Non-Linear Computation of the Equatorial Upwelling*

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Abstract: The equatorial data of the vertical profile of zonal components of velocity, wind stresses on the surface of the sea, and the slope of isobaric surfaces including the sea surface are used to derive upwelling and sinking, or the vertical components of current at the equator ($\varphi=0^\circ$). Only thing to be assumed is the coefficient of eddy viscosity. The non-linear hydrodynamical equations were linearized exactly on the Equator by these arrangements to an ordinary differential equations to compute the vertical components of equatorial flow. Numerical result is given.

1. Introduction

The author has been working theoretically on the explanation of the equatorial current system of the Pacific for several years (HIDAKA, 1962, 1963, 1965) on the assumption that the effects of vertical motion can be disregarded. However, the equatorial regions of the oceans have been frequently watched with keen interest by theoretical oceanographers as the sites of intense upwelling. Moreover, it was pointed out the the field accelerations play an important rôle at the Equator where the Coriolis force vanishes. (ARTHUR, 1960)

Thus the author now intends to explain the equatorial upwelling of the Pacific, making use of the geopotential distances, vertical distribution of zonal component of equatorial flow (Equatorial Undercurrent) determined by direct measurements, the components of wind stresses on the sea surface and the effects of Coriolis force and vertical eddy viscosity, and taking into account the non-linear terms due to field accelerations. Thermodynamic processes and effect of horizontal mixing are disregarded.

2. Hydrodynamical equations at the Equator

We suppose a meridional section passing through the equator. Take x -, y -, z -axes positive eastward, northward and downward and let the velocity components in these directions be u , v and w respectively. Further assume that the east-west variation of velocity

components is small compared with that in the meridional and vertical directions.

This condition is not necessarily satisfied all the way along the equator. The following discussion therefore applies in some longitudes where this assumption is approximately valid.

For this reason and to make the problem simple, terms

$$u \frac{\partial u}{\partial x}, \quad u \frac{\partial v}{\partial x}, \quad \text{etc.}$$

are neglected, except for the east-west pressure gradient:

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} \quad \text{or} \quad -\frac{\partial D}{\partial x},$$

which is approximately uniform all the way along the equator. Under these assumptions the steady state hydrodynamical equations for a planetary flow will be, taking into account* the Coriolis component due to vertical velocity component w ,

$$\left. \begin{aligned} \frac{\mu}{\rho} \frac{\partial^2 u}{\partial z^2} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z} + 2\omega \sin \varphi v \\ + 2\omega \cos \varphi w - \frac{\partial D}{\partial x} = 0, \\ \frac{\mu}{\rho} \frac{\partial^2 v}{\partial z^2} - v \frac{\partial v}{\partial y} - w \frac{\partial v}{\partial z} - 2\omega \sin \varphi u \\ - \frac{\partial D}{\partial y} = 0, \end{aligned} \right\} \quad (1)$$

while the equation of continuity is

* Suggested by Dr. Takashi ICHIYE, the Lamont Geological Observatory, Columbia University. However, this consideration does not give any significant contribution in the result.

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$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (2)$$

where D is the geopotential distance of an isobaric surface counted upward from a sufficiently deep reference level where the motion of water can be neglected, ω the angular velocity of the Earth, φ the geographic latitude, ρ the density of sea water, while μ is the coefficient of vertical eddy viscosity supposed uniform.

The problem now is how to solve the equations (1) and (2) subject the boundary conditions:

$$z=0: -\frac{\partial u}{\partial z} = \tau_x; -\mu \frac{\partial v}{\partial z} = \tau_y \quad (3)$$

and

$$z=h: u=v=0 \quad (4)$$

where h is the depth of the reference level.

It is difficult to say down to what depth the vertical velocity w will exist. It will not vanish at the surface as long as there are slope on, or evaporation from the sea surface. Nevertheless, we assume

$$z=0 \text{ and } z=h: w=0 \quad (5)$$

to the first approximation, because we cannot expect perceptible upwelling or sinking at the sea surface and at the reference level supposed sufficiently deep.

If we eliminate $\frac{\partial v}{\partial y}$ between the equation of continuity (2) and the second equation of (1), we have

$$\frac{\mu}{\rho} \frac{\partial^2 v}{\partial z^2} - w \frac{\partial v}{\partial z} + \frac{\partial w}{\partial z} v - 2\omega \sin \varphi u - \frac{\partial D}{\partial y} = 0 \quad (6)$$

where $\frac{\partial D}{\partial y}$ can be computed from observations.

