Dynamic Inelastic Failure of Structures*

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This article focuses on the behaviour of structures which are subjected to dynamic loads (impact, blast, pulse) which produce large inelastic strains and material failure. Particular attention is given to the elementary and strain energy density failure criteria and some recent experimental results on the dynamic inelastic failure of structural members.

Key Words: Dynamic Inelastic Failure, Structural Failure Criteria, Strain Energy Density Criterion, Shear Failure, Tensile Tearing

1. Introduction

This paper focuses on the behaviour of ductile metal structures which are subjected to dynamic loads which produce large inelastic strains and material failure. In particular, it examines the criteria for the failure or rupture of the material which would precipitate a dynamic structural failure. This particular topic is important for energy absorption estimates, hazard assessments and safety calculations of structural components and systems subjected to large dynamic loads throughout engineering⁽¹⁾⁽²⁾.

Two major lines of enquiry could be followed to clarify the dynamic inelastic failure of structures which are caused by material failure. One major approach uses micro-mechanics as exemplified by Bammann et al.⁽³⁾ who used finite-element codes to examine the failure of several dynamic problems. This approach appears attractive and will no doubt become more so as computers become ever more powerful. However, it is important to remark that these methods require several material constants which are difficult to obtain experimentally. For example, a Taylor-Quinney coefficient of 0.9 is used in the basic equations by Bammann et al.⁽³⁾ which implies that 90 per cent of the plastic work is

dissipated as heat. This value is similar to an average factor of 0.85, approximately, which was found in some impact tests on mild steel specimens⁽⁴⁾. However, this factor is dependent on the energy absorbed even in a static tensile test⁽⁵⁾.

Another interesting recent finite-element study was published by Knoche and Needleman⁽⁶⁾ on the effect of size on the ductility of dynamically loaded round bars in tension. Again, however, the numerical scheme incorporates a number of assumptions including, for example, a time-independent value of 0.90 for the Taylor-Quinney coefficient, a critical void volume fraction of 0.12 and employs parameters in the plastic flow potential which were obtained from similar quasi-static studies.

No doubt with the passage of time the various assumptions required currently in the use of micro-mechanics for the dynamic inelastic failure of structures will decrease as a greater body of experimental knowledge becomes available.

The other major method for predicting failure uses the principles of structural mechanics which have been used successfully throughout engineering. Of course, it is evident that many failures are initiated locally, whereas a structural mechanics approach uses global quantities such as generalised stresses (moments, forces) and generalised strains (curvatures, membrane strains). This appears to be a fundamental contradiction. Nevertheless, these methods do allow for some local behaviour in a structural member because, typically, the plane

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sections assumption gives rise to maximum stresses and strains at a discrete location on the surface of a structural member.

The actual spatial and temporal characteristics of a dynamic loading are often not known accurately. The dynamic material properties are also sometimes not available, especially for large plastic strains and high strain rates. Thus, estimates predicted by the structural mechanics approach are often adequate for many designs and for almost all preliminary designs, particularly outside the defense and nuclear establishments. On the other hand, numerical predictions of sophisticated finite-element methods using micro-mechanics can provide a misleading impression of accuracy unless great care is taken and they are accompanied by a considerable amount of experimental validation.

This article focuses on recent research work published on the dynamic inelastic failure of structural members, principally from the perspective of structural mechanics. The knowledge gained from this approach can be incorporated into numerical finite-element codes, as exemplified by Holmes et al.,⁽⁷⁾ who used a curve which relates the failure strain to the associated stress triaxiality.

Section 2 of this article introduces the elementary failure criterion and illustrates various failure characteristics with the aid of the equations for a fully clamped rigid, perfectly plastic beam, which is loaded impulsively. The strain energy density failure criterion and some recent experimental results on structural failure are discussed in sections 3 and 4, respectively. It is shown how some of the experimental results may be interpreted using the elementary failure criterion. The dynamic perforation of plates is a particular type of failure, as mentioned in section 5. This article is completed with a discussion and conclusions in sections 6 and 7, respectively.

