In the present note, we study some properties of a finite group whose lattice of subgroups is lower semi-modular. We, however, use no result of the general theory of lattices.

I give my hearty thanks to Mr. M. Suzuki for his kind remarks and advices.

NOTATIONS: \( S_p(X) = S_p(X), H_p(X) = H_p(X), C(X), C_m(X), G(X) \) and \( g(X) \) denote a \( p \)-Sylow subgroup, a \( p \)-Sylow complement, the centre, the hypercentre, the commutator subgroup and \( \langle p \rangle \)-subgroup of a group \( X \) respectively; \( (X) \) may be often omitted.

\( \mathcal{N}_G(X) \) denotes the normalizer of a subgroup \( X \) in a group \( G \).

1. On the \( p \)-nilpotency.

DEFINITION 1. A finite group is called \( p \)-nilpotent if it has a normal \( p \)-Sylow complement.

PROPOSITION 1. Let \( G \) be a group whose order has at least three distinct prime factors and let \( P \) be one of them. Then \( G \) is \( p \)-nilpotent if every proper subgroup of \( G \) is so.

PROOF. Let \( G \) be a group which satisfies our condition. If \( G \) is not \( p \)-normal in GRUN's sense, there exist a \( p \)-subgroup \( P \) and a \( p \)-regular element \( A \) in \( G \) such that \( A \) induces a non-identical automorphism into \( P \), by virtue of a theorem of W. Burnside. Since \( P \cdot \{A\} \) is non-\( p \)-nilpotent, we have \( G = P \cdot \{A\} \). Let \( A = A_1A_2 \cdots A_r \) be the Sylow decomposition of \( A \). Then \( r \geq 2 \) by our condition.

Clearly \( G \neq P \cdot \{A\} \), whence \( P \cdot \{A_i\} = P \cdot \{A_j\} \). Therefore \( G = P \cdot \{A\} \) which is a contradiction. Hence \( G \) is \( p \)-normal. Now by a theorem of O. Grun, \( S_p(\theta/\sigma(\theta)) \cong S_p(\mathcal{N}(C(S_p))/\mathcal{N}(C(C(S_p)))) \) if \( G = G(S_p) \), since the latter is \( p \)-nilpotent by our condition.

If \( G = G(S_p) \), since the latter is \( p \)-nilpotent by our condition, \( S_p(\mathcal{N}(C(S_p))/\mathcal{N}(C(C(S_p)))) \cong \mathbb{Z} \) whence \( S_p(\mathcal{N}(C(S_p))/\mathcal{N}(C(C(S_p)))) \cong \mathbb{Z} \). Therefore, \( G = G(S_p) \) whence it is easily verified that \( G \) is \( p \)-nilpotent. If \( G = G(S_p) \) and \( S_p \neq C(S_p) \), then induction argument can be applied to \( G/G(S_p) \) and we can see that \( G/C(S_p) \) is \( p \)-nilpotent whence it is easily verified that \( G \) is \( p \)-nilpotent. Finally if \( G = G(S_p) \) and \( S_p = C(S_p) \), then there exists, by a theorem of I. Schur, one \( H_p \) in \( G \). Since \( G = S_p \cdot H_p \), by our condition, \( S_p \cdot S_p \cdot H_p = S_p \cdot H_p \) whence \( G = S_p \cdot H_p \). Therefore, of course, \( G \) is \( p \)-nilpotent.

PROPOSITION 2. Let \( G \) be a non-\( p \)-nilpotent group whose every proper subgroup is \( p \)-nilpotent. Then \( G = S_p \cdot S_t \) where \( S_p \) is normal, \( S_t = \{Q\} \) is cyclic, non-normal. And every proper subgroup of \( G \) is nilpotent. In particular it is soluble. The converse is also valid.