As is seen from most observations of Equatorial Undercurrent, the distribution of u is very exactly symmetric about the Equator. This observational fact favors us another condition:

$$\varphi=0: \frac{\partial u}{\partial y} = 0. \quad (7)$$

Thus (1) becomes

$$\frac{\mu}{\rho} \frac{d^2 u_0}{dz^2} - w_0 \frac{du_0}{dz} + 2\omega w_0 - \frac{\partial D_0}{\partial x} = 0 \quad (8)$$

and

$$\frac{\mu}{\rho} \frac{d^2 v_0}{dz^2} - w_0 \frac{dv_0}{dz} + \frac{dw_0}{dz} v_0 - \frac{\partial D_0}{\partial y} = 0 \quad (9)$$

where the suffix 0 means that all quantities hereafter refer to the Equator ($\varphi=0^\circ$). The equation (9) will not be used here in this research. However, it is written down here because it serves to compute v_0 from w_0 in later computations.

3. Computation of vertical component from the profile of zonal component

There are a number of very detailed measurements of the Equatorial Undercurrent, or the zonal component at the Equator. John A. Knauss combined his own direct measurements into a composite vertical profile. (KNAUSS, 1960, p. 274)

We have from (8),

$$w_0 = -\frac{\frac{\mu}{\rho} \frac{d^2 u_0}{dz^2} - \frac{\partial D_0}{\partial x}}{\frac{du_0}{dz} - 2\omega}. \quad (10)$$

We shall be able to derive the vertical motion w_0 from this formula by numerical differentiation. This equation will fail at a depth at which $\frac{du_0}{dz} = 2\omega$. This singularity can be avoided by differentiation (8) with respect to z , and putting $\frac{du_0}{dz} = 2\omega$, or by using the formula:

$$w_0(z) = \frac{\frac{\mu}{\rho} \frac{d^2 u_0}{dz^2} - \frac{\partial^2 D_0}{\partial z \partial x}}{\frac{d^3 u_0}{dz^3}}. \quad (11)$$

However, numerical differentiation of higher orders make the result more unreliable due to the accumulation of error of computation. There has been, in addition, a further need of computing the meridional component v from the equation (9). For these reasons, observed vertical distribution of $u_0(z)$ and the slope of isobaric surfaces were expanded in series

$$u_0(z) = \sum_{n=1}^{\infty} u_{2n-1} \cdot \cos \frac{(2n-1)\pi z}{2h}, \quad (12)$$

and

$$\frac{\partial D_0}{\partial x} = \sum_{n=1}^{\infty} \left(\frac{\partial D}{\partial x} \right)_{2n-1} \cdot \cos \frac{(2n-1)\pi z}{2h}, \quad (13)$$

since both $u_0(z)$ and $\frac{\partial D_0}{\partial x}$ have to vanish at the reference level $z=h$.

As to the vertical component $w_0(z)$ which is

not known as yet, we may assume

$$w_0(z) = \sum_{n=1}^{\infty} w_{2n} \sin \frac{2n\pi z}{2h}, \quad (14)$$

because this component is supposed to vanish both at the sea surface and at the reference level.

Further assume

$$\frac{d^2 u_0}{dz^2} = \sum_{n=1}^{\infty} \cos \frac{(2n-1)\pi z}{2h} \cdot \frac{2}{h} \int_0^h \frac{d^2 u_0}{d\xi^2} \cos \frac{(2n-1)\pi \xi}{2h} d\xi; \quad (15)$$

then we have by Stokes' method

$$\begin{aligned} \frac{2}{\mu} \int_0^h \frac{d^2 u_0}{d\xi^2} \cos \frac{(2n-1)\pi \xi}{2h} d\xi \\ = \frac{2}{h} \cdot \frac{\tau_x}{\mu} - \frac{(2n-1)^2 \pi^2}{4h^2} u_{2n-1}, \end{aligned} \quad (16)$$

where τ_x is the eastward component of wind stress defined by the surface condition (3), or $z=0$: $-\mu \frac{du_0}{dz} = \tau_x$. Thus (15) becomes

$$\begin{aligned} \frac{d^2 u_0}{dz^2} = \sum_{n=1}^{\infty} \left\{ \frac{2}{h} \frac{\tau_x}{\mu} - \frac{(2n-1)^2 \pi^2}{4h^2} u_{2n-1} \right\} \\ \cdot \cos \frac{(2n-1)\pi z}{2h}. \end{aligned} \quad (17)$$