Nomenclature

k : Proportion of structural thickness required for a transverse shear failure
 w_s : Transverse shear displacement
 H : Thickness of a structural member
 I : Total impulse

$2L$: Span of a beam
 M_0 : Fully plastic bending moment of the cross-section of a structural member (per unit length)
 Q_0 : Fully plastic transverse shear force of the cross-section of a structural member (per unit length)
 R : Radius at the boundary of a circular plate
 V_0 : Impulsive velocity
 W_f : Maximum permanent transverse displacement
 ϵ : Strain
 $\dot{\epsilon}$: Strain rate
 $\epsilon_{\max}$: Maximum strain
 ϵ_r : Uniaxial rupture strain
 θ : Strain energy density
 θ_c : Critical value of θ
 λ : Non-dimensional kinetic energy, equations (2) and (13)
 ν : $Q_0 L / 2M_0$
 ρ : Density of material
 σ : Stress
 σ_0 : Uniaxial yield stress
 Φ : Dimensionless pulse defined by equation (15)

2. Elementary Failure Criteria

It is well known that theoretical methods of analysis using the idealisation of a rigid-plastic material have been developed successfully to predict the behaviour of structural members subjected to large static loads. Similar rigid-plastic methods of analysis may also be used to obtain the response of structures subjected to large dynamic loads⁽¹⁾⁽²⁾. In these analyses, the permanently deformed profile of a structural member, for example, is predicted when assuming that the material remains ductile, regardless of the strain magnitudes. This is known as a mode I response. In the particular case of a rigid, perfectly plastic beam in Figure 1 with a thickness, H , which is fully clamped across a span, $2L$, and subjected to a uniform impulsive velocity, V_0 , which produces finite-displacements, the dimensionless maximum permanent transverse displacement is

$$W_f/H = \{(1 + 3\lambda/4)^{1/2} - 1\}/2, \dots\dots\dots (1)$$

where

$$\lambda = \rho V_0^2 L^2 / M_0 \quad (2)$$

is a non-dimensional initial kinetic energy, $M_0 = \sigma_0 H^2 / 4$ and where ρ and σ_0 are the density and yield stress for the material respectively.

It is evident that a material failure might intervene during a mode I ductile response of an actual structure before it absorbs all of the external dynamic energy and reaches a permanently deformed shape.

The first theoretical rigid-plastic analysis to cater for failure⁽⁸⁾ examined an impulsively loaded beam and identified two types of failure. The plastic deformation is exhausted when the maximum uniaxial tensile strain on the surface of the beam at the supports ($\epsilon_{\max}$) reaches the rupture strain in a standard tensile test (ϵ_r), or

$$\epsilon_{\max} = \epsilon_r, \quad (3)$$

which is a mode II response. It might be shown⁽¹⁾⁽⁸⁾ that this failure occurs for the fully clamped beam in Figure 1 when the dimensionless kinetic energy is

$$\lambda^{1/2} = 2[1 + \{4 + 2\epsilon_r(2L/H)^2 - 4L/H\}^{1/2}] / 3^{1/2} \quad (4)$$

provided $3\lambda/4 \gg 1$ and when idealising the behaviour of a plastic hinge according to Nonaka⁽⁹⁾.

A transverse shear failure with relatively small transverse displacements develops at the supports for large impulsive velocities. This failure is known as a mode III response and is assumed to occur when

$$w_s = kH, \quad (5)$$

where w_s is the transverse shear displacement and $k < 1$ is an experimentally determined factor⁽¹⁰⁾. A rigid, perfectly plastic analysis for the fully clamped beam loaded impulsively in Figure 1 predicts⁽¹⁾⁽⁸⁾ that a mode III transverse shear failure occurs when

Fig.1 Fully clamped beam subjected to a uniform impulsive velocity, V_0

$$\lambda^{1/2} = 2\nu(2k/3)^{1/2}, \quad (6)$$

where $\nu = Q_0 L / 2M_0 > 3$ and $Q_0 \approx \sigma_0 H / 2$. The parameter $\nu = L/H$ for the particular case of a beam with a rectangular cross-section so that equation (6) gives

$$\lambda^{1/2} = (2L/H)(2/3)^{1/2} \quad (7)$$

when assuming that $k=1$. It can be shown that

$$W_f/H = 2w_s/H = 2k \quad (8)$$

when a mode III transverse shear failure occurs. It should be noted that this type of transverse shear failure is much more likely to develop under dynamic loads than under static loads⁽¹⁾⁽²⁾. It transpires that equations (1), (4) and (6) provide good agreement with the experimental data for the mode I, II and III responses, respectively⁽¹⁾⁽⁸⁾⁽¹⁰⁾.

