Impact Problems of Balls in Tennis and Golf*

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This paper reviews some of the work that has been carried out on golf ball and tennis ball impacts. The emphasis is on the motion of the ball rather than equipment and an attempt is made to focus on ball impacts with turf. It was found that the majority of work has looked at golf ball impacts with the club and that viscoelastic models of impact are used by many authors to simulate the ball/surface collision, although the identification of the viscoelastic parameters has remained a problem for most researchers. Two studies of ball impacts on turf were outlined; golf ball impacts on golf greens and tennis ball impacts on synthetic turf. For golf ball impacts, it was found that the hardness of the turf was an important parameter while for tennis ball impacts the hardness of the ball was important. The harder surface, or ball, causes the ball to slip throughout impact causing less topspin on the ball after impact and allowing the ball to rebound fast at a shallow angle. It was concluded that more work needs to be carried out on the material parameters of balls and turf if suitable models are to be formulated.

**Key Words:** Sports Engineering, Tennis, Golf, Ball, Impact, Surface, Rebound

1. Introduction

Sport is being criticised more and more that it is being dominated by technology. It is important, therefore, that the effect of improvements of design on sport is determined. This paper focuses on the mechanics of tennis and golf ball impacts during the ball/surface interaction and a greater understanding of the impact with the playing surface will identify the important ball and surface parameters. This could, in turn, identify the appropriate maintenance procedures for turf or surface, say, or modifications to the ball which would allow the game to be modified. This would particularly apply to tennis where the modern grass game is dominated by the serve.

2. Models of ball impacts

There has been much work carried out on ball impacts with implements such as clubs and rackets while there is a significant lack of information on impacts with the playing surface. Daish*1 applied classical mechanics to study balls impacting on surfaces. His assumptions were that the ball and the ground deformed minimally, that there were two individual cases of pure slip and pure sticking and that the coefficients of friction $\mu$ and restitution $e$ determined the process of impact. The fact that the predictions of the model did not agree with reality were explained by the fact that the ball actually creates a saucer-like depression in the surface during impact and that the ball rebound angle is modified by the front inclined face of the depression. Brody*1 used a similar approach to investigate the way in which a tennis ball bounces during play. The work indicated what happens under classical conditions and related this anecdotally to what is seen in practice. Hertz*3 established his theory of contact in which the force experience by two isotropic elastic bodies with perfectly smooth surfaces under a static compressive load was proportional to the deformation to the power of 1.5. Almost one hundred years later Maw et al.*4 developed the Hertzian theory for oblique impacts of an elastic sphere on a half space. It was assumed that the contact area consisted of sticking and slipping regions with a constant coefficient of friction in the slipping regions. Maw et al.*5 tried later to verify their theories although the results are inconclusive. Other researchers, notably Hutchings et al.*6*7 and Rickerby et al.*8, modelled rigid spheres impacting obliquely with an elastic–plastic half space. Penetration of the spheres into the surface was allowed and the force on the sphere was proportional to the contact area.

Not surprisingly, economics has dictated that the majority of the research has been carried out on golf ball impacts with golf clubs. Lieberman*9 used rigid body theory to assess the effect of impact.
conditions on spin rate by projecting golf balls at a fixed plate. Lieberman and Johnson\textsuperscript{10} found that the coefficients of restitution and friction alone were not sufficient to analyse golf ball impacts. Instead, they considered a five parameter viscoelastic model for golf ball impacts normal to a rigid plate. The model was extended to oblique impacts by the addition of a torsion model\textsuperscript{11}. The normal model was extended into the “Simon model”\textsuperscript{12,13} which needs three parameters for two-piece balls and five parameters for wound balls. The parameters were found by fitting the equation of motion to force/time data for balls impacting on a rigid metal block.

Ujihashi\textsuperscript{14} also used viscoelastic parameters to model golf ball impacts on a steel block and found that the predicted force/time curves agreed well with experiments. It is apparent that the major problem with viscoelastic models, or indeed any modelling, is the appropriate identification of the viscoelastic parameters and contents. This has only really been addressed by Johnson and Lieberman\textsuperscript{10,11,13} through fitting of their theoretical curves to experimentally obtained force/time curves.

