On the Redistribution of Heat in the North Pacific Ocean*

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Abstract: An analysis is made of the processes maintaining the temperature distribution in the surface layer of the North Pacific Ocean in the presence of time- and space-variable heat input. Considered are horizontal and vertical advection, mixing processes and heat storage. The excess heat input into the North Pacific Ocean is balanced by a vertical circulation of about 8 million m³/sec. Horizontal circulation transports heat from the areas of heat gain to the areas of heat loss. This circulation is calculated from the distribution of heat input and surface temperature and is shown to require a transport of only 10 million m³/sec in the sub-tropical anticyclonic gyre. Consequently, all horizontal heat transports take place in a shallow surface layer only. Comparison between required and observed heat input allows conclusions about the location and intensity of upwelling areas and the contribution of horizontal eddy diffusion. Seasonal variations of surface temperature and heat input are compared, and demonstrate that a mixed layer of 30 to 70 m depth is affected. A complete local storage of the entire seasonal variation of heat input is possible when a variable mixed layer depth is allowed, but the late-winter maximum of mixed layer depth would occur too late. Horizontal advection explains part of the phase difference between heat input and surface temperature. It can be estimated that the contributions of heat advection and of variations in mixed layer depth are of the same order of magnitude with regard to their effect on surface temperature.

1. Introduction

The analysis of the distribution of heat in the surface layer of the ocean is a very complex problem because several processes having effects of approximately the same order of magnitude are involved. These processes are the energy exchange at the sea surface, the heat storage in the surface layer, horizontal advection, vertical advection by upwelling or sinking, and mixing processes. An analysis of the interaction of these various processes is, however, necessary to understand how the distribution of temperature in the surface layer is maintained. Moreover, any analysis and prediction of anomalies in the surface temperature field which is not based entirely on statistical and empirical relationships will require an understanding of the interaction of these various processes. Using the equations for heat advection and diffusion, and the data on heat exchange and surface temperature in the

North Pacific Ocean compiled by WYRTKI (1966a), we will attempt to give an analysis of the processes that maintain the temperature distribution in the presence of heat sources and sinks. The contribution of each individual component and the interaction between any two of them will be separately treated to analyze and understand their importance in this complex system.

2. Equations of Heat Advection and Diffusion

In order to discuss the processes involved in the redistribution of heat in the ocean, it seems to be advisable to write the governing equations in the most general form, using integral relationships. The first equation is that of conservation of heat in a given volume (Fig. 1)

$$\int_0^D \left(T \vec{v} \cdot \vec{n} - A_h \frac{\partial T}{\partial n} \right) dz dl + \int_{\text{Bottom of cylinder}} \left(w T - A \frac{\partial T}{\partial z} \right) dx dy + \int_{\text{Surface}} Q dx dy$$

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Fig. 1. Notations used in equations (1) and (2).

$$= \frac{\partial}{\partial t} \iiint_{\text{Volume}} T dz dx dy \quad (1)$$

where $\vec{v}$ is the horizontal velocity vector, T the temperature, $\vec{n}$ a unit vector perpendicular to the vertical walls of the enclosed volume, w the vertical velocity at the bottom of the cylinder, A and A^h the vertical and horizontal eddy diffusion coefficients, and D the depth of the cylinder as illustrated in Fig. 1. It should be noted that the depth z is positive downwards, and ascending velocities w are negative. The heat exchange q at the sea surface, which is ordinarily expressed in $\text{cal cm}^{-2} \text{ day}^{-1}$, as in our Figures, must be transformed into units of $Q = q/\rho c_p$, where ρ is the density of the sea water and c_p its specific heat, in order to avoid employing constants through all equations. The first integral describes the horizontal flow of heat through the mantle of the cylinder by advection and horizontal mixing; the second term, vertical advection and mixing; the third term, the heat input or loss; and the right-hand side of the equation, the time change of heat content of the volume. The only important restriction that is made is that the temperature T is considered constant within the depth D , but both T and D may be functions of x , y , and t . The second equation is that of continuity, which in analogous form can be written as