Equations (1), (4) and (7) for the mode I, II and III responses of the beam in Figure 1 are plotted in Figure 2 for an aluminium alloy beam with $\epsilon_r = 0.15$ and $2L/H = 30$. It is evident that the largest transverse displacements are associated with a mode II tensile tearing failure. The total transverse displacements become smaller for higher values of $\lambda^{1/2}$ due to the influence of transverse shear sliding at the beam supports. The transverse shear displacement increases with $\lambda^{1/2}$ to reach a maximum value for a mode III transverse shear failure with the smallest associated total transverse displacement. It should be noted that a mode III failure occurs at the end of the first phase of motion leaving a failed beam with a dimensionless kinetic energy, $1-2/\nu$, some of which is absorbed in bending deformations as the beam flies through space.

These elementary failure criteria have been used to predict the failure of beams subjected to impact loads⁽¹⁰⁾ and generalised for predicting the failure of a wider class of structural members. They have also assisted in the interpretation and classification of the failure modes in blast loaded plates⁽¹¹⁾.

Further details on the elementary failure criteria for dynamically loaded structures are given in References (10) and (12).

Wen et al.⁽¹³⁾ have modified the elementary failure criterion and examined the failure of fully clamped steel and aluminium alloy beams subjected to mass impact loadings⁽¹⁴⁾⁽¹⁵⁾. An equivalent strain is used to cater for the influence of the transverse

shear strain on the uniaxial rupture strain for a mode II failure. A similar theoretical approach has been used by Wen⁽¹⁶⁾ to study the failure of impulsively loaded beams⁽¹⁷⁾.

An examination⁽¹⁸⁾ of the transverse shear failure of rigid, linear hardening beams under an impulsive velocity loading, identified three different failure modes. One of these included a criterion similar to equation (5). The other two were defined as a critical transverse shear strain and an adiabatic shear failure for high strain rates.

3. Strain Energy Density Failure Criterion

Clift et al.⁽¹⁹⁾ have studied the initiation of failure in a range of simple metal forming operations using a finite-element technique and nine different failure criteria. These authors have obtained numerical predictions of the site for the initiation of failure at the experimental value of deformation at failure. This was achieved by locating the node in the mesh which had the largest accumulated value of the failure criterion function. These predicted sites were then compared with the failure initiation sites observed experimentally. It transpires that only the criterion based on the critical value of generalised plastic work per unit volume predicted successfully the failure initiation sites found experimentally in all of the metal forming operations.

It is assumed that rupture occurs in a rigid-plastic structure when the absorption of plastic work (per unit volume), θ , reaches the critical value

$$\theta = \theta_c, \dots\dots\dots (9)$$

where θ contains the plastic work contributions related to all of the stress components. The maximum possible value of θ_c is taken as

$$\theta_{cm} = \int_0^{\epsilon_{f0}} \sigma_d(\epsilon, \dot{\epsilon}_m) d\epsilon, \dots\dots\dots (10)$$

where $\sigma_d(\epsilon, \dot{\epsilon}_m)$ is the dynamic engineering stress-strain curve, which is obtained from a dynamic uniaxial tensile test for a given $\dot{\epsilon}_m$, and where ϵ_{f0} is the engineering rupture strain (zero gauge length). In general, both ϵ_{f0} and σ_d could depend on the magnitude of the strain rate. For a rigid, perfectly plastic material, equation (10) simplifies to

$$\theta_{cm} = \sigma_{d0} \epsilon_p, \dots\dots\dots (11)$$

where

$$\epsilon_p = \frac{1}{\sigma_{d0}} \int_0^{\epsilon_{f0}} \sigma_d(\epsilon, \dot{\epsilon}_m) d\epsilon \dots\dots\dots (12)$$

where σ_{d0} is the mean dynamic flow stress.