Integrating both sides of the expression (17) from the surface $z=0$ down to a level z , we have, making use of the condition (3),

$$\begin{aligned} \frac{du_0}{dz} = \frac{\tau_x}{\mu} + \sum_{n=1}^{\infty} \left\{ \frac{2}{h} \frac{\tau_x}{\mu} - \frac{(2n-1)^2 \pi^2}{4h^2} u_{2n-1} \right\} \\ \cdot \frac{2h}{(2n-1)\pi} \sin \frac{(2n-1)\pi z}{2h} \Big|_0^z \\ = -\frac{\tau_x}{\mu} \left\{ 1 - \frac{2}{h} \sum_{n=1}^{\infty} \frac{2h}{(2n-1)\pi} \sin \frac{(2n-1)\pi z}{2h} \right. \\ \left. - \sum_{n=1}^{\infty} \left\{ \frac{(2n-1)\pi}{2h} u_{2n-1} \right\} \sin \frac{(2n-1)\pi z}{2h} \right\}, \end{aligned} \quad (18)$$

because

$$\begin{aligned} \int_0^z \frac{d^2 u_0}{dz^2} dz = \frac{du_0}{dz} - \frac{du_0}{dz} \Big|_{z=0} \\ = \frac{du_0}{dz} + \frac{\tau_x}{\mu}. \end{aligned}$$

But we have

$$\frac{2}{h} \sum_{n=1}^{\infty} \frac{2h}{(2n-1)\pi} \sin \frac{(2n-1)\pi z}{2h} = 1, \quad (19)$$

and the above expression becomes

$$\frac{du_0}{dz} = \sum_{n=1}^{\infty} \left\{ -\frac{(2n-1)\pi}{2h} u_{2n-1} \right\} \sin \frac{(2n-1)\pi z}{2h}. \quad (19)$$

Further, it follows from (18)

$$2\omega = 2\omega \cdot \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin \frac{(2n-1)\pi z}{2h},$$

so that we have

$$\begin{aligned} -w_0(z) \frac{du_0}{dz} + 2\omega w_0(z) \\ = w_0(z) \sum_{n=1}^{\infty} \left\{ \frac{(2n-1)\pi}{2h} u_{2n-1} + \frac{4}{(2n-1)\pi} \cdot 2\omega \right\} \sin \frac{(2n-1)\pi z}{2h}. \end{aligned} \quad (20)$$

Substituting from (14), this expression becomes

$$\begin{aligned} \frac{\pi}{4n} \sum_{n=1}^{\infty} \left[\sum_{s=1}^{\infty} \left\{ (2s-2n+1) u_{2s-2n+1} \right. \right. \\ \left. \left. + \frac{8}{(s-2n+1)\pi^2} \cdot 2\omega h - (2s+2n-1) u_{2s+2n-1} \right. \right. \\ \left. \left. - \frac{8}{(2s+2n-1)\pi^2} \cdot 2\omega h \right\} \cdot w_{2s} \right] \cos \frac{(2n-1)\pi z}{2h} \end{aligned} \quad (21)$$

Again substituting (13), (17) and (20) in the equation (8), the following expressions are obtained between $w_2, w_4, \dots$ and $u_1, u_3, u_5, \dots$ as follows:

$$\begin{aligned} \sum_{s=1}^{\infty} \left\{ 2s-2n+1 u_{2s-2n+1} + \frac{8}{(2s-2n+1)\pi^2} \cdot 2\omega h \right. \\ \left. + (2s+2n-1) u_{2s+2n-1} - \frac{8}{(2s+2n-1)\pi^2} \cdot 2\omega h \right\} w_{2s} + \frac{8}{\pi} \frac{\tau_x}{\rho} \\ - \frac{4h}{\pi} \left(\frac{\partial D_0}{\partial x} \right)_{2n-1} - \frac{(2n-1)\pi}{h} \frac{\mu}{\rho} u_{2n-1} = 0, \\ (n, s=1, 2, 3, \dots) \end{aligned} \quad (22)$$

provided $u_{-j} = u_j$ where j is an odd positive integer. These are necessary conditions for the equation (8) to be satisfied, though not sufficient condition.

