial y \partial z^2} + 6\beta^2 \frac{\partial^2 w}{\partial z^2} \simeq 0$$

$$\frac{\partial^2 w}{\partial y^2} + \frac{f^2}{gs} \frac{\partial^2 w}{\partial z^2} \simeq 0 \quad (4.12)$$

Hence

$$\frac{\partial^2 w}{\partial z^2} - \frac{gs l^2}{f^2} w \simeq 0$$

so that

$$w \simeq w_H \exp \left[-\frac{\sqrt{gs} l}{f} (z-H) \right] \quad (4.13)$$

For the smaller values of y ,

$$\frac{\partial^2 w}{\partial z^2} \simeq -\frac{gs}{6\beta^2} \frac{\partial^4 w}{\partial y^4} \quad (4.14)$$

Hence

$$\frac{\partial^2 w}{\partial z^2} + \frac{gs l^4}{6\beta^2} w \simeq 0$$

of which the solution will be

$$w = w_H \cos \frac{\sqrt{gs} l^2}{\sqrt{6}\beta} (z-H) \quad (4.15)$$

Putting $gs = 6 \times 10^{-5} \text{ sec}^{-2}$, $l = 0.8 \times 10^{-7}$,

in (4.13), we have $w(z)/w(0) \simeq e^{-1}$

for $z = \frac{f}{6.4 \times 10^{-10}} \text{ cm}$. At latitude 6° ,
 $z \simeq 190 \text{ m}$; and at 10° , $z \simeq 315 \text{ m}$.

Putting $w \simeq w_H \cos \frac{2\pi}{L_e} (z-H)$ in (4.15)

$$L_e \simeq 2\pi \times 10^4 \text{ cm} \simeq 628 \text{ meters}$$

Thus, the vertical velocity does not change its sign with depths at higher latitudes, whereas it does so very near the equator, provided that is given by something like $W_H(0) \cos ly$.

As seen from (3.1), the width of the Equatorial Undercurrent is somewhat broader than the width of the surface upwelling.

The half width of the Undercurrent is the distance between the equator and the maximum sinking which occurs at 3.6° .

From (1.7),

$$\left(A \frac{\partial u}{\partial z} \right)_H = -\frac{3}{5} \tau_x (1 - P\xi) \quad (4.16)$$

As the solution must approach very closely to $P\xi \rightarrow 1$ for large values of ξ , shearing stress $(A\partial u/\partial z)_H$ becomes negligible at latitudes well apart from the equator. Practically the stress is quite small beyond $\xi = \xi_M \sim 1.45$ and is negligible beyond $\xi = \xi_t \sim 2.3$, corresponding to the outer boundaries of the surface upwelling region and the

Fig. 2C. Latitudinal distribution of the east-west component of velocity: expressed by the dimensionless parameters.

Undercurrent, respectively. Hence, the undercurrent exists in the region where $(A\partial u/\partial z)_H$ has a significant magnitude compared with the surface wind stress. The computed curve for $1-P\xi$ is given in Fig. 2C, and the computed values of the core velocity of the Undercurrent are $96 \text{ cm}\cdot\text{sec}^{-1}$ at the equator, $75 \text{ cm}\cdot\text{sec}^{-1}$ at 1°N and S , $36 \text{ cm}\cdot\text{sec}^{-1}$ at 2°N and S , $8 \text{ cm}\cdot\text{sec}^{-1}$ at 3°N and S and $2 \text{ cm}\cdot\text{sec}^{-1}$ westward at 4°N and S .

The meridional circulation will be obtained by the solution of (3.10) as well as (4.3).

For $z > H$,

$$v = -\frac{\pi}{2} v_0(y) \sin \frac{2\pi}{L_e} (z-H) \quad (4.17)$$

where $v_0(y)$ denotes the average meridional velocity in the surface layer given by (3.10). Thus, there is equatorward flow beneath the surface layer and this flow leads to the subsurface convergence at latitudes lower than 2.3° and to the divergence beyond the latitude of 2.3° .