$$\int_0^D \vec{v} \cdot \vec{n} dz dl - \int \int w dx dy = 0 \quad (2)$$

These two equations govern the distribution of heat or temperature by advection and mixing in the presence of heat sources and sinks. The use of only these two equations indicates that

the problem will be treated in a kinematic way, and that arguments on the dynamical causes of the circulation involved will not be discussed nor will the feedback from the ocean to the atmosphere. Dividing the two equations (1) and (2) by the surface area of the cylinder and going to the limit for infinitesimal small surface area, the equations can be written in the well-known form of differential equations

$$\frac{\partial}{\partial t} TD + \frac{\partial}{\partial x} V^x T + \frac{\partial}{\partial y} V^y T - A^h D \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right\} + \left\{ wT - A \frac{\partial T}{\partial z} \right\}_D = Q \quad (3)$$

$$\frac{\partial V^x}{\partial x} + \frac{\partial V^y}{\partial y} + w = 0 \quad (4)$$

if volume transports within the surface layer defined by

$$V^x = \int_0^D u dz \quad V^y = \int_0^D v dz \quad (5)$$

are introduced.

First, the stationary case will be discussed, which describes the maintenance of the average annual temperature distribution in the surface layer by advection and mixing in the presence of the average annual heat sources and sinks. Second, the response of the temperature in the surface layer to a variable heat input will be analyzed with regard to advection and a variable depth of the surface layer. For this purpose the average annual variation of surface temperature will be used, because it represents the most regular variation of any oceanic property and has the largest amplitude. After this time-dependent process is understood, an attempt will then be made in another study to apply the knowledge obtained to describe the formation, migration, persistence and decay of anomalies in the temperature field.

3. Excess heat gain of the North Pacific Ocean

The distribution of the average annual heat exchange at the surface of the North Pacific Ocean is shown in Fig. 2 according to WYRTKI (1965a); he calculated that the total heat input is positive. It seems logical to divide the North

Fig. 2. Average annual heat exchange at the surface of the North Pacific Ocean in $\text{cal cm}^{-2} \text{ day}^{-1}$.

Pacific Ocean from the South Pacific Ocean at the latitude of the Equatorial Countercurrent (6°N), which separates the two large subtropical anticyclonic gyres. North of 6°N the total heat gain of the North Pacific Ocean is $2 \times 10^{14} \text{ cal sec}^{-1}$. A stationary thermal structure requires that, in an area where a thermocline is developed, the turbulent downward flow of heat is balanced by ascending movements of cooler water. WYRTKI (1961) has shown that for a simple thermohaline circulation the divergence of the heat flux must vanish within the thermocline, which results in the equation

$$wT - A \frac{\partial T}{\partial z} = \text{const} = wT_B \quad (6)$$

The vertical heat flux itself must be constant, and the value of the constant can be fixed by the temperature T_B at great depths, where the temperature gradient vanishes.

Such a simple model of a thermohaline circulation can then be applied to the North Pacific Ocean, using equations (1) and (2). Since for our purposes the ocean can be considered closed at the north, the transport of water can occur only across the southern boundary, which we have set at 6°N . The equation of continuity (2) transforms then into the statement

$$-\iint_{\text{Southern boundary}} \vec{v} \cdot \vec{n} dz dl = -\iint_{\text{Entire ocean}} w dx dy = C \quad (7)$$

where C is the strength ($\text{cm}^3 \text{ sec}^{-1}$) of the circulation. Using the relationship (6) and assuming that no turbulent heat transport occurs across the heat equator because the north-south temperature gradient vanishes there, equation (1) reduces under stationary conditions to the simple statement

$$-\iint_{\text{Southern boundary}} T(\vec{v} \cdot \vec{n}) dz dl - \iint w T_B dx dy + \iint Q dx dy = 0 \quad (8)$$