PROOF. Let \( G \) be a group which satisfies our condition. Follow the proof of PROPOSITION 1. First it is evident that the order of \( G \) is \( p \)-nilpotent by PROPOSITION 1. Therefore if \( G \) is not \( p \)-normal, then \( A = A_1 \), using the same notations as in the proof of PROPOSITION 1, and this proves PROPOSITION 2. Now assume that \( G \) is \( p \)-normal. Then \( \mathcal{N}(C(S_p)) = G \cdot S_t \) and if \( \mathcal{N}(C(S_p)) = G \), \( G \) is \( p \)-nilpotent, as is easily seen by virtue of the proof of PROPOSITION 1. If \( G = S_p \cdot S_t \), induction can be applied to \( G/C(S_p) \) and we can easily prove Proposition 2. Finally if \( C(S_p) = S_p \), then \( G = S_p \cdot S_t \) and if \( S_t = T \cdot U \) where \( T \) and \( U \subseteq S_t \), since \( G = S_p \cdot T \) and \( S_p \cdot T = S_p \cdot T \) and \( S_p \cdot U = S_p \cdot U \) whence \( G = S_p \cdot S_t \) which is a contradiction. Therefore \( S_t \) is cyclic and this proves PROPOSITION 2. The converse is obvious.

REMARK 1. Similar results as PROPOSITION 1 and 2 have been obtained by many authors, for instance, O. Schmid, D. Kol'yanowsky, S. Tschnikhin and K. Iwasawa. And our result is a slight modifi-
cation of theirs. But it seems to me that our formulation is a little more general and applicable than antecedents. (Cf. M. SUZUKI\(^{(2)}\)).

**PROPOSITION 3\(^{(0)}\).** A simple non-abelian group \(G\) has a proper subgroup which satisfies the condition in PROPOSITION 2 for every prime factor \(p\) of its order.

**PROOF.** Clearly \(G\) is not \(p\)-nilpotent. Therefore \(G\) has at least one non-\(p\)-nilpotent subgroup, for instance, \(G\) itself. Choose a minimal one of such subgroups. Then it is a group of PROPOSITION 2 and soluble. Therefore it does not coincide with \(G\).

**PROPOSITION 4\(^{(0)}\).** Let the order \(g\) of a group \(G\) have just \(n\) distinct prime factors. If \(G\) has at most \(n-1\) non-isomorphic proper non-nilpotent subgroups, \(G\) is soluble.

**PROOF.** Clearly we may assume that \(G\) is \(p\)-nilpotent for some \(p\) which is a prime factor of \(g\), as is easily seen by virtue of the proof of PROPOSITION 3. The \(p\)-Sylow complement has clearly at most \(n-2\) non-isomorphic proper non-nilpotent subgroups. Now for \(n=1\) \(G\) is nilpotent. Therefore we can easily prove PROPOSITION 4 by induction for \(n\).

2. **On \((C)\)-groups.**

**DEFINITION 2.** A finite group is called a \((C)\)-group if every maximal subgroup of any subgroup has a prime index.

**PROPOSITION 5.** A \((C)\)-group is \(p\)-nilpotent for the least prime factor \(p\) of its order. In particular, it is soluble.

**PROOF.** Let \(G\) be a group satisfying the condition in PROPOSITION 2. If \(G\) is a \((C)\)-group, then \(G\) has a subgroup \(H\) of index \(p\) as a maximal subgroup containing \(S_q\) and \(H\) is normal since \(p\) is the least. Since \(H\) is nilpotent, \(S_q(H)=S_q(G)\) is normal in \(H\) and therefore in \(G\). This is a contradiction.

**REMARK 2.** Groups of this type investigated first by O. ORE\(^{(2)}\) and complemented by G. ZAPPA\(^{(3)}\) and K. IWASAWA\(^{(4)}\).

We shall refer only to

**PROPOSITION 6.** A minimal normal subgroup of a \((C)\)-group has a prime order. Therefore, it has a chief series each of whose factors is of a prime order. The converse is also true.

**PROOF.** We shall show that \(G\) has a normal subgroup of order \(p\) where \(p\) is the maximum prime factor of the order of \(G\). If \(C(S_q)\neq H\) where \(C(S_q)\neq H\) has a normal subgroup of order \(p\) and clearly this is also normal in \(G\). Assume that \(C(S_q)\neq H\). Then \(G\) has a subgroup \(M\) of index \(p\) as a maximal subgroup containing \(H\). If \(S_q(M)=\mathfrak{e}\) then \(M\) has a normal subgroup of order \(p\) and this clearly is also normal in \(G\). Finally if \(S_q(M)=\mathfrak{e}\) then \(S_q(G)\) is normal in \(G\) and of order \(p\). The remainder and the converse are obvious.

3. **On \((LM)\)-groups.**

**DEFINITION 3.** A finite group \(G\) is called an \((LM)\)-group if every intersection of two distinct maximal subgroups of any subgroup is maximal respectively in such two maximal subgroups.