The solution of the simultaneous equations (22) will give $w_2, w_4, w_6, \dots$ in terms of $\mu, \rho; \tau_x, \omega$ and $u_1, u_3, u_5, \dots; \left(\frac{\partial D_0}{\partial x} \right)_1, \left(\frac{\partial D_0}{\partial x} \right)_3, \left(\frac{\partial D_0}{\partial x} \right)_5, \dots$ determined by harmonic analysis according to the expressions (12) and (13). The vertical velocity component $w_0(z)$ will be then given by (14), or

$$w_0(z) = \sum_{n=1}^{\infty} w_{2n} \sin \frac{2n\pi z}{2h} \quad (14)$$

4. Numerical computation

The zonal velocity of the Equatorial Undercurrent was measured by several scientists since Townsend CROMWELL, R. B. MONTGOMERY and E. D. STROUP first discovered this current in the Pacific in September 1951 (CROMWELL, MONTGOMERY and STROUP, 1954). In 1959, John A. KNAUSS made a composite profile of this current (140° E) by combining various observations by himself (KNAUSS, 1960). The values of $u_0(z)$ was read off from Knauss's diagram (KNAUSS, 1960, p. 274) every 50 m from the surface down to 1,000 m level. The result is given in the Table I below. At the same time, the average values of $\frac{\partial D_0}{\partial x}$ for the same levels over 1,000 db surface were computed from the observational data occupied by Hawaiian oceanographers in the equatorial waters during the "Hugh M. Smith" cruises of 1950, 1951 and 1952 and other data occupied during the IGY and IGC periods. (average of those from 140° E to 160° W) (Mid-Pacific Oceanography I, II, III, and IV, U. S. Department of the Interior Fish and Wildlife Service, Special Scientific Report-Fisheries No. 54, 131 and 135; IGY Oceanographic Report No. 3, parts IIa.) The values of τ_x were averaged from the Hawaiian observations which are available at a spacing of 1° of meridional arc.

It should be stressed that the correspondence in the area and time observations between the numerical values of $u_0(z)$, $\frac{\partial D_0}{\partial x}$ and τ_x is important in this sort of computation in order to explain the actual phenomena. However, the author has to confess that it was not easy to select the data all occupied simultaneously and at the same longitude. It should be merely taken as an example of numerical computation for the availability of the method described. Therefore it should be borne in mind that the result would not necessary be always consistent.

The author hopes that someday he would be possible to use the data occupied at the same time and at the same longitude, so that he would be able to explain actual phenomena more precisely.

The harmonic analysis of $u_0(z)$ and $\frac{\partial D_0}{\partial x}$

Table 1. Vertical distribution of u_0 (observed) and $\frac{\partial D_0}{\partial x}$ (computed over 1,000 lb reference level from observations)

depth z (meters or decibars)	u_0 (after Knauss) (cm/sec)	$\frac{\partial D_0}{\partial x}$ (cm/sec ²)
0	-30	-5.22×10^{-5}
50	130	-4.39 "
100	150	-2.99 "
150	80	-1.29 "
200	48	-0.09 "
250	30	-0.36 "
300	18	-0.26 "
350	8	-0.21 "
400	-3	-0.16 "
450	-9	-0.14 "
500	-11	-0.11 "
550	-12	-0.11 "
600	-13	-0.10 "
650	-13	-0.10 "
700	-14	-0.10 "
750	-12	-0.09 "
800	-10	-0.08 "
850	-9	-0.06 "
900	-8	-0.04 "
950	-5	-0.02 "
1,000	0	0

according to the series:

$$u_0(z) = \sum_{n=1}^{20} u_{2n-1} \cdot \cos \frac{(2n-1)\pi z}{2h}$$

and

$$\frac{\partial D_0}{\partial x} = \sum_{n=1}^{20} \left(\frac{\partial D_0}{\partial x} \right)_{2n-1} \cdot \cos \frac{(2n-1)\pi z}{2h},$$

gives the values for $u_1, u_3, u_5, \dots$ and $\left(\frac{\partial D_0}{\partial x} \right)_1, \left(\frac{\partial D_0}{\partial x} \right)_3, \dots$ as compiled in Table 2.

With these coefficients, two series give exactly the same values for $u_0(z)$ and $\frac{\partial D_0}{\partial x}$ for the 20 levels compiled in Table 1.