With the assumption that the surface flow across the southern boundary is everywhere of the same high temperature T_0 , the combination of (7) and (8) gives

$$C(T_0 - T_B) = \iint Q dx dy \quad (9)$$

The excess heat input into the North Pacific Ocean is then compensated by an inflow of cold Deep and Bottom water into the North Pacific Basin, by ascending movements, and by an outflow of warm surface water. The resulting circulation has a strength of about $8 \times 10^{12} \text{ cm}^3 \text{ sec}^{-1}$ as shown by WYRTKI (1965a) and is

driven by a heat input of 2×10^{14} cal sec⁻¹. The outflow of warm water seems to occur through the Southeast Asian Waters and in the vicinity of the Galapagos Islands. In the deep ocean below the main thermocline the ascending movements connected with this circulation are probably quite evenly distributed over the entire ocean basin. In higher levels, however, they appear to be concentrated in the eastern tropical Pacific Ocean (WYRTKI, 1966b), in the Gulf of Alaska (TULLY and BARBER, 1960), and in the upwelling area off California. However, no conclusions can be drawn from this simple model about the actual location of the areas of ascending movements near the surface. Only the analysis of the horizontal distribution of heat sources and sinks may offer the possibility to verify their location from a consideration of the heat exchange. The described thermohaline circulation will absorb the net heat input at the sea surface, but will not contribute to the equalization of the horizontal differences in the heat input. To effect such an equalization, horizontal circulation is necessary.

4. The redistribution of heat by horizontal advection

Another important fact to be learned from Fig. 2 concerns the location of the major heat sources and sinks in the annual average heat exchange. Within the North Pacific subtropical anticyclonic gyre the ocean gains heat along the coast of California (about 0.2×10^{14} cal sec⁻¹), and in the central and western part of the trade-wind region (about 1.9×10^{14} cal sec⁻¹). Consequently heat has to be transported by horizontal advection from the eastern and southern parts of the subtropical anticyclone to its northwestern part, and cool water into the area off California. The differential between the areas of heat gain and heat loss is about 3×10^{14} cal sec⁻¹ and is only slightly larger than the heat driving the thermohaline circulation discussed above. Since the temperature differential between the southern and the northern parts of the subtropical anticyclone is about 14°C which is only half of that of the thermohaline circulation, the necessary horizontal circulation will have to be of approximately the same strength as the vertical circulation discussed

above. A corresponding schematic model was discussed by WYRTKI (1965a) and required a horizontal circulation of about 10×10^{12} cm³ sec⁻¹.

The horizontal circulation in the surface layer will be largely responsible for the transfer of heat between areas of heat gain and heat loss, and for the maintenance of the average annual temperature distribution. To investigate this process we will use the heat budget equation in the form of (3) for a stationary temperature distribution, and the relationship (6) for the vertical heat flow. With these assumptions (3) transforms to

$$\frac{\partial}{\partial x} V^x T + \frac{\partial}{\partial y} V^y T - A^h D \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right\} + w T_B = Q \quad (10)$$

The horizontal circulation in the surface layer represented by the transport vector V according to (5) can be divided into two parts, $V = V_1 + V_2$, with one part being free of divergence, $\text{div } V_1 = 0$, and the other part containing vertical movements, $\text{div } V_2 = -w$. With these notations (10) can be written in the form

$$\frac{\partial}{\partial x} V_1^x T + \frac{\partial}{\partial y} V_1^y T = Q + A^h D \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right\} + w(T - T_B) - \left\{ V_2^x \frac{\partial T}{\partial x} + V_2^y \frac{\partial T}{\partial y} \right\} \quad (11)$$

Because $\text{div } V_1 = 0$, this part of the horizontal circulation can be represented by a stream function according to

$$V_1^x = -\frac{\partial \phi}{\partial y} \quad V_1^y = \frac{\partial \phi}{\partial x} \quad (12)$$

and (11) can be written in the form

$$\frac{\partial \phi}{\partial x} \frac{\partial T}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial x} = Q - Q_0 \quad (13)$$

with

$$Q_0 = V_2^x \frac{\partial T}{\partial x} + V_2^y \frac{\partial T}{\partial y} - A^h D \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right\} - w(T - T_B)$$