The impact of golf balls with turf were modelled by Haake\textsuperscript{15} using three viscoelastic parameters. The parameters were estimated using results from impacts of golf balls on golf greens. The model was used to predict the motion of different golfer’s shots on golf greens and their subsequent travel although the total travel of the ball was not validated. Finite element models have also been used for modelling ball impacts. Iwata et al.\textsuperscript{16} used Dyna 3D to model the impact of a graphite club head with an experimental golf ball projectile. The club was divided into 1204 elements while the projectile consisted of 808 elements. The analysis concentrated upon the design of the club rather than the ball and ignored the role of the shaft. Mars\textsuperscript{17} also used Dyna 3D to demonstrate the ability of the software to model a golf ball entering a vertical hole. The same software package was used by Haake and Ward\textsuperscript{18,19} to model golf ball impacts with turf with limited success. Experimental verification of models is relatively simple on hard surfaces but becomes much more difficult on natural surfaces such as turf and this is addressed in the following section.

3. Experiments

Experimentation on ball impacts has inevitably followed the same path as the modelling of ball impacts. In many cases, the experimentation has been performed purely to verify models. Daish\textsuperscript{1} backed up his theoretical treatments of ball impacts and motion by recording impacts using high speed video. Very little of the results were verified, however, and the work takes on the form of a treatise rather than a justification.

Maw et al.\textsuperscript{10} used a series of air-table experiments to validate their theories. A flat disc-shaped puck was propelled on the air table by a heavy pendulum toward a clamped block of similar material. Stroboscope photography was used to record the pucks’ motion. High velocities could not be achieved and they assumed, therefore, that large local velocities at the point of impact, that is, a slow puck with a lot of spin impacting at an angle almost normal to the surface, was the same as a puck impacting with no spin at a fairly large angle to the surface.

Hutchings et al.\textsuperscript{7} projected hardened steel spheres of 9.5 mm diameter on ductile mild steel targets and the measured depth of the craters formed allowed them to verify their models of impact. Rickerby et al.\textsuperscript{9} found that a good description of the contact area between the sphere and the surface was required to gain good agreement between the experiments and their model.

Tatara\textsuperscript{20} tried to verify some parts of the Hertz theory in relation to tennis ball impacts by filming impacts of balls with each other and the ground at 5,000 frames per second. Tatara found that the results could not be attributed to the Hertz theory above and suggested that another process was also taking place. Interestingly, Tatara noticed distinct oscillations of the ball after impact.

Thorpe and Canaway\textsuperscript{21} used photometric methods to measure the pace and bounce of tennis balls on different court surfaces. A pneumatic ball projector was used to project tennis balls with zero
spin while a mechanical ball projector with two counter-rotating drums and a vertical peg at the mouth produced impacts with spin. The spin was given to the ball when it hit the construction on leaving the ball projector. This was not particularly precise, however, and gave no continuous variation of the spin. A cine camera running at 64 frames per second recorded the impacts. Correlation and multiple regression techniques were used to study the relationships between velocity, angle and spin without adding to the mechanical understanding of the impact. It was concluded that an unmeasured (or unexplained) process was occurring during impact which was out of the scope of the project.

Gobush measured the normal and tangential forces acting on golf balls during impact with a rigid plate. A three component force transducer was mounted on a rectangular steel block weighing 100 lbs. Balls with different constructions were projected at the block at different angles and forces recorded with time. It was found that the tangential force reversed its direction just before the end of impact due to the internal ‘winding up’ of the ball. This may cause the backspin of the ball to be lower than might otherwise have been expected. The characteristic force/time curves were used later by other researchers for model verification, notably Lieberman and Johnson.

Ujihashi performed similar experiments on normal impacts using a single accelerometer behind a cylindrical steel target of 30 mm diameter. The deflection of the ball was also measured using a high speed camera allowing force deflection/curves to be determined for the ball. The hysteresis loses in the ball were found to be much larger at 47 ms\(^{-1}\) than at 30 ms\(^{-1}\). There have not been many studies of ball impacts on turf apart from Thorpe and Canaway and Tatara. An attempt has been made to address this fact by the author which is described in the following two sections.