In view of the success observed by Clift et al.⁽¹⁹⁾, the energy density failure criterion was employed to study the dynamic inelastic failure of beams⁽²⁰⁾⁽²¹⁾. The authors of Reference (10) have observed that, although the elementary rigid, perfectly plastic method of analysis gives useful predictions for the overall features of the failure behaviour of beams subjected to uniform impulsive and impact loadings, the energy density failure criterion provides greater detail of the behaviour. It is also capable of further development for a wide range of dynamic problems in the fields of structural crashworthiness, hazard assessments and impact engineering and has been used to predict the dynamic inelastic failure of circular plates⁽²²⁾. Nevertheless, further experimental and theoretical studies are required to assess whether or not this criterion may be used with confidence for predicting the dynamic inelastic failure of a wider range of structures.

Kawata has continued his long term interest in the mechanical properties of a range of materials under dynamic loads. In particular, the absorbed energy per unit volume, which is required for the strain energy density failure criterion, is obtained by Kawata et al.⁽²³⁾⁽²⁴⁾ for several materials subjected to uniaxial tension for strain rates up to 10^3 s^{-1} . It appears⁽²⁴⁾ that the total energy absorbed to failure is not significantly changed for several aluminium alloys over a range of strain rates between 10^{-3} s^{-1} and 10^3 s^{-1} , approximately.

Some values for the dynamic energies absorbed by several steels were obtained at a strain rate of 2500 s^{-1} by Miura et al.⁽²⁵⁾

It is evident that further experimental tests are required on a range of materials in order to obtain the strain energies which can be absorbed up to failure for various stress states and strain rate magnitudes.

Finally, it should be noted that the strain rate is zero when the motion of a structure ceases and,

therefore, the influence of strain rate on the strain energy density failure criterion might not be too important when failure just initiates in this case. However, failure could also develop during the structural response when the strain rate is not zero. Moreover, the strain rate sensitivity of the material would influence the structural response throughout its motion and, therefore, effect the conditions of the structure at the initiation of failure.

4. Experimental Test Results on the Dynamic Inelastic Failure of Structures

Systematic studies on the dynamic inelastic failure of structures, due to material failure, have been conducted only in relatively recent years, despite the importance of this topic for assessing the safety of a wide range of engineering systems subjected to large dynamic, impact and blast loads.

In 1973, Menkes and Opat⁽¹⁷⁾ conducted tests on the failure of impulsively loaded beams and identified modes I to III which were studied theoretically⁽⁸⁾, as noted in § 2 and shown in Figure 2. Subsequently, the failure of beams due to impact loadings was examined and found to be more complex than a uniform impulsive velocity loading⁽¹²⁾⁽¹⁴⁾⁽¹⁵⁾⁽²⁶⁾. It transpires that impact loadings produce localised material failures in mild steel and aluminium alloy beams, as well as the mode II and mode III failures observed in beams subjected to uniformly distributed impulsive loadings.

Recent studies have also been conducted on the dynamic inelastic failure of ductile metal plates subjected to blast loadings which may be idealised as a uniform impulsive velocity. Teeling-Smith and Nurick⁽¹¹⁾⁽²⁷⁾ have examined the failure characteristics of circular plates which are made from a 1.6 mm thick cold rolled mild steel and fully clamped around a boundary having a 100 mm diameter. It transpires that similar failures to the mode I, II and III failures, which were identified for beams, also occur for these plates.

Many theoretical rigid-plastic solutions for structures having a thickness, H , and a characteristic dimension, L_c , subjected to uniform impulsive velocities, V_0 , might be expressed in terms of a

dimensionless kinetic energy of the form⁽¹⁾.

$$\lambda = \rho V_0^2 L_c^2 / M_0, \dots\dots\dots (13)$$

where $M_0 = \sigma_0 H^2 / 4$, σ_0 is the yield stress and ρ is the density of the material. Equation (13) may also be written

$$\lambda^{1/2} = 2I / \{\pi R H^2 (\rho \sigma_0)^{1/2}\}, \dots\dots\dots (14)$$

for a circular plate with $L_c = R$, where R is the support radius and I is the total impulse.