5. A model in which $\partial u/\partial x$ is important

It is again assumed that the vertical velocity has the maximum at $z=H$. The presence of the equatorward flow in the layer below $z=H$ would be consistent with the occurrence of subsurface convergence, only if the relative vorticity decreases as the water moves southward either through

diffusion or advection of the vorticity. In the absence of such effects of friction or accelerations, on the other hand, the longitudinal variation in the zonal component of current could maintain the subsurface convergence. This could be the case when eastward current decreases its speed toward the eastern boundary of the ocean. The meridional component of flow directs poleward in this case and the divergence due to this component of flow on and very near the equator could be overcompensated by the eastward current. It should be noticed that the equator is an exceptional place as to the relation between the geostrophic horizontal divergence and the meridional flow. Except for the lowest latitudes, the poleward flow may produce the geostrophic convergence and hence maintain the upward motion of water above (STOMMEL 1956; YOSHIDA and MAO, 1957).

This can immediately be seen from the following equation derived by differentiating (4.1) with y :

$$-f \frac{\partial^2 w}{\partial z \partial y} - 2\beta \frac{\partial w}{\partial z} - \beta \frac{\partial u}{\partial x} \simeq -A \frac{\partial^4 u}{\partial y^2 \partial z^2} \quad (5.1),$$

in which

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}$$

is used.

Put

$$2\beta = 2 \times 10^{-13}, \quad w = 10^{-3}, \quad |\partial/\partial z| \sim 10^{-4}, \\ u = 10^2, \quad |\partial/\partial x| \sim (5 \times 10^8)^{-1}$$

and then

$$\left| 2\beta \frac{\partial w}{\partial z} \right| = 4 \times 10^{-13} \times 10^{-7} = 4 \times 10^{-20} \\ \left| \beta \frac{\partial u}{\partial x} \right| = 2 \times 10^{-13} \times 10^2 \times \frac{1}{5} \times 10^{-8} = 4 \times 10^{-20}.$$

For the scales of motions corresponding to the Undercurrent, it will be seen that the friction term in the right hand side of (5.1) is quite comparable with these two terms, provided that A is of the order of 10 in its magnitude (C.G.S.). However, in this model, we may assume that the model is nearly geostrophic in this layer below $z=H$, in contrast to the model given in the preceding sections.

Anyhow, from (5.1) ignoring the mixing term,

$$\frac{\partial u}{\partial x} = -\left(2 \frac{\partial w}{\partial z} + \frac{f}{\beta} \frac{\partial^2 w}{\partial z \partial y}\right) \quad (5.2)$$

At the equator,

$$\frac{\partial u}{\partial x} = -2 \frac{\partial w}{\partial z}$$

If we put

$$\frac{\partial w}{\partial z} \simeq \frac{w(D) - w(H)}{D - H} \\ \simeq \frac{1 \times 10^{-3} - (-1 \times 10^{-3})}{4 \times 10^4} \simeq \frac{1}{2} \times 10^{-7} \text{ sec}^{-1},$$

$$\text{then } \frac{\partial u}{\partial x} \simeq -1 \times 10^{-7} \text{ sec}^{-1}$$

Assuming $u=0$ at the eastern boundary,

$$u(x) \sim -1 \times 10^{-7} x$$

With $x=5 \times 10^8$,

$$u(x) \sim 50 \text{ cm} \cdot \text{sec}^{-1}$$

As $u(x)$ refers to the *average* eastward velocity at the equator, the obtained value will be reasonable.

The idea given above could in fact be explained by a small modification from the MUNK's mass transport theory (1950). Based on the simple vorticity equation as Munk assumes for the central and eastern portions of the oceans;

$$-f \frac{\partial w}{\partial z} + \beta v = \frac{\partial}{\partial z} \left(A \frac{\partial \zeta}{\partial z} \right) \quad (5.3)$$

we may now divide the ocean into two layers of which the interface refers to the depth of no horizontal divergence.

The upper layer (with the subscript 1) may correspond to the upper mixed layer in which the wind mixing is effective, and the lower layer (with the subscript 2) is assumed to be frictionless.