The 20 equations for computing $w_2, w_4, \dots, w_{40}$ are as follows:

$$\begin{aligned} & (\bar{u}_1 + 3\bar{u}_3)w_2 + (3\bar{u}_3 + 5\bar{u}_5)w_4 + (5\bar{u}_5 + 7\bar{u}_7)w_6 + \dots \\ & + (39\bar{u}_{37} + 0)w_{40} \\ & = -\frac{8}{\pi} \frac{\tau_x}{\rho} + \frac{4h}{\pi} \left(\frac{\partial D_0}{\partial x} \right)_1 + \left(\frac{9\pi}{h} u_1 \right) \frac{\mu}{\rho}, \\ & (-1\bar{u}_1 + 5\bar{u}_5)w_2 + (\bar{u}_1 + 7\bar{u}_7)w_4 + (3\bar{u}_3 + 9\bar{u}_9)w_6 + \dots \\ & + (37\bar{u}_{37} + 0)w_{40} \end{aligned}$$

Table 2. 20 Coefficients in the Expansions in Terms of $\cos \frac{(2n-1)\pi z}{2h}$ for u_0 and $\frac{\partial D_0}{\partial x}$.
($n=1, 2, 3, \dots, 20$)

$2n-1$	u_{2n-1} (cm/sec)	$\left(\frac{\partial D_0}{\partial x}\right)_{2n-1}$ (cm/sec ²)
1	37.97145	-1.32162×10^{-5}
3	44.03876	-1.02983 "
5	18.20369	-0.87126 "
7	6.56432	-0.65931 "
9	-1.88448	-0.48882 "
11	-5.24597	-0.34498 "
13	-8.70113	-0.22131 "
15	-10.61137	-0.13134 "
17	-13.40103	-0.06846 "
19	-13.29632	-0.02842 "
21	-12.96634	-0.00454 "
23	-12.02045	0.00918 "
25	-10.90045	0.01566 "
27	-10.10755	0.01593 "
29	-8.87627	0.01074 "
31	-7.40474	0.00346 "
33	-6.38032	-0.00947 "
35	-5.49087	-0.02506 "
37	-4.77900	-0.03390 "
39	-4.71103	-0.03710 "

$$\begin{aligned}
 &= -\frac{8}{\pi} \frac{\tau_x}{\rho} + \frac{4h}{\pi} \left(\frac{\partial D_0}{\partial x} \right)_3 + \left(\frac{25\pi}{h} u_3 \right) \frac{\mu}{\rho}, \\
 &(-3\bar{u}_3 + 7\bar{u}_7)w_2 + (-\bar{u}_1 + 9\bar{u}_9)w_4 \\
 &\quad + (\bar{u}_1 + 11\bar{u}_{11})w_6 + \dots + (35\bar{u}_{35} + 0)w_{49} \\
 &= -\frac{8}{\pi} \frac{\tau_x}{\rho} + \frac{4h}{\pi} \left(\frac{\partial D_0}{\partial x} \right)_5 + \left(\frac{49\pi}{h} u_5 \right) \frac{\mu}{\rho}, \\
 &(-5\bar{u}_5 + 9\bar{u}_9)w_2 + (-3\bar{u}_3 + 11\bar{u}_{11})w_4 \\
 &\quad + (-\bar{u}_1 + 13\bar{u}_{13})w_6 + \dots + (33\bar{u}_{33} + 0)w_{40} \\
 &= -\frac{8}{\pi} \frac{\tau_x}{\rho} + \frac{4h}{\pi} \left(\frac{\partial D_0}{\partial x} \right)_7 + \left(\frac{81\pi}{h} u_7 \right) \frac{\mu}{\rho}, \\
 &\dots\dots\dots (23)
 \end{aligned}$$

$$\begin{aligned}
 &(-35\bar{u}_{35} + 39\bar{u}_{39})w_2 + (-33\bar{u}_{33} + 0)w_4 \\
 &\quad + (-31\bar{u}_{31} + 0)w_6 + \dots + (13\bar{u}_{13} + 0)w_{40} \\
 &= -\frac{8}{\pi} \frac{\tau_x}{\rho} + \frac{4h}{\pi} \left(\frac{\partial D_0}{\partial x} \right)_{37} + \left(\frac{1369\pi}{h} u_{37} \right) \frac{\mu}{\rho}, \\
 &(-37\bar{u}_{37} + 0)w_2 + (-35\bar{u}_{35} + 0)w_4 \\
 &\quad + (-33\bar{u}_{33} + 0)w_6 + \dots + (u_1 + 0)w_{40} \\
 &= -\frac{8}{\pi} \frac{\tau_x}{\rho} + \frac{4h}{\pi} \left(\frac{\partial D_0}{\partial x} \right)_{39} + \left(\frac{1521\pi}{h} u_{39} \right) \frac{\mu}{\rho},
 \end{aligned}$$

where

$$(2n-1)\bar{u}_{2n-1} = (2n-1)u_{2n-1} + \frac{8}{(3n-1)\pi^2} \cdot 2\omega h \quad (24)$$