The left-hand side of this equation contains horizontal advection by a divergence-free horizontal circulation only. The right-hand contains the heat sources and sinks and the effects resulting from vertical movements, from the horizontal flow connected with them and from horizontal mixing. Equation (13) can be used to calculate the stream function ϕ if the distribution of temperature T and of the heat sources

$Q - Q_0$ is known, under the condition that for a closed circulation system

$$\iint (Q - Q_0) dx dy = 0 \quad (14)$$

This shows quite clearly that any heat surplus in an ocean basin must be removed by processes included in the second term on the right-hand side of equation (13). This is basically a vertical circulation described previously, because the integrated effects of horizontal mixing vanish.

A solution to (13) for ϕ can be obtained by integrating any of the three ordinary differential equations

$$\frac{dx}{\frac{\partial T}{\partial y}} = \frac{dy}{-\frac{\partial T}{\partial x}} = \frac{d\phi}{Q - Q_0} \quad (15)$$

Since the term $\frac{\partial T}{\partial x}$ is usually small, the calculation will have to be performed along parallels

$$\phi(x) = \int_0^x \frac{Q - Q_0}{\frac{\partial T}{\partial y}} dx \quad \text{with} \quad \int_0^L \frac{Q - Q_0}{\frac{\partial T}{\partial y}} dx = 0 \quad (16)$$

which is equivalent to $\phi(0) = \phi(L) = 0$. Such a computation has been performed for the North Pacific Ocean and the result is shown in Fig. 3. Values of Q_0 , which is a function of y , agree very closely with the average heat gain or loss

at each parallel, but there is no way to draw any conclusion about the variation of Q_0 with longitude.

The agreement of the computed stream function shown in Fig. 3 with the actual circulation in the North Pacific subtropical anticyclone is surprising, if one considers that it is based exclusively on the effects of horizontal advection. Even the westward intensification of the anticyclone is apparent. There are certain obvious discrepancies with the observed circulation, however. First, the North Equatorial Current appears to be much farther north than it actually is. Second, the circulation in the Gulf of Alaska is very weak, and third, the Oyashio does not appear at all.

However, the calculated stream function represents only that part of the circulation which contributes to heat transports, while a circulation which involves water of the same temperature would not appear. The horizontal circulation required to transport heat from the areas of heat gain to those of heat loss is surprisingly small. Only 10 million $\text{m}^3 \text{sec}^{-1}$ are necessary to transport about $1.5 \times 10^{14} \text{ cal sec}^{-1}$ between these two regions. ZHUKOV (1964) calculates the response of the surface temperature distribution in the North Atlantic Ocean to two different atmos-

Fig. 3. Stream function computed from heat exchange according to equation (16).
Transports in million $\text{m}^3 \text{sec}^{-1}$.

pheric circulation patterns, and determines the strength of the circulation in the subtropical anticyclonic gyre to 14 and 18 million m^3/sec in the upper 200 m. His values are in the same order of magnitude as ours and demonstrate again that only a small fraction of the total horizontal circulation contributes to heat advection.

The results of the calculation of the stream function from the distribution of heat sources and sinks seem to be encouraging enough to pursue the study of horizontal heat advection further. It is very simple to use equation (13) to calculate the heat exchange Q^* , which would be necessary to maintain a given temperature distribution T in the presence of a given circulation ϕ , according to

$$\frac{\partial \phi}{\partial x} \frac{\partial T}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial x} = Q^* \quad (17)$$

The resulting Q^* can then be compared with the actual heat input function Q . The difference between Q and Q^* will indicate areas where heat is used by processes involving vertical movements or by horizontal mixing, according to the second term on the right-hand side of equation (13).