Vertical integration of (5.3) for each of the upper and the lower layer gives

$$-f w_H + \beta M_{v1} = \text{curl}_z \tau \quad (5.4)$$

$$f w_H + \beta M_{v2} = 0 \quad (5.5)$$

where $w(0) = w(D) = 0$,

$$M_{v1} = \int_0^H v dz, \quad M_{v2} = \int_H^D v dz$$

Differentiating (5.4) and (5.5) with y , and putting $f=0$, we obtain

$$\beta \left(\frac{\partial M_{v1}}{\partial y} - w_H \right) = \frac{\partial}{\partial y} \text{curl}_z \tau \quad (5.6)$$

$$\beta \left(\frac{\partial M_{v2}}{\partial y} + w_H \right) = 0 \quad (5.7)$$

Considering that $w_H = u_H \frac{\partial H}{\partial x} + v_H \frac{\partial H}{\partial y}$, we must have $\text{div}_H/M_1 = \text{div}_H/M_2 = 0$ (5.6) and (5.7) can be written

$$\frac{\partial M_{x1}}{\partial x} = -\frac{1}{\beta} \frac{\partial \text{curl}_s \tau}{\partial y} - w_H \quad (5.8)$$

$$\frac{\partial M_{x2}}{\partial x} = w_H \quad (5.9)$$

The integration with the boundary condition that $M_{x1} = M_{x2} = 0$ at the eastern edge of the ocean $x = x_e$ yields

$$M_{x1} = \frac{1}{\beta} \int_x^{x_e} \frac{\partial \text{curl}_s \tau}{\partial y} dx + \int_x^{x_e} w_H dx \quad (5.10)$$

$$M_{x2} = - \int_x^{x_e} w_H dx \quad (5.11)$$

The equations (5.9) and (5.11) represent nearly the same things as (5.3). In the annual mean, the term

$$\int_x^{x_e} \frac{\partial \text{curl}_s \tau}{\partial y} dx$$

is positive, so that the zonal component of the total transport along the equator is eastward in MUNK's result. REID's result (1948) shows, on the other hand, the westward transport at the equator, since he deals with the conditions during October to November when his wind data gives negative values of $\frac{\partial \text{curl}_s \tau}{\partial y}$. The eastward transport at the equator in MUNK's result might suggest the presence of the Equatorial Undercurrent rather than that of the surface Equatorial Countercurrent extending down to a little south of the equator.

From (5.2), if we simply put

$$\begin{aligned} \partial w / \partial z &\simeq -w_H / D_0, \\ \frac{\partial u}{\partial x} &= -\frac{1}{f\beta} \frac{\partial}{\partial y} \left(f^2 \frac{\partial w}{\partial z} \right) \\ &\simeq \frac{1}{f\beta D_0} \frac{\partial}{\partial y} (f^2 w_H) \end{aligned} \quad (5.12)$$

Hence

$$u \simeq -\frac{x}{f\beta D_0} \frac{\partial}{\partial y} (f^2 w_H) \quad (5.13)$$

It is seen that the eastward flow ($u > 0$) occurs in the regions very near the equator and the width of the eastward current is in this case somewhat *narrower* than that of the surface upwelling.

If we use w_H obtained in the section 3, it is shown that the outer boundary of the Undercurrent is nearly at $\xi = 0.95$ which corresponds to 1.5° N and S approximately. (5.13) can be written

$$u = -\frac{3}{5} \frac{\tau_x}{\beta L \beta^2} \frac{x}{D_0} \frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi^2 \frac{dP}{d\xi} \right)$$

If we put $\tau_x = -0.5$, $x = 5 \times 10^8$, $D_0 = 3 \times 10^4$,

$$u = 10^2 \left\{ \frac{1}{\xi} \frac{d}{d\xi} \left(\xi^2 \frac{dP}{d\xi} \right) \right\} \text{cm. sec}^{-1}$$

Latitude: $\varphi(o)$	ξ	$u(\text{cm. sec}^{-1})$
0	0	120
0.3	0.2	112
0.6	0.4	89
0.9	0.6	59
1.3	0.8	30
1.6	1.0	-10
1.9	1.2	-37
2.2	1.4	-52

On this model, the Undercurrent is much narrower than obtained on preceding model. The observed width of the Current may appear to be closer to this present model.

6. Similarity to the costal upwelling

Some analogy can be seen between the equatorial upwelling and the coastal upwelling. Let us assume the similar equations and the coastal boundary on one edge of the ocean.

The equations are

$$-fv = -\frac{\partial P}{\partial x} \quad (6.1)$$

$$fu = -\frac{\partial p}{\partial y} + A \frac{\partial^2 v}{\partial z^2} \quad (6.2)$$

$$sw = A \frac{\partial^2 p}{\partial z^2} \quad (6.3)$$

where all symbols denote the quantities which are the same as in preceding sections, and the coast is assumed to be at $x = L$.