**PROPOSITION 7.** An \((LM)\)-group is \(p\)-nilpotent for the least prime factor \(p\) of its order. In particular, it is soluble.

**PROOF.** Let \(G\) be a group as in PROPOSITION 2. Assume that \(G\) is an \((LM)\)-group and we kick out a contradiction. To do this we use an induction argument. If \(S_p\) is not minimal normal we take such \(P\) contained in \(S_p\) and consider \(G/T\). Then a contradiction easily tumbles out. Hence we may assume that \(S_p\) is minimal normal. Further if \(S_q\) is of order \(q\), a maximal subgroup \(T\) of \(S_q\) is normal in \(G\). If we observe \(G/T\), a contradiction easily tumbles out. Hence we may assume that \(S_q\) is not of order \(q\). Then \(S_q\) and \(S_t\) are maximal in \(G\) and obviously \(S_q(S_t)=\mathfrak{e}\). Since \(G\) is assumed to be an \((LM)\)-group, \(S_p\) must be of order \(p\) and \(S_q\) is normal in \(G\) since \(p<q\). This is a contradiction.

**PROPOSITION 8.** An \((LM)\)-group is a \((C)\)-group. The converse is not true.

**PROOF.** Assume that every proper subgroup of \(G\) is a \((C)\)-group. We show first that the number of prime factors of \(G/M\) is invariant by the choice of maximal subgroup \(M\). In fact, let \(N\) be another maximal
Hence we may assume that $\theta \in S,$ or $\Gamma_{S,L} \subseteq \Lambda_{S,L} \subseteq \Lambda_{S,L}^3 \subseteq \Lambda_{S,L}^3$ whence the assertion is obvious. If there exists no such $N$, then $G$ is cyclic and the assertion is trivial. Now since $G$ is soluble, $G:M$ is prime and $G$ is $(C)$. Thus induction completes our proof.

PROPOSITION 9. Let the order of a group $G$ have the following prime factor decomposition: $pq^2$ where $p > q > 3$. Then $G$ is a $(C)$-group, except the case that $G \cong \mathbb{Z}_4$. Further $G$ is a $(C)$-group and not an $(LM)$-group if and only if $H_1$ induces an automorphism of order $p^2$ or $p^2$ into $S_1$.

PROOF. If $S_1$ is not normal then $M(S_1) = S_1$, therefore, $\bar{e} = 1 \mod p$ where $p > 3$, $q = 2$ and $G \cong \mathbb{Z}_4$. Hence $S_1$ is normal if not $\bar{e} = 1 \mod p$. This proves our first assertion. Assume that $G$ is not isomorphic to $\mathbb{Z}_4$. Now $\theta$ is nilpotent by a theorem of 0.ORE(0) and if $\theta \neq 1$ or more generally $G$ is not irreducible, then $G$ is clearly an $(LM)$-group. Further assume that $G$ is not an $(LM)$-group. Therefore $S_1 = \{e\}$. Then $H_1$ is cyclic and is considered as a group of automorphisms of $S_1$, as is easily seen. Conversely, assume that this is the case. Putting $S_1 = \{A\}$ we have $H_1 \cap H_1 = \{e\}$. Therefore $G$ is not an $(LM)$-group.

REMARK 3. PROPOSITION 9 was suggested to the author by Mr. S. SATO and I give him my hearty thanks. (Cf. S. SATO(0))

PROPOSITION 10. Assume that the order of a group $G$ have the following prime factor decomposition: $pq^2$ where $p > q > 3$. Then $G$ is an $(LM)$-group, if $S_1S_2 = S_1 \times S_2$ or $S_1S_2 = S_1 \times S_2$.

PROOF. Assume that the assertion is true for all groups of smaller order. Now $G$ is nilpotent by a theorem of 0.ORE(0) and if $\theta \neq 1$ or $S_1(\theta)$ is distinct from $e$. Further if $S_1(\theta) = S_1$ or $S_2(\theta) = S_2$ then $S_1S_2 = S_1 \times S_2$ or $S_1S_2 = S_1 \times S_2$ and if $S_2(\theta) \neq S_2$ then induction can be applied to $G/S_2$ and $S_1S_2 = S_1 \times S_2$. Since $S_1(\theta)$ and $S_1(\theta)$ are distinct, $S_1(\theta) \cap S_1(\theta) = e$ whence we may assume that $\theta \neq S_1$. Hence $S_1$ is abelian. Putting $S_1 = \{A\}$, we consider $H_1 \cap H_1$ then this contains $S_1$ or $S_2$, whence we can easily see that $S_1$ or $S_2$ is normal in $G$. Therefore $S_1S_2 = S_1 \times S_2$. Thus induction completes the proof.