4. Golf ball impacts on turf

A three year study, sponsored by the Royal and Ancient Golf Club of St Andrews, was carried out to determine the important parameters in golf ball impacts with turf. A ball projection apparatus was designed to project spinning golf balls with up to 650 rads\(^{-1}\) backspin and 30 ms\(^{-1}\) velocity directly onto the turf. The path of the ball was recorded using stroboscope photography and the experiments carried out in a large darkened enclosure on golf greens in-situ (Figure 1). Over one thousand impacts were recorded on eighteen golf greens around the UK and related to turf characteristics which were measured simultaneously. These characteristics included ball bounce, Clegg impact hardness and measurement of soil moisture content and soil particle size from cores removed from the turf and analysed later. The ground cover and species composition was also recorded using an optical frame point quadrat. Figure 2 shows a typical impact of a golf ball impact on turf. Each image is composed of a double exposure in which the first image is a grid placed in the plane of impact. The grid is used as a frame of reference from which to analyse the ball’s motion. The second exposure is of the stroboscope image of the ball’s path before and after impact. In this case the ball enters from the right at 23.6 ms\(^{-1}\) at an angle of 50° to the horizontal and with 125 rads\(^{-1}\) backspin. The ball slows down after impact to 3.8 ms\(^{-1}\) and rebounds at 72° to the horizontal. The backspin has been modified to topspin of 120 rads\(^{-1}\).

A graph of rebound spin versus rebound velocity for all impacts is shown in Figure 3. It can be seen
that most impacts lie on the line indicating rebound velocity/radius or the spin required for the ball to roll off the surface. Most impacts, regardless of surface, initial spin, speed or angle tended, therefore, to roll off the surface. Only those with backspin over approximately 300 rads⁻¹ or at shallow angles tended to slip throughout impact.

Figure 4 shows the ratio of outgoing to incident velocity, ratio of the tangents of the outgoing angle to the incident angle, the change of spin during impact and the depth of the pitchmark created for ‘soft’ and a ‘hard’ green against increasing incident backspin.

The incident angle and velocity were set at 45° and 22 ms⁻¹ respectively but variation in actual projected values gave mean and standard deviations of 45.5±1.0° and 22.0±0.8 ms⁻¹.

The variation in the parameters of the projected ball requires that the outgoing velocity and angle be normalised to the incoming values and that the change of spin is analysed rather than absolute values.

The hardness of the green was measured using a Clegg Impact Hardness tester which measures the peak deceleration of an impact hammer on turf dropped from a height of 300 mm. The average and standard deviation peak deceleration for all the greens tested in this study was 563±135 ms⁻². The ‘soft’ green was measured to have a peak deceleration of 284 ms⁻² and the ‘hard’ green a peak deceleration of 716 ms⁻². Each value of peak deceleration was an average of twenty readings across the turf in the region of testing.

It can be seen in Figure 4 that, for the soft green, an increase in the backspin of the incoming ball causes the rebound velocity to decrease, the rebound angle to increase and the change of spin during impact to increase. The depth of the pitchmark stays approximately constant at 3 to 7 mm. The trends are mostly the same for a hard green but it appears that something unusual happens at large backspins. There are some impacts above incident backspins of 400 rads⁻¹ which rebound faster and at a lower angle than might be expected. Above 400 rads⁻¹ the change in spin during impact decreases indicating that the impacts are very close to the situation where backspin may be retained after impact. This indicates that the ball has slipped throughout impact. The pitchmark depths for the hard green are shallower than for the soft green and it was noted that they were also longer in length. Although the number of greens studied was not statistically large enough (18), it was found that the turfs on which balls tended to slip through-
Fig. 4  Rebound velocity, angle, spin and depth of the pitchmark for golf balls impacting at 22 ms\(^{-1}\), at 45° to the horizontal and with increasing backspin, for a soft and a hard green
out impact and retain backspin were those that were relatively hard and were characterised by free draining sandy soils and grasses predominantly of a Bent/Fescue mixture. Conversely, the turfs that caused the ball to rebound with rolling spin and did not allow spin tended to be characterised by less will drained clay type soils with a dominance of Poa Annua (Annual Meadow Grass) on them.