From (6.2)

$$fu = -\frac{\partial p}{\partial y} + \frac{\tau_y + \left(A \frac{\partial v}{\partial z} \right)_H}{H} \quad (6.4)$$

From (6.1) and (6.3),

$$A \frac{\partial^3 v}{\partial z^3} = \frac{gs}{f} \frac{\partial w}{\partial x} \quad (6.5)$$

Assuming

$$w = w_H \sin \frac{\pi}{2H} z$$

and

$$v = v_0 \cos \frac{\pi}{2H} z$$

(6.4) becomes

$$A \left(\frac{\pi}{2H} \right)^3 v_0 = \frac{gs}{f} \frac{\partial w_H}{\partial x}$$

Hence

$$\left(A \frac{\partial v}{\partial z} \right)_H = - \left(\frac{2H}{\pi} \right)^2 \frac{gs}{f} \frac{\partial w_H}{\partial x}$$

Assuming $\partial u / \partial x = -w_H / H$, and $\partial p / \partial y \approx 0$, (4.4) becomes

$$\frac{\partial^2 u}{\partial x^2} - \frac{f^2}{gH_1 \Delta \rho} u = - \frac{f}{gH_1 \Delta \rho} \frac{\tau_y}{H}$$

where

$$gH_1 \Delta \rho = \frac{4gsH^2}{\pi^2} \quad (6.7)$$

This is the equation analogous to (3.10) for the equatorial upwelling.

The main difference is that the CORIOLIS parameter f can be regarded constant in the case of coastal upwelling.

The characteristic length in this case is $\sqrt{gH_1 \Delta \rho} / f$ which is something like the radius of deformation for the baroclinic motions. In this case, we must have $u=0$ for $x=0$

and $u \rightarrow \frac{\tau_y}{fH}$ for $x \rightarrow -\infty$.

Then, the solution is

$$u = \frac{\tau_y}{fH} \left[1 - e^{-\frac{f}{\sqrt{gH_1 \Delta \rho}}(L-x)} \right] \quad (6.8)$$

in case when $\tau_y = \text{constant}$.

$$\text{Then } w_H = \frac{\tau_y}{\sqrt{gH_1 \Delta \rho}} e^{-\frac{f}{\sqrt{gH_1 \Delta \rho}}(L-x)} \quad (6.9)$$

Along the coast, assuming

$$v = v_M \sin \frac{2\pi}{4H} (z-H),$$

one obtains

$$v_M = - \left(\frac{2H}{\pi} \right) \frac{\tau_y}{A}$$

Putting $gs=10^{-4}$ and $H=5 \times 10^3$, we have $gH_1 \Delta \rho = 10^3 \text{ cm}^2 \cdot \text{sec}^{-2}$

For $f = \frac{1}{2} \times 10^{-4}$,

$$\frac{f}{\sqrt{gH_1 \Delta \rho}} = 1.6 \times 10^{-6} \text{ cm}^{-1} \quad (6.10)$$

For $\tau_y = -0.5$

$$w_H = \frac{\tau_y}{\sqrt{gH_1 \Delta \rho}} = -1.6 \times 10^{-2} \text{ cm} \cdot \text{sec}^{-1} \quad (6.11)$$

Assuming $A=50$,

$$v_M \approx 33 \text{ cm} \cdot \text{sec}^{-1} \quad (6.12)$$

For instance, off the California coast, the undercurrents of such speeds have not been observed.

The width of the upwelling and hence the undercurrent will* be very small, less than 50 km as seen from (6.10), so that the current is difficult to be detected by ordinary observations. Furthermore, the shallow depths adjacent to the coast may prevent the development of such undercurrent. Even though this might really exist, future direct measurements of the current could only detect it. The presence of the geostrophic northward current (opposite to the surface wind direction) below the thermocline has been reported in the upwelling region off California coasts (SVERDRUP et al., 1942; YOSHIDA and MAO, 1957).

The analogy may essentially be in the deep penetration of the vertical mixing in the narrow boundary zone along the equator and along the coast. The occurrence of such deep mixing may be due to the essential balance between this mixing and pressure gradient in the direction of the dominant current at the boundary, where the component of the CORIOLIS force must vanish. At some distances from the boundaries, the pressure gradient would rather balance the CORIOLIS force, approaching the geostrophic balance.