Now, Teeling-Smith and Nurick⁽¹¹⁾ have used this dimensionless parameter in the form

$$\Phi = \lambda^{1/2} / 2 \dots\dots\dots (15)$$

to identify the threshold conditions for a mode II tensile tearing failure, as indicated in Figure 3. The experimental results plotted in this figure with respect to Φ show that the slope of the dimensionless maximum permanent transverse displacements, W_f/H , changes at $\Phi \simeq 47$. Thus, those experimental results having $\Phi < 47$, are associated with a mode I response, which involves large ductile deformations and no material failure, while those with $\Phi > 47$ suffer a mode II tensile tearing failure. A mode III transverse shear failure is more difficult to determine because of lack of experimental results in this region, but the threshold value is estimated to lie within the range $85 \leq \Phi \leq 95$.

It is evident that the general form of the experimental results in Figure 3 for a circular plate is similar to Figure 2 for the impulsive loading of a fully clamped beam. If $L_c = L$ for the beam in Figure 1, then equation (13) becomes equation (2) so that dividing the abscissa in Figure 2 by 2 gives Φ according to equation (15). Thus, plotting the experimental values for W_f/H against Φ allows the transition from a wholly ductile mode I response

Fig. 2 Modes I, II and III for the beam in Fig. 1 with $2L/H = 30$, $\epsilon_r = 0.15$ and $k = 1$

Fig. 3 Failure modes for fully clamped, thin, circular steel plates ($H=1.6$ mm, $R=50$ mm) subjected to a uniform impulsive loading⁽¹¹⁾. X: mode I; $\otimes$: mode II (partial tearing); $\circ$: mode II (complete tearing)

to a mode II tensile tearing type failure to be identified from a change in the slope. Smaller maximum permanent transverse displacements are associated with a mode III failure mode in both Figures 2 and 3. The decrease in W_f/H between the mode II and mode III failures is associated with a combined tensile tearing and a transverse shear failure and a residual kinetic energy after severance.

The dimensionless transverse shear displacement, w_s/H , for a mode III failure [equation (5)] remains constant for values of Φ larger than the threshold value [equations (6) and (7)]. This theoretical observation might allow the threshold of a mode III transverse shear failure to be identified in future experimental test programmes.

It should be noted that the theoretical predictions in Figure 2 are plotted for rigid, perfectly plastic, strain rate insensitive beams having $\epsilon_r=0.15$, whereas the experimental results in Figure 3 are for strain rate sensitive steel plates. Adiabatic shear failures are not considered for larger values of Φ ⁽¹⁸⁾.

It has been noted⁽²²⁾ that some in-plane displacements occurred at the plate boundaries of the specimens examined by Teeling-Smith and Nurick⁽¹¹⁾. This phenomenon would have the effect of delaying the onset of a mode II failure so that the experimental data might provide an overestimate of the threshold impulse required for a structural failure. Subsequently, Nurick and colleagues⁽²⁸⁾⁽²⁹⁾ have reported experimental test results on thin steel circular plates with several different clamped boundary conditions. It is observed that the perma-

Fig. 4 Failure modes for fully clamped, thin, square steel plates ($H=1.6$ mm, $L=44.5$ mm) subjected to a uniform impulsive loading⁽³¹⁾. $\circ$: mode I; $\blacktriangle$: mode II*; $\blacksquare$: mode II a; $\square$: mode II b-III

nent transverse displacements at the threshold conditions for a mode II failure are larger for plates having radiused clamps than for those with sharp edges. Good agreement is now obtained with the theoretical rigid-plastic predictions for a fully clamped plate when using an energy density failure criterion⁽²²⁾.

Further experimental studies have also been conducted on the failure of square steel plates subjected to uniform impulsive loadings⁽²⁸⁾⁽³⁰⁾⁽³¹⁾. It is observed that three different kinds of failure develop within the mode II tensile tearing mode. These are identified as II* for partial tearing around the boundary, while modes II a and II b are associated with complete tearing but have increasing and decreasing maximum permanent transverse displacements with increase in impulse, as indicated in Figure 4. The mode II failure results in Figure 3 for circular plates with partial tearing around a short sector of the boundary are now identified as II*, while II a responses were not observed. The mode II results in Figure 3 are now re-defined as mode II b.