From surface current maps and from theoretical

models of ocean circulation such a stream function, representing the transports in the surface layer, has been constructed and is shown in Fig. 4. With this function and the average annual surface temperature, the distribution of Q^* has been computed according to (17) and is shown in Fig. 5. A comparison with the actual distribution of heat exchange (Fig. 2) shows remarkable agreement. The area of heat gain off California and that of heat loss off Japan are clearly shown and are numerically correct, if the amplitude of the stream function is chosen as 10 million $\text{m}^3 \text{sec}^{-1}$. Some discrepancies between the two distributions can be explained by other processes. The large area of heat gain in the western Pacific between 10° and 25°N is not shown in the map of Q^* . This may be explained by a downward diffusion of heat, which is not sufficiently balanced by ascending movements, or by a loss of warm surface water from the southwest Pacific, as postulated earlier. Probably both of these factors are contributing to the heat loss from this area. Off Hokkaido, northern Japan, an area of heat gain would be required to maintain the temperature distribution, and it will be shown in the next section that horizontal mixing across the Kuroshio may supply the necessary heat.

Fig. 4. Stream function ϕ , representing the transports in the surface layer of the North Pacific Ocean, used in subsequent calculations. Transports in million $\text{m}^3 \text{sec}^{-1}$.

Fig. 5. Distribution of Q^* calculated according to equation (17) in $\text{cal cm}^{-2} \text{ day}^{-1}$. This is the heat input required to maintain the observed temperature distribution in the presence of the circulation shown in Fig. 4.

In the Gulf of Alaska and off the coast of North America as far south as 35°N a heat loss is required, which indicates either that the chosen stream function advects too much warm water, or that upwelling occurs in these parts of the ocean. TULLY and BARBER (1960) have shown that upwelling takes place in the Gulf of Alaska at a rate of $w = 7 \times 10^{-5} \text{ cm sec}^{-1}$. With an average temperature difference between the surface and the subsurface water of 4°C this upwelling would require a heat input of about $24 \text{ cal cm}^{-2} \text{ day}^{-1}$, as shown in this map. Off the coast of Oregon, where upwelling is very strong, the heat deficit is even larger. Off southern California the situation is different and will be explained later, when horizontal mixing is discussed.

Between California and Hawaii there is a large area with a slight heat loss indicated in Fig. 2, but the calculations of Q^* show that the area should have a heat gain. This implies that either our stream function as calculated advects too much cool water from the California Current, or advection of warm water from the eastern tropical Pacific Ocean actually contributes considerably to the warming of this area.

The stream function we have selected does not allow much freedom for modification without

becoming unrealistic, and experiments with slightly different patterns have shown that slight modifications of the stream function have relatively little effect on the distribution of Q^* . From this analysis we can conclude that the advection of heat between areas of heat gain and areas of heat loss in the North Pacific Ocean is accomplished by a relatively weak horizontal circulation of only 10 million $\text{m}^3 \text{ sec}^{-1}$, which takes place in a fairly shallow surface layer. Water below this layer is merely circulated with no temperature changes.

5. The effect of large-scale horizontal mixing

Next we will consider an ocean where the distribution of temperature is controlled exclusively by the effect of horizontal eddy diffusion. In this case equation (3) reduces to

$$-A^h D \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right\} = Q \quad (18)$$

if the stationary case is considered. The Laplacian of the average annual temperature distribution has been calculated and is shown in Fig. 6. Large values are in the order of 10×10^{-14} , and occur in the area of the Kuroshio, in higher latitudes, and along the eastern boundary of the

Fig. 6. The Laplacian of the average annual surface temperature distribution in 10^{-14} degree cm^{-2} .

ocean. An estimate of the magnitude of A^h , using a value of $Q=10^{-3}$ cal cm^{-2} sec^{-1} , which is equal to about 115 cal cm^{-2} day^{-1} , and a mixing-layer depth of 50 m, gives a horizontal exchange coefficient $A^h=2 \times 10^6$ cm^2 sec^{-1} . The numerical value of this coefficient appears to be very reasonable, but the distributions of the Laplacian (Fig. 6) and of the average annual heat exchange Q (Fig. 2) show only partial agreement, although discrepancies can be explained by considering the effects of advection.