For $w_0(z)$ the series were also confined to 20 terms, or

$$w_0(z) = \sum_{n=1}^{20} w_{2n} \sin \frac{2n\pi z}{2h} \quad (25)$$

from which only $w_2, w_4, w_6, \dots, w_{40}$ are to be determined. The constants used are

$$\begin{aligned}
 \frac{\tau_x}{\rho} &= -1.381 \text{ c.g.s} \\
 h &= 1,000 \text{ m} = 10^5 \text{ cm}
 \end{aligned} \quad (26)$$

where τ_x was computed from the same sources as $\frac{\partial D_0}{\partial x}$.

The computation gives the values of w_{2n} always in the form:

$$w_{2n} = w_{2n}^{(I)} + \frac{\mu}{\rho} w_{2n}^{(II)} \quad (27)$$

where $w_{2n}^{(I)}$ and $w_{2n}^{(II)}$ have all opposite signs and are given as compiled in Table 3.

Table 3. Values of $w_{2n}^{(I)}$ and $w_{2n}^{(II)}$ where

$$w_{2n} = w_{2n}^{(I)} + \frac{\mu}{\rho} w_{2n}^{(II)}.$$

$2n$	$w_{2n}^{(I)}$	$w_{2n}^{(II)}$
2	-0.0182521	0.001008230
4	0.0040192	-0.000212213
6	0.0106797	-0.000770698
8	0.0066470	-0.000490487
10	-0.0055932	0.000362601
12	-0.0009663	0.000156503
14	0.0058515	-0.000385958
16	0.0109251	-0.000633034
18	0.0009974	-0.000055837
20	-0.0049822	0.000313080
22	0.0012172	-0.000052499
24	0.0063975	-0.000482817
26	0.0037582	-0.000227687
28	-0.0064666	0.000393289
30	-0.0068338	0.000400349
32	0.0015132	-0.000243892
34	0.0075428	-0.000568787
36	-0.0041715	0.000176598
38	-0.0080682	0.000589728
40	-0.0240596	0.001329814

Substituting the numerical values for w_{2n} thus computed in (13), we have the vertical component as follows:

$$\begin{aligned}
 w_0(z) &= \sum_{n=1}^{20} w_{2n}^{(I)} \sin \frac{2n\pi z}{2h} \\
 &\quad - \frac{\mu}{\rho} \sum_{n=1}^{20} w_{2n}^{(II)} \sin \frac{2n\pi z}{2h}
 \end{aligned}$$

Table 4. Values of $w_0^{(I)}(z)$ and $w_0^{(II)}(z)$ where
 $w_0(z) = w_0^{(I)}(z) + \frac{\mu}{\rho} w_0^{(II)}(z)$.

depth z (in meters)	$w_0^{(I)}(z)$ (cm/sec)	$w_0^{(II)}(z)$ (cm/sec)
0	0	0
50	0.0163465	-0.001268386
100	0.0198463	-0.000977431
150	-0.0060240	-0.000080141
200	-0.0015959	-0.000026192
250	0.0001305	-0.000271689
300	0.0001267	-0.000106977
350	-0.0040455	0.000407458
400	-0.0075882	0.000295999
450	-0.0873501	0.005812691
500	-0.0143933	0.000737528
550	-0.0227223	0.001333157
600	-0.0125914	0.000557827
650	-0.0140123	0.000666346
700	-0.0115637	0.000350065
750	-0.0044663	0.000272965
800	-0.0082027	0.000439174
850	-0.0063556	0.000348959
900	-0.0153325	0.000575689
950	-0.0041115	0.000617574
1,000	0	0

or

$$w_0(z) = w_0^{(I)}(z) + \frac{\mu}{\rho} w_0^{(II)}(z) \quad (28)$$

The numerical values of $w_0^{(I)}(z)$ and $w_0^{(II)}(z)$ are compiled in Table 4.