A model of the ball/turf impact was formulated which considered the interaction of the ball with the turf as being viscoelastic. The assumptions of the model were that (1) the ball was rigid and elastic in comparison to the turf; (2) the turf consisted of two layers, the first comprising the grass and roots sand, the second consisting of the lower soil structure; (3) the upper layer was considered as a spring and a damper while the lower layer was considered as a damper only; (4) the forces were proportional to the contact area of the ball with the turf\(^{(15)}\). The model predicted correctly the trends and the threshold indicating that the concept of a double layer turf system is correct. The viscoelastic parameters, however, are difficult to determine for turf and the absolute numbers from the model are unlikely to be correct. As stated earlier further work on the dynamic stress-strain characteristics of turf (and golf balls) needs to be carried out.

5. Tennis ball impacts on synthetic turf

A short study was carried out to find the important parameters for tennis ball impacts with synthetic turf\(^{(17,18)}\). Although there is plenty of anecdotal evidence on speeds in the modern game through the use of speed guns, there is little data on the spins applied to tennis balls.

A short study was carried out with an amateur player to determine velocities, spins and angles from a forehand shot. It was found that a typical shot rebounded from the racket at 23 ms\(^{-1}\) at 9° to the horizontal in an upwards direction and with 51 rads\(^{-1}\) topspin. Using a trajectory model this gave an estimate of the velocity, angle and spin of a ball impacting with the surface at the end of the forehand shot i.e. 16 ms\(^{-1}\), 22.4° to the horizontal and 30 rads\(^{-1}\) topspin. Using this information a set of velocities, angles and spins were studied using three different types of ball (Table 1).

The last column indicates the static deflections of the balls subjected to an 80 N compressive load. The stiffest ball was the unpressurised ball while the least stiff was the low pressure ball.

The three types of ball consisted of a relatively stiff unpressurised ball and two balls of similar construction but different internal pressures.

These balls were projected directly onto a synthetic grass system using a Jugs ball projection machine. The synthetic surface consisted of artificial grass of 10 mm depth adhered in a vertical orientation to a plastic backing. In normal play the surface is laid loose over concrete and is filled with sand and kept in place by the weight of the sand. A 0.25 m\(^2\) area of turf was placed on a concrete block and adhered at its edges to simulate the boundary conditions of the turf in play. Six hundred millilitres of dried silica sand was brushed into the surface and the whole turf system surrounded by plastic buffers to catch any sand that might be ejected. After each ball impact the surface was brushed and after 9 impacts any ejected sand was

<table>
<thead>
<tr>
<th>Velocity (ms(^{-1}))</th>
<th>Angle (°)</th>
<th>Spin (rads(^{-1}))</th>
<th>Ball</th>
<th>Compression (mm) (80 N static load)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>20</td>
<td>+100</td>
<td>Unpressurised</td>
<td>4.6 - 4.8</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>0</td>
<td>Low pressure</td>
<td>6.9 - 7.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-100</td>
<td>High pressure</td>
<td>6.4 - 6.5</td>
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</tbody>
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returned to the turf. The motion of the tennis balls on impact with the turf were recorded using an Ektapro high speed video running at 1 000 frames per second using a synchronised stroboscope to provide a short 10 µs flash to reduce image blur.

The results of the impacts are shown in Figure 5. The graph represent balls projected from the left hitting the turf at (0, 0) and rebounding to the right. The length of each line represents the velocity of the ball for that particular set of parameters and each line is an average of three impacts. Figure 5 shows results for impacts at 20 ms⁻¹ at 20 and 25° degrees to the horizontal for the three ball constructions studied.

To explain the graphs, consider Figure 5(b) which shows impacts at 20 ms⁻¹ at 20 and 25° to the horizontal and with no spin (note that the vertical scale has been exaggerated). To the right of (0, 0),

![Graph](image)

**Fig. 5** Impact results for unpressurised, low pressure and high pressure tennis balls. The ball enters from the left at 20 ms⁻¹ at 20 or 25° to the horizontal and rebounds to the right. The length of each line represents the velocity of the rebounding ball.
i.e. for the rebounding ball, the solid line represents the high pressure ball, the dashed line the low pressure ball and the dotted line the unpressurised ball. Figure 5(b) indicates that for impacts at 20 ms\(^{-1}\) and with no spin, there is little difference between the balls. If backspin is added, however, Figure 5(a), it can be seen that the balls tend to rebound faster (since the lines are longer) and shallower. The unpressurised ball rebounds shallower and faster than either the low pressure or intermediate ball at both 20 and 25°.