The undercurrents in both cases may occur

$$\begin{aligned} * \text{ From } A \frac{\partial^3 v}{\partial z^3} &= \frac{gs}{f} \frac{\partial w}{\partial x}, \\ - \left(\frac{\pi}{2H} \right)^3 A v_M &= \frac{gs}{f} \frac{\partial w_H}{\partial x} = \frac{s\tau_y}{H_1 \Delta \rho} e^{-\frac{f}{\sqrt{gH_1 \Delta \rho}}(L-x)} \\ v_M &= - \frac{gs}{A} \left(\frac{2H}{\pi} \right)^3 \frac{\tau_y}{gH_1 \Delta \rho} e^{-\frac{f}{\sqrt{gH_1 \Delta \rho}}(L-x)} \\ &= - \frac{2H}{A\pi} \tau_y e^{-\frac{f}{\sqrt{gH_1 \Delta \rho}}(L-x)} \end{aligned}$$

within the zones where such deep mixing is considerable.

Concluding Remarks

These seems to be in the oceans *no* such well-fixed current like the CROMWELL Current which is fixed on the equator and confined within only one to two degrees from the equator with a beautiful symmetry about the equator! For instance, the Equatorial Counter Current is relatively narrow current, but this is found to reveal considerable degree of seasonal fluctuation in its position as well as in the intensity. The position is fixed only in a statistical mean within a certain latitudinal range, principally depending on large-scale wind stress distributions. As far as we can refer to current ever observed, it is likely to suppose that the phenomenon of the Cromwell Current is that characteristic to the *equator itself*. It is quite impressive that the waters near the equator, particularly in the subsurface layer, seem to "feel the equator very accurately". Why should they be able to do this? The answer is simply that the equator has a singularity of vanishing vorticity due to the Earth's rotation! It may be of interest to see that the width of the equatorial upwelling as well as the Undercurrent is determined essentially by the characteristic factor related to the density stratification or by the *mean vertical stability*. With the reasonable value for the mean vertical stability, the width of the upwelling is computed to be between 2.3° S and 2.3° N. Under certain assumption, the latitudinal distributions of w and v are found reasonable. It is found that the layer of convergence will have a thickness of about 200 meters below the depth of thermocline.

The eastward current is present within this layer of convergence, and the core of the current is at the mid-depth of the layer along the equator. Of course, the theoretical model may be too crude to obtain the vertical distributions of the eastward current. The magnitude of the Undercurrent

at the core is found to be $100 \text{ cm} \cdot \text{sec}^{-1}$ and the total eastward transport becomes $40 \times 10^6 \text{ m}^3 \cdot \text{sec}^{-1}$, provided that the eddy viscosity as well as the eddy diffusivity is assumed to be about 20.

The present model may suggest that vertical mixing extends down to the layers below thermocline near the equator.

The eastward current exists in the regions where vertical mixing penetrates deep. The Undercurrent may thus be interpreted to be maintained by the vertical shearing stress exerting over the surface $z=H$ near the equator.

As far as we have seen, the characteristic density features across the equator may be considered as indicative of the Undercurrent. The Undercurrent is contained over the layer of convergence. The broad easterly winds over the equatorial region must be responsible for the Undercurrent. If this may only be the essential conditions for the Undercurrent, it will be very likely that the Current may exist also in the Atlantic. It is very curious that an observational data taken by the Japanese in the eastern portion of the equatorial Indian Ocean indicates very clearly the similar thermal feature across the equator as seen in the eastern Pacific. (YAMANAKA, 1958). The observations were taken during January. Although the wind conditions are not reported, the usual conditions over these regions seem to differ largely from those in other oceans

The similarity between the density distributions in the equatorial region and the coastal region seems to be a manifestation of real similarity in dynamical processes involved. Owing to the shallow depths adjacent to the coast, the coastal undercurrent may not develop as the equatorial undercurrent, so that such striking phenomena of the strong undercurrent may not exist in actuality. However, as pointed out by the present author before, the density distributions seem to indicate that the geostrophic northward flow does occur along the coast of California during upwelling.

Direct current measurements must be made to detect such narrow subsurface counter-current.

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