Thus it is seen that both upwelling and sinking, or the vertical flows at the Equator consist of two parts, that is, the term $w_0^{(I)}(z)$ due to mass distribution and wind stress on the sea surfaces and the second part proportional to $\frac{\mu}{\rho}$, or depending on the magnitude of eddy viscosity. This fact means that the intensity of upwelling and sinking at the equator primarily depends upon the magnitude of $\frac{\mu}{\rho}$, since there are, in equatorial regions, no large annual variation in mass distribution and wind stress at the surface of the sea.

The vertical flow $w_0(z)$ was computed for $\frac{\mu}{\rho} = 5, 10, 15.02748, 18.09243, 50, 100, 200$ and 500 c.g.s. The value $\frac{\mu}{\rho} = 18.09243$ was chosen in place of 20, because this value makes $w_{40} = 0$.

Table 5. Intensity of Vertical Flow, $w_0(z)$ in 10^{-3} cm/sec, or Upwelling (negative sign) and Sinking (positive sign) at the Equator for Several Different Values of $\frac{\mu}{\rho}$.

depth z (in meters)	$\frac{\mu}{\rho}$ =0	$\frac{\mu}{\rho}$ =5	$\frac{\mu}{\rho}$ =10	$\frac{\mu}{\rho}$ =15.02749	$\frac{\mu}{\rho}$ =18.09	$\frac{\mu}{\rho}$ =50	$\frac{\mu}{\rho}$ =100	$\frac{\mu}{\rho}$ =200	$\frac{\mu}{\rho}$ =500
0	0	0	0	0	0	0	0	0	0
50	1.6	10.0	3.7	-2.7	-5.7	-47.1	-110.5	-237.3	-617.8
100	2.0	-15.0	10.1	5.2	-2.2	-29.0	-77.9	-175.6	-468.9
150	-6.0	-6.4	-6.8	-7.2	-7.5	-10.0	-14.0	-22.1	-34.0
200	-1.6	-1.7	-1.9	-2.0	-2.1	-2.9	-4.2	-6.8	-11.5
250	0.1	-1.2	-2.6	-4.0	-4.8	-13.5	-23.0	-54.2	-135.7
300	0.1	0.4	-0.9	-1.5	-1.8	-5.2	-10.6	-21.3	-53.4
350	-4.0	-2.0	0	2.1	3.3	16.3	36.7	77.4	199.7
400	-7.6	-6.1	-4.6	-3.1	-2.2	7.2	22.0	51.6	140.4
450	-8.7	-58.3	-29.2	0	17.8	203.3	493.1	1075.2	2819.0
500	-14.4	-10.7	-7.0	-3.3	-1.1	22.5	59.4	133.1	354.4
550	-22.7	-16.1	-9.3	-2.7	1.4	43.9	110.6	243.9	643.9
600	-12.6	-9.8	-7.0	-4.2	-2.5	15.3	43.2	99.0	266.3
650	-14.0	-10.7	-7.3	-4.0	-2.0	19.3	52.6	119.3	319.2
700	-11.6	-9.8	-8.1	-6.3	-5.2	5.9	23.4	58.4	163.5
750	-4.5	-3.1	-1.7	-0.4	0.5	9.2	22.8	50.1	132.0
800	-8.2	-6.0	-3.8	-1.6	-0.2	13.8	35.7	79.6	211.4
850	-6.4	-4.6	-2.6	-1.1	0	11.1	28.5	63.4	168.1
900	-5.3	-12.5	-9.6	-6.7	-4.9	13.5	42.2	99.8	272.5
950	-4.1	-1.0	-2.1	5.2	-0.8	26.8	57.6	119.4	304.7
1,000	0	0	0	0	0	0	0	0	0

The result is compiled in Table 5. The discrepancies in the order of magnitudes between those values and those of Table 2 of previous paper (HIDAKA, 1965) will be due mostly to the uncertainty of numerical differentiations.

5. Discussion on the result

Table 5 gives the result of final computation, or the vertical flow in cm/sec, giving the intensity of upwelling, (negative sign) and sinking (positive sign) for several different values of $\frac{\mu}{\rho}$. When $\frac{\mu}{\rho}$ is small ($0 < \frac{\mu}{\rho} < 20$), there is upwelling in most depths except at shallow layers just below the surface. The magnitude of upwelling is between 10^{-3} and 10^{-2} cm/sec. For $\frac{\mu}{\rho} = 18.09242$ which makes $w_{40} = 0$, the equatorial water upwells in most layers except there is a rather intense sinking around 450 m level (0.0178 cm/sec). (Fig. 2) To remove this intermediate sinking, we can assume $\frac{\mu}{\rho} = 15.02748$ for which $w_0(z)$ vanishes at 450 m level. (Table 5 and Fig. 1)