In order to improve the dynamic resistance to failure of steel plates, some experimental data has been reported on the impulsive loading of circular⁽³²⁾ and square⁽³³⁾ plates with one stiffener across the middle. Some data has also been reported⁽³⁴⁾ on the failure of rectangular plates having two parallel stiffeners.

5. Perforation of Plates

Most of the experimental and theoretical studies on the perforation of plates have focused on the influence of a relatively small mass travelling with a high velocity which is of particular interest to the defense community. However, there are many practical situations in which an engineer requires the threshold perforation conditions for a relatively large mass travelling at a lower velocity up to 10–15 m/s, approximately.

A plate sustains highly localised damage in the high velocity case and is, therefore, very difficult to analyse because of the influence of temperature and other effects on the material properties. A relatively large mass travelling with a low velocity, on the

Fig. 5 Perforation of a fully clamped mild steel circular plate ($H=6$ mm, $R=76.2$ mm) struck by a blunt-ended mass with a 17.85 mm diameter, weighing 81 kg and travelling with an impact velocity of 4.98 m/s⁽³⁵⁾

Fig. 6 Variation of the dynamic (E_p) to static (E_s) perforation energies with perforation velocities (V_p) for fully clamped circular mild steel plates. Further details are given in Reference⁽³⁶⁾

other hand, causes an impact event with a much longer duration which allows the external dynamic energy to be absorbed over a greater region of a plate. In particular, global displacements might develop to allow a significant proportion of the dynamic energy to be absorbed in membrane stretching before perforation occurs, as indicated in Figure 5. This is also illustrated in Figure 6 which shows the change in perforation energy with the impact velocity of blunt cylindrical masses striking fully clamped circular mild steel plates⁽³⁶⁾. It is evident that this behaviour is related to the phenomena in Figures 2 to 4 which show the change with dimensionless impulsive velocity of the maximum permanent transverse displacements associated with the various failure modes of fully clamped beams, circular plates and square plates. Thus, the phenomenon of perforation for the high and low velocity cases is quite different and, therefore, requires different methods of analysis.

Recently, several authors^{(35)–(39)} have considered the high mass-low velocity case and presented experimental results for the perforation of steel and aluminium alloy plates. Semi-empirical equations⁽³⁶⁾⁽⁴⁰⁾⁽⁴¹⁾ have been developed for design purposes and several promising theoretical developments have been published^{(42)–(46)}. These theoretical methods were derived using the standard equations for thin plates and elementary failure criteria for the material which are similar to those discussed in § 2. It transpires that these theoretical analyses often predict good agreement with experimental data and will replace the existing empirical equations with greater confidence when more information becomes available, in due course, on the failure criteria.

6. Discussion

It is evident from the previous sections that there is considerable uncertainty on the criterion which should be used to predict the dynamic inelastic failure of structures. Several promising approaches have been observed to provide good agreement for certain structures, but the range of validity when applied to other structures is not clear in many cases.

Often the uniaxial rupture strain recorded in a static uniaxial tensile test is used by designers and in numerical finite-element codes to predict the failure of complex dynamic structural systems with multi-dimensional associated stress fields which are spatially complicated and time-dependent. Moreover, the stress field in a uniaxial tensile test specimen at failure is not uniform, as shown by Bridgman. Furthermore, it is well-known that the magnitude of the uniaxial rupture strain depends strongly on the associated stress triaxiality (or hydrostatic pressure). In addition, the uniaxial rupture strain depends on the strain rate magnitude, as observed by Kawata et al.⁽⁴⁷⁾⁽⁴⁸⁾ and others. Thus, even this simple failure criterion when generalised to an equivalent strain requires further experimental data, not least for multi-dimensional stress states.

It was noted in § 2 that transverse shear effects were much more important for structures subjected to dynamic loads than in similar structures loaded statically. Thus, an equivalent rupture strain criterion, which is based only on static tensile test data, is not likely to be suitable for predicting the dynamic inelastic failure of those structural systems which have significant transverse shear strains as well as tensile strains contributing to a material failure.