Heat diffuses into areas where the Laplacian is positive, and a heat sink should be situated in those areas. This is the case for the region of the northwestern Pacific Ocean to the north of the Kuroshio, where the Laplacian is positive and a heat loss occurs. In the Kuroshio region itself, off Japan, the Laplacian is negative and not balanced by heat input, and one has to assume that the heat required is provided by advection of warm water. In the western parts of the trade-wind region a strong heat source is situated, and parts of this heat seem to be lost from the region by horizontal eddy diffusion, as indicated by the negative values of the Laplacian between 20° and 30°N. Off the coast of California the positive Laplacian indicates a diffusion of heat into the region, which can be balanced only by upwelling of colder water, since the area

in itself gains considerable heat at the sea surface anyway. Along the west coast of North America, north of 40°N, the Laplacian is negative, and heat diffuses out of the area. Off Oregon this loss is balanced by positive values of the heat exchange while farther north in the Gulf of Alaska advection of warm water probably compensates the loss.

In the upwelling area along the west coast of North America a very interesting difference between the heat budgets north and south of 40°N seems to exist. The area off Oregon and Washington gains heat at the sea surface and by horizontal advection from the west (Fig. 5) and loses heat due to upwelling and horizontal diffusion. On the other hand, the area off California gains heat from the atmosphere and by horizontal diffusion, but loses heat by upwelling and cool advection.

The discussion of the influence of large-scale horizontal eddy diffusion demonstrates quite clearly that this effect cannot control the transfer of heat from the main heating to the main cooling areas. Regionally, on a smaller scale, horizontal eddy diffusion can certainly influence the heat budget of an area appreciably, as discussed above. Consequently it cannot be disregarded as a contributing factor in a calculation of the redistribution of heat in the oceans, especially in areas where isotherms are strongly

curved and crowded. In a comprehensive model describing the redistribution of heat, horizontal eddy diffusion will have to be considered.

6. The seasonal variation of heat input and surface temperature

When considering seasonal variations of the heat input, the basic question is to determine how much of the heat is used locally to change the heat content of the water column, and how much is carried away by advection and diffusion. A first impression of the order of these effects can be obtained from a comparison of the amplitudes of the seasonal variations of heat exchange and surface temperature. The amplitude of the annual wave of the heat exchange Q_1 at the sea surface is shown in Fig. 7 and can be compared with the amplitude of the annual wave of sea surface temperature T_1 drawn by WYRTKI (1965b) and based on the same data. There is a pronounced similarity between both distributions. Amplitudes of both parameters increase from the equator northwards, and have largest values in the northwest Pacific Ocean. In higher latitudes amplitudes decrease. The largest amplitudes of the heat exchange are found in the Kuroshio region off Japan and stretch east to the south of 40°N , while largest amplitudes of the surface temperature are slightly to the north of 40°N .

On the basis of this comparison it is inferred that there will be a strong local balance between heat input and heat storage, if the phase difference between these two parameters is close to 90° . The phase differences between the annual waves of heat input and surface temperature shown in Fig. 8 are indeed between 70° and 90° over most of the ocean, with exceptions only in the area between Japan and the Philippines and in the eastern tropical Pacific. This indicates that large parts of the heat input are absorbed locally. The deviation of the phase difference from 90° is very likely due to advection and seasonal variations of the depth of the mixed layer.

In a first attempt to analyze these relationships more closely we will consider an ocean without advection and diffusion and a time-independent depth of the mixed layer. A perfect local balance between heat input and temperature is governed by the equation

$$D \frac{\partial T}{\partial t} = Q \quad (19)$$

Since both temperature and heat input are periodic functions of time with the amplitudes T_1 and Q_1 , the quotient Q_1/T_1 should be proportional to the depth of the affected layer if their phase difference is 90° . The value $D = Q_1/\omega T_1$, representing the depth of the layer affected by the

Fig. 7. The amplitude Q_1 of the annual wave of heat exchange at the sea surface in $\text{cal cm}^{-2} \text{ day}^{-1}$.