For larger values of $\frac{\mu}{\rho}$, upwelling is confined in the layers shallower than 300 m, whereas sinking predominates in deeper layers down to the reference surface ($h=1,000$ m). The intensity of both upwelling and sinking tends to asymptotic vertical distribution, expressed by the formula:

$$w_0(z) = \frac{\mu}{\rho} w_0^{(II)}(z),$$

when $\frac{\mu}{\rho}$ is large. For $\frac{\mu}{\rho} = 50$ c.g.s., the upwelling has a maximum velocity $w_0(z) = -0.0471$ cm/sec. at 50 m, and sinking a maximum $w_0(z) = 0.2033$ cm/sec. at 450 m. (Fig. 3) For $\frac{\mu}{\rho} = 100, 200$ and 500 c.g.s., maximum values of upwelling exist at 50 m layer, having values of 0.1105, 0.273 and 0.6178 cm/sec. upwards, while the maximum values of sinking are 0.4931, 1.0752 and 2.8190 cm/sec. respectively at 450 m level.

There will be questions on the real existence of these large values of sinking at 450 meter level, because they are quite unreasonable magnitudes for a vertical motion of water even

if we permit the particular circumstances which may prevail at the equator. This result may suggest that there cannot be large eddy viscosity in equatorial regions.

In a previous paper by the author (HIDAKA,

Fig. 1. Computed upwelling (shaded area) and sinking for $\frac{\mu}{\rho} = 15.027$.

Fig. 2. Computed upwelling (shaded area) and sinking for $\frac{\mu}{\rho} = 18.09$.

Fig. 3. Computed upwelling (shaded area) and sinking for $\frac{\mu}{\rho} = 50$.

1965), an anticipation of an intense upwelling at the equator is suggested. The same applies in the present consideration. It should be remembered, however, that the deeper part of Knauss's composite vertical profile of the Equatorial Undercurrent is less trustable, because of the slow horizontal flows and the difficulty in directly measuring them. Therefore, no guarantee will be given for a real existence of such a big vertical flows.

Professor Kozo YOSHIDA, University of Tokyo, does not seem to be quite convinced with my numerical results. To make a rough check of the results, he suggests possible computations of vertical velocities from the two equations not used in the present paper: One from the equation of heat conservation and the other from the equation of continuity. Yoshida's preliminary calculation appears to show certain similarity between my dynamical results and his from the thermal equation.

5. Summary

The intensity of vertical flow at the equator was computed from the hydrodynamical equations by using actual observations of zonal component of the Equatorial Undercurrent and

as many terms in the hydrodynamical equations and observed elements as possible. It was concluded that the equatorial vertical flow depends largely on $\frac{\mu}{\rho}$, to a larger value of which ($\frac{\mu}{\rho} > 50$ c.g.s.) there predominates upwelling in the layers shallower than 300 meters and sinking predominates in deeper layers. The magnitudes of these vertical flows are of the order of 10^{-3} to 10^{-2} cm/sec., or larger, thus suggesting intense upwelling and sinking at the Equator.

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赤道湧昇流の非線型計算

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要旨 赤道に於ける湧昇流や沈降流を流体力学の運動方程式に於ける非線型項を省略しないで計算する方法を考えてみた。東方に x を、北方に y を、下方に z をそれぞれ正に採り、問題を簡単にするために、海面が西から東に下降するための東西圧力傾度は勿論省略出来ないが、速度成分の東西傾度は省略出来ると仮定して方程式を簡略にし、連続の条件及び $\varphi=0$ という条件に依ってだけの非線型二次微分方程式を導びき、これに各層の圧力傾度の他に赤道潜流の実際の観測値や風の応力の平均

値等を代入して正弦函数の級数として鉛直流を計算し得た。その結果は $w(z)=w^I(z)\frac{\mu}{\rho}w^{II}(z)$ の如き $\frac{\mu}{\rho}$ の一次式で表されるが、勿論 $\frac{\mu}{\rho}$ の如何なる値に対しても實際を表わすとは考えられない。尚圧力傾度等（圧面の傾度）、風の応力、潜流の東西成分等が同時に同程度が測られたわけではないから結果は単に計算の可能性を示すのみで、これに依って實際現象を説明する段階に未だ達していない。