If topspin is added to the impact, Figure 5(c), the balls tend to rebound steeper than with no spin. Again, the unpressurised balls rebound at the shallowest angle and with the largest velocity.

If Figure 5 is studied closely it can be seen that an increase in incoming velocity increases the rebound speed and an increase in the angle decreases the rebound speed. There is certainly far more energy lost at 25 ms\(^{-1}\) than at 20 ms\(^{-1}\) as can be seen by the coefficient of restitution in Table 2. An increase in velocity reduces the coefficient of restitution \(e = -\frac{v_{\text{rebound}}}{v_{\text{impact}}}\) when either backspin or topspin are used. The non-spinning impacts show no conclusive variation. Table 2 indicates more clearly that the unpressurised ball rebounds faster than both the high pressure and low pressure balls at the lower speed of 20 ms\(^{-1}\). There is also an indication that the effect of the addition of backspin or topspin is more marked with the unpressurised ball than either of the pressurised balls.

The amount of spin of the ball after impact is an indication of slip and roll during the impact event. If the horizontal velocity of the ball matches the peripheral velocity of the ball due to its spin then it must be rolling. If the horizontal velocity is much faster or much slower than the peripheral velocity then the ball must slip. Table 3 shows the difference between the horizontal velocity of the ball and the peripheral velocity of the ball due to its spin calculated using \(u = v_{\text{rebound}}r\) where \(v_{\text{rebound}}\) is the spin of the ball after impact and \(u\) is the peripheral velocity of the ball after impact and assuming that \(r = 33\) mm. A positive value indicates that the ball as a whole was travelling faster across the surface at the end of impact than it was spinning with the result that at the contact between the ball and the surface it was slipping with the forces acting to increase the spin of the ball. A negative result indicates that the ball was “over-spinning” i.e. the peripheral velocity of the ball was faster than the velocity of the ball across the sur-

| Table 2 | Coefficient of restitution \(e = -\frac{v_{\text{rebound}}}{v_{\text{impact}}}\) for three different tennis balls at an impact angle of 20° on synthetic turf |
|---|---|---|---|---|---|---|---|---|
| Unpressurised Ball | Low Pressure Ball | High Pressure Ball |
| 20 ms\(^{-1}\) | 25 ms\(^{-1}\) | 20 ms\(^{-1}\) | 25 ms\(^{-1}\) | 20 ms\(^{-1}\) | 25 ms\(^{-1}\) |
| Backspin -100 rads\(^{-1}\) | 0.697 | 0.631 | 0.656 | 0.636 | 0.673 | 0.652 |
| No Spin | 0.646 | 0.645 | 0.633 | 0.634 | 0.634 | 0.667 |
| Topspin +100 rads\(^{-1}\) | 0.666 | 0.643 | 0.654 | 0.638 | 0.645 | 0.636 |

| Table 3 | Difference between the horizontal velocity after impact and the peripheral velocity of the ball after impact in ms\(^{-1}\) calculated using \(u = v_{\text{rebound}}r\) for three different balls at an impact angle of 20° on synthetic turf |
|---|---|---|---|---|---|---|
| Unpressurised Ball | Low Pressure Ball | High Pressure Ball |
| 20 ms\(^{-1}\) | 25 ms\(^{-1}\) | 20 ms\(^{-1}\) | 25 ms\(^{-1}\) | 20 ms\(^{-1}\) | 25 ms\(^{-1}\) |
| -100 rads\(^{-1}\) | 7.7 | 5.3 | 4.0 | 4.9 | 6.3 | 5.0 |
| Zero Spin | 3.3 | 3.3 | 0.8 | 1.4 | 1.5 | 3.3 |
| +100 rads\(^{-1}\) | 0.3 | 0.2 | -1.5 | -0.7 | 0.0 | -0.3 |
face. The main assumption in these calculations is that the ball was undeformed since the undeformed value of the radius used in the calculation. Since this was clearly not true during the majority of the impact, the results can only be indicators of the state of roll or slip at the end of the impact.