The strain energy density failure criterion discussed in § 3 is potentially more universal than the elementary failure criterion since it retains the influence of all contributions to the strain energy. It has been found to predict good agreement with experimental results in the metal forming field⁽¹⁹⁾ and for the failure of several simple structural members loaded dynamically^{(10)(20)~(22)}. However, further experimental work and theoretical development is required in order to gain confidence in the accuracy of this criterion for a wide range of dynamic inelastic structural systems.

A major difficulty with this failure criterion lies with the actual volume of material to be used in the criterion. Some theoretical studies^{(10)(20)~(22)} use a volume of material which is based on the plastic hinge length and the structural thickness. However, a numerical finite-element study for a beam

impact problem⁽⁴⁹⁾⁽⁵⁰⁾ has observed that the strain energy varies significantly around the failure site and is not uniform through the beam thickness due to bending effects, as shown in Figure 15(c) in Reference (50). Thus, it is difficult to select a critical volume in this case. It might be expected that the critical volume should equal that observed in a tensile test. However, the strain energy density in a tensile test specimen at rupture in Figure 5 in Reference (50) is also complex and is quite different from that in the impacted beam. A rational procedure is suggested in Reference (50) for obtaining the correct volume to use in the energy density failure criterion for an impacted beam and other structures. It is assumed that the plastic strain energy density varies linearly between the impact point and the centroid of a nearby element. Failure occurs when the plastic strain energy density, at a characteristic distance from the impact point, equals the critical value which is obtained from a uniaxial tensile test.

The recent studies of Kawata et al.⁽²³⁾⁽²⁴⁾ need extending in order to obtain more information on the variation of the strain energy density at failure with increase in the strain rate.

The numerical finite-element study in References (49) and (50) examined in considerable detail some earlier⁽¹⁵⁾ experimental results on the failure of metal beams impacted by large masses travelling up to 11.5 m/s. The static and dynamic material properties were obtained for the material from which these beams were made and the numerical finite-element method examined the beams which had just failed in the experimental study. The response of a beam was computed for exactly the same conditions recorded experimentally at the failure threshold. In order not to prejudice the observations, no failure was incorporated into the finite-element method.

The numerical procedure allowed a wide range of parameters to be computed at the threshold of failure for an actual beam. Of interest to this paper, Reference (50) presented values for the maximum axial and shear strains, equivalent plastic strain, maximum axial and shear stresses, von Mises and Tresca stresses, hydrostatic pressure,

maximum axial force, maximum transverse shear force and maximum bending moment. It transpires that the maximum membrane force in a beam cross-section is the most promising quantity to use for predicting a tensile tearing failure. The maximum Tresca or von Mises stresses and the maximum plastic strain energy density are worthy of further study for the development of a shear failure criterion. A failure index based on an interaction yield surface is also promising for the development of a global failure criterion.

The current state of knowledge for the dynamic inelastic failure of structures is shown to be wanting by this paper, even for predicting the failure threshold for simple structural members. Thus, until more knowledge is gained, the failure criterion for the dynamic inelastic local or global failure of the pressurised pipelines⁽⁵¹⁾ in Figure 7, for example, and many other important practical structural problems, must, perforce, be obtained on an *ad hoc* basis. It is clear that theoretical analyses and

numerical schemes must be developed in partnership with experimental studies which characterise the dynamic properties of the material as well as examine the conditions for structural failure.

This article has focused on the topic of dynamic failure from the viewpoint of structural mechanics. As mentioned in the Introduction, the perspective from a micro-mechanics viewpoint is being examined by many authors. However, these methods are also handicapped by a paucity of experimental data and, in any event, are not suitable, at least within the near future, for most preliminary designs and many final designs in industry, except for the defense and nuclear establishments.

7. Conclusions

It is evident from this article that some useful progress has been made in recent years in predicting the inelastic failure of structures caused by large dynamic loads. However, various shortcomings of the failure criteria are highlighted, though many of these limitations might be removed by careful experimental studies. It is suggested that further progress in the development of the dynamic inelastic failure criteria for engineering structures can occur only through a close partnership between experimental studies, theoretical analyses and numerical studies.

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(a) Local failure at the impact position

(b) Global failure at the supports

Fig. 7 Pressurised steel pipelines with fully clamped ends subjected to local impact loadings⁽⁵¹⁾

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