PROPOSITION 11. Let $G$ be a $(C)$-group whose order $G$ has at least four distinct prime factors. If every proper subgroup is an $(LM)$-group, then $G$ is so, too.

PROOF. Let $M$ and $N$ be any two distinct maximal subgroups of $G$. We have to show that $M \cap N$ is maximal in $M$ and $N$. Now if $M$ and $N$ are not conjugate $M \cap N = G$, by a theorem of OORE, whence we can easily see that $G:M = N:M \cap N$. Further, assume that $G$ is not an $(LM)$-group, $G$ is distinct from $e$. Further if $G$ is normal in $G$ and not an $(LM)$-group, then since $G$ is soluble, $G:M = N:M \cap N$ prime. Therefore $M \cap N$ is clearly maximal in $M$ and $N$. Hence we may assume that $N = M^*$ for some element $x$ of $G$. Now let $g$ have the following prime factor decompositions:

$$
\begin{align*}
1 < p_1 < p_2 < \ldots < p_r
\end{align*}
$$

If $G/M^* = F_1$, then $G/S_1 = \not\subseteq M/S_1$ and $S_1 = \not\subseteq F_1$. Therefore $G = M^*$. Now assume that the assertion is true for all groups of smaller order. If $e < x$ then $S_1(M)$ is normal in $G$ and we can apply induction argument to $G/S_1(M) \equiv M/S_1(M)$ and $N/S_1(M)$.

Then we can see that $G/S_1$ is an $(LM)$-group and the assertion clearly holds. Hence we may assume that $e = 1$ and put $S_1 = \{A\}$ and $x = A$. Now PROPOSITION 10 can be applied to this case: We have $S_1^A = S_1$ except at most one $k (1 < k < r)$, where $M = S_2, \ldots, S_r$. Finally consider $S_1 \cap S_k$. Since $S_1(\theta) \subseteq S_1(\theta)$ is an $(LM)$-group, $S_1(\theta) \cap S_k$ is maximal in $S_k$ and $S_1(\theta)$. Hence $M \cap N$ is clearly maximal in $M$ and $N$. Therefore PROPOSITION 11 has been completely proved by induction.

PROPOSITION 12. Let $G$ be a soluble group. Then $G$ is a $(C)$- or an $(LM)$-group according to that $G/C_G$ is a $(C)$- or an $(LM)$-group. The converse is also true.

PROOF. First assume that $G/C_G$ is a $(C)$-group. We use induction for the length of the upper central series. Then we may assume that $C_G = C^G$. If $M$ is any maximal subgroup of $G$ and $G/C_G$ is a $(C)$-group, $G:M = C^G$. If $M^G$
then $M$ is normal in $G$ and obviously we have $G/M = \text{prime}$. Next we assume that $G/C_\beta$ is an (LM)-group. As above we may assume that $C = e$. Let $M$ and $N$ be any two distinct maximal subgroups of $G$. If $M$ and $N$ are not conjugate, we have $MN = NM = G$ by a theorem of O.ORE \cite{1} and easily see $G:M = N:M = G$ prime and $G:MN$ is normal in $G$. Therefore $M \cap N$ is clearly maximal in $M$ and $N$. Hence we may assume that $M$ and $N$ are conjugate. Then $M$ and therefore $N \supset C$, since if not we can easily see that $M$ is normal and $M = N$ which is a contradiction. Hence $G/C \supseteq C$ and $N/C$ is an (LM)-group, $M \cap N$ is clearly maximal in $M$ and $N$. Therefore induction proves PROPOSITION 12. The converse is trivial.

PROPOSITION 13. Let $G$ be a (C)- but non-(LM)-group whose every proper subgroup is an (LM)-group. Then $G$ has a homomorphic image which is a group as in PROPOSITION 9.