Fig. 8. The phase difference $\Delta\phi$ between the annual waves of heat input and surface temperature in degrees.

seasonal variation of heat exchange, is shown in Fig. 9. In the North Pacific Ocean this calculated depth varies between about 60 m in the North Equatorial Current and 30 m in higher latitudes. It can be most readily compared with the depth of the mixed layer during summer, because the heat contained above the summer thermocline is a measure for the heat storage in the upper layer of the oceans. The only map of the depth of the mixed layer during summer, which is based on observations and of which we are aware, is that by PATTULLO and COCHRANE (1951). For the three summer months their maps show mixed-layer depths of 50 to 80 m in the North Equatorial Current between Hawaii and the Philippines and depths of about 20 m north of 40°N. The greater depths of the affected layer between Japan and Formosa of about 70 m are not shown in their observations nor are similar depths off California. However, the agreement between the calculated and observed depths of the layer affected by the annual wave of heat input is good enough to demonstrate that most of the seasonal variation of heat exchange is absorbed locally in a fairly shallow seasonal summer thermocline which is superimposed on the permanent thermocline. Because of this agreement, and because the depth of the mixed layer is not

constant during the year, it seems to be worthwhile to pursue the study further. When the depth of the mixed layer is variable during the year, the equation describing a local balance between heat storage and heat input is

$$\frac{\partial}{\partial t} \int_0^{\infty} T dz = Q \quad (20)$$

In a very simple way the vertical temperature distribution can be approximated by a function such as $T = T_0 \exp(-z^2/4D^2)$ where T_0 is the surface temperature and D a measure for the depth of the mixed layer. In our formula D is the depth at which the temperature T is 78 % of the surface value, and can be regarded as a reasonable measure for the depth of the mixed layer. Using this expression for T , equation (20) becomes

$$\frac{\partial}{\partial t} \sqrt{\pi} T_0 D = Q \quad (21)$$

and $D(t)$ can be determined from known values of T_0 and Q .

These calculations have been performed for each ten-degree square of the North Pacific Ocean and are shown in Fig. 10. It is apparent that, between 10° and 30°N, the seasonal variation of the mixed layer remains small, while north of

Fig. 9. The depth of the layer affected by the seasonal variation of heat input according to equation (19). $D=Q_1/\omega T_1$ in meters.

30°N sizeable amplitudes occur. In general, the amplitude of the seasonal variation as well as the depth of the mixed layer increases from east to west. The shallowest mixed layers occur in summer, usually August, in complete agreement with the observations. The deepest mixed layers appear in April, two months later than observed. This discrepancy indicates that the heat content of the ocean is not in a perfect local balance with the heat input, and that other effects, chiefly advection, contribute to its variation. It is well known from other investigations (PAT-TULLO, 1957 and BRYAN and SCHROEDER, 1960) that the amplitude of local heat storage exceeds significantly the amplitude of heat input. Our own computations (not shown here) have confirmed their results, which suggests that advection must be the cause.

To investigate the effect of advection on the distribution of temperature in the mixed layer, an equation similar to (17) but time-dependent can be used:

$$D \frac{\partial T}{\partial t} + \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial T}{\partial y} = Q \quad (22)$$

where ϕ is a time-independent stream function, as shown in Fig. 4, representing the circulation in the surface layer. If T and Q can be

represented by their first harmonics in the form $T=T_1 \exp(-i\omega t)$ and $Q=Q_1 \exp(-i\omega t + \Delta\psi)$, the value of the phase difference $\Delta\psi$ between the annual waves of heat input and temperature can be computed by

$$tg \Delta\psi = \frac{\omega D T_1}{\frac{\partial \phi}{\partial y} \frac{\partial T_1}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial T_1}{\partial y}} \quad (23)$$

It should be emphasized that this phase difference is independent of the amplitude of Q .

The results of this computation, using the stream function from Fig. 4 and the observed amplitude of the annual variation of surface temperature T_1 are shown in Fig. 11. This phase difference is less than 90° in the Kuroshio current system and in the northern portions of the California Current, in agreement with the observed phase difference shown in Fig. 8. Consequently advection must be the most important factor in determining the temperature distribution in the strong boundary currents. Elsewhere the phase difference is larger than observed, being >90° to the south of 30°N and about 90° to the north of 30°N. This shows that advection is not the decisive factor controlling the phase difference between heat input and surface temperature in regions of zonal flow but that the variations of the depth of the mixed

Fig.10. Calculated seasonal variation of the mixed-layer depth according to equation (21) for each ten-degree square (vertical scale in meters, horizontal scale in months; February, May, August and November are marked).