Table 3 shows that balls impacting at 20° and with no spin just slip at the end of impact. If 100 rads⁻¹ topspin is added then the peripheral and horizontal velocities are almost matched and the balls roll off the surface at the end of impact. If 100 rads⁻¹ of backspin is added then slip most certainly occurs at the end of impact. The unpressurised ball is much more likely to slip throughout impact than the pressurised ball and the low pressure ball shows the least amount of slip. The results in Tables 2 and 3 are only for impacts at 20°. The same trends however, are also shown at 25° although the results are not presented here.

The differences between the balls can be partly explained by the compression results in Table 1. In terms of deflection for a given load the unpressurised ball deflects least while the low pressure ball deflects most.

A hypothesis for the scenario that occurs during impact which explains the results is shown as follows. This requires that one understands that when slipping the coefficient of friction (the ratio of the forces parallel to the surface to the forces normal to the surface) is dynamic and lower in value than when the ball rolls. There is little relative slip when the ball rolls and the coefficient of friction is high. Deformation rapidly occurs as the ball slides across the surface and the large contact area and the downward momentum of the ball causes the tangential forces on the ball to increase causing topspin to develop. If these forces are large enough then the ball rolls off the surface at the end of impact.

If the ball slips throughout impact then the frictional forces are relatively low since the coefficient of friction is dynamic. Once the ball starts to roll, the coefficient of friction increases to its static value and the frictional forces increase. The frictional forces act to decrease the horizontal velocity and increase the topspin. A ball which slips throughout impact would be expected, therefore, to rebound relatively fast at a relatively low angle (since the horizontal velocity remains relatively high). Balls which would tend to do this are those which do not deform much during impact i.e. the unpressurised ball. This is indeed what is seen experimentally. The relatively soft pressurised balls, both low and high pressure, tend to rebound the steepest and tend to rebound the slowest. The balls deform more during impact giving a larger contact area and larger frictional forces. This makes them more likely to roll at the end of impact causing the low velocity and steep rebound. Conversely, the relatively stiff unpressurised ball deforms least and tends to slip throughout impact.

In Table 2 it can be seen that the unpressurised ball rebounds approximately 4% faster than the high pressure ball and 2% faster than the low pressure ball. Indications are that the rebound angles of the unpressurised ball are about 10% less than the high pressure ball although this is less easy to judge.

It is clear that a greater range of velocities, angles and spins need to be studied in relation to tennis ball impacts of turf. To do this, the ranges seen in play need to be measured. Details of contact areas, deformation of the ball and contact times would also aid in the understanding of what happens during impact. The compression of the ball has a large influence on what happens during impact and this undoubtedly will be studied in more detail in the future.

6. Final Discussion

The analysis of ball impacts on turf requires large numbers of impacts to ensure that the variation in the turf or the balls doesn't mask differences due to the variation in incoming velocity, angle, spin or ball construction. It is still possible, however, to find differences between turf given a standard ball or between balls given a standard turf. It is interesting to note that the extent of the slipping or rolling phases of the impact are important to the ball's post-impact motion. For golf balls on natural turf the ball will roll at the end of impact on a soft turf and slip at the end of impact on a hard turf.
The ball will rebound slowly and steeply and with topspin on the soft turf and relatively fast and shallow on the hard turf. In some cases backspin may be retained on the hard turf.

A similar effect is seen with tennis balls on synthetic turf. A hard ball such as an unpresurised ball rebounds faster and shallower than softer pressurised balls. The hard ball deforms less during impact tending to slip more while the pressurised balls deform more and rebound slower and at a steeper angle.

Most of the impact models and experiments reviewed consider ball impacts with implements or equipment. Viscoelastic models may prove the most useful when considering ball impacts with the turf. The critical factor, however, is the measurement of the strength of the turf in terms of strain rate and thus the velocity of impact. Without specific measurement of these turf parameters, the current models will be mere approximations.

References


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