Fig. 11. Computed phase difference $\Delta\psi$ between the annual wave of heat input and surface temperature in the presence of advection according to equation (23).

layer are of greater importance. Consequently a knowledge of heat input and circulation alone is not sufficient to calculate the distribution of temperature in the surface layer, but additional information on the depth of the mixed layer must be available. From the foregoing calculations about the phase difference between heat input and temperature variations, it can be concluded that the deviation of this phase difference from

90° can be caused either by horizontal advection or by a variable depth of the mixed layer, and that the contributions of these two factors are of the same order of magnitude according to the observed conditions in the ocean and have to be fully taken into account. Since our present data do not include information on the variations of mixed-layer depth, the study cannot be pursued further. It is planned, however, to treat data

on the heat storage in the North Pacific Ocean in a similar way to obtain quantitative information on the contribution of advection.

A similar approach has been used by ZHUKOV and TERIAEVA (1965) to calculate the annual variation of temperature distribution in the upper 200 m of the North Atlantic Ocean. They estimate surface circulation from wind stress and vertical heat flux from the temperature gradient at 200 m depth, but neglect vertical advection which is shown here to be of primary importance. Their results can only be expressed in terms of heat storage (average temperature 0-200 m depth), not as surface temperatures.

7. Conclusions

1. The excess heat input at the surface of the North Pacific Ocean is balanced by a vertical circulation of approximately 8 million m^3/sec , representing a thermohaline type of circulation.

2. The regional differences in the average annual heat input are equalized by a horizontal circulation of about 10 million m^3/sec in a comparatively shallow surface layer.

3. Horizontal eddy diffusion can contribute considerably to heat transports in areas of strong temperature gradients.

4. Horizontal and vertical advection and horizontal mixing contribute to the maintenance of the temperature distribution in various areas of the North Pacific Ocean in different ways. Some of these differences are discussed and explained.

5. Large portions of the seasonal variation of the heat input are stored locally. If a variable mixed-layer depth is considered, even the entire heat input could be stored in a mixed layer varying between 30 and 70 m, but there is some disagreement in the phase relations.

6. Phase shifts between heat input and temperature are chiefly due to horizontal advection, but the complete problem can be solved only by introducing heat storage or mixed-layer depth as an additional observed factor; the observed surface temperature alone is not sufficient.

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北太平洋における熱の再配分について

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要旨 時間、空間的に変化する熱供給を受けながら、北太平洋表面層の温度分布を維持する過程を分析する。ここでは、水平及び鉛直移流、混合過程、熱の貯留を考える。

北太平洋へ入る（表面からの）熱供給の過剰分は、約 800 万 m^3/sec の鉛直大循環と釣合っている。熱を得る海域から熱を失う海域へ、水平循環によって熱が輸送される。この循環は熱供給と表面温度から計算され、亜熱帯高気圧性循環においてわずかに 1,000 万 m^3/sec の輸送を要するのみであることが示される。従って、すべての水平熱輸送は薄い表面層のみで行なわれる。必要とされる熱供給を観測されたものと比較すると湧昇域の場所

と強さ、および水平渦動拡散の寄与がどのくらいであるかについて結論を与えることが出来る。表面水温と熱供給の季節変化を比較し、30~70 m の深さの混合層が影響を受けることが示される。もし混合層の深さが変えるとすると熱供給の季節変化分全体をその場所で貯留することが出来るが、晩冬にあらわれる混合層厚の最大は、もっと時期が遅れることになる。熱供給と表面水温の間の位相の違いの一部は水平移流で説明出来る。したがって、表面水温に対する効果という点で、熱の移流と混合層厚の変化とはほぼ同程度の寄与をしていると推定出来る。