PROOF. Follow the proof of PROPOSITION 11. Let $G$ be a group which satisfies our condition. We may replace $G/C_w$ by $G$ by virtue of PROPOSITION 11; therefore we may assume that $C = e$. Now the order of $C$ has the following prime factor decomposition:

$$p^a q^b r^c s^d,$$

and $G$ has a homomorphic image of order $p^a q^b r^c s^d$ or $p^a r^c s^d$ as is easily seen in virtue of the proof of PROPOSITION 11. Therefore we may assume that the order of $G$ is $p^a q^b r^c s^d$ or $p^a r^c s^d$. Now $G/H(\beta)$ satisfies the same condition that $G$ does. Therefore we may assume that $G \supseteq S_\beta$. Hence $H_\beta$ is abelian. Now put $S_\beta = \{A\}$ and let $K$ be a maximal subgroup of $H_\beta$, and we consider $K \cap K_\beta$. Since $S_\beta K$ is an (LM)-group, $K \cap K_\beta$ is maximal in $K$. On the other hand, it is contained in $C$. Therefore $K \cap K_\beta = e$. Therefore $H_{\beta}$ is of order $r^c s^d$ or $r^c s^d$ and this completes the proof of PROPOSITION 13.

Next we shall apply the method, by which HALL \cite{2} studied complemented (C)-groups, to general (LM)-groups. And this is proposed by Mr. N. SUZUKI.

PROPOSITION 14. Let $G = G_1 \times G_2$ be a solvable group. Every maximal subgroup $M$, such that $M \not\supseteq G_1$ and $G_2$, is normal.

PROOF. Assume that the assertion is true for all groups of smaller order. And we shall prove PROPOSITION 14 by induction. Now if $M \supseteq G_2 = e$ and $M \cap G_1 = e$, then $G_1 \supseteq G_1 / G_2 = G_2$, and $G_2 = e$, where $p$ is a prime factor of the order of $G$. Therefore $G : e = G_1 : e = G_2 / G_1 = e$, and, in particular, $G_2$ is an $p$-group. Hence $M$ is obviously normal in $G$. Then we may assume that $M \cap G_1 = N \supseteq e$. Since $M \supseteq N \supseteq M \cap G_1 = N \supseteq e$. Since $N \supseteq G_1$, or $N \supseteq G_2$, or $N \supseteq G_1$, or $N \supseteq G_2$. Hence $M$ is normal in $G$. Therefore we can apply induction to $G/N \supseteq G_1 / N \supseteq G_2 / N$. Hence $M$ is normal in $G$. And induction completes the proof of PROPOSITION 14.

PROPOSITION 15. If $G_1$ and $G_2$ are (LM)-groups, then $G = G_1 \times G_2$ is so, too.

PROOF. Let the assertion be secured for groups of smaller order. And we shall prove PROPOSITION 15 by induction argument.

First it is trivial that $G$ is a (C)-group by PROPOSITION 6. Let $M$ and $N$ be any two distinct maximal subgroups of $G$. If $M$ and $N$ are not conjugate, then $MN = NM = G$ by a theorem of 0.ORE \cite{1}. Hence $G : M = N : M = G : MN$ are prime and $G : N = N : M = G : MN$ prime. If $M$ and $N$ are conjugate, then $M$ and $N$ contain $G_1$ or $G_2$, whence it is clear that $M \cap N$ is maximal in $M$ and $N$. Now it is sufficient to show that every proper subgroup of $G$ is an (LM)-group. So we shall assume that there exists at least one non-(LM) subgroup in $G$; let $H$ be a minimal one. Then every proper subgroup of $H$ is an (LM)-group. Now we may assume that $G : H = G_1 : H = G$. For if not, say $G : H \neq G$ then induction can be applied to $G : H = G_1 : (G_1 \cap G : H)$ and we see that $G : H$ is an (LM)-group. Then, of course, $H$ is (LM) which is a contradiction. Further we may assume that any minimal normal subgroup of $G$ which is contained in $G_1$ or $G_2$ is contained in $H$. For if not, it is evident that $N(H \cap L)$ is normal in $G$. Since $H \cap L$ is distinct from $L$ and since $L$ is minimal, $H \cap L = e$. Then induction can be applied to $G : H = G_1 : G : H \cap L \cong H$ and we see that $G : H$ is (LM). Then, of course, $H \cap L \cong H$ is (LM) which
is a contradiction. In particular H contains a minimal normal subgroup P which is contained in G or G2, say G2 of order P, where P is the maximum prime factor of the order of G. The existence of P is secured since G is a (C)-group. Now as is easily seen in virtue of the proof of PROPOSITION 11 there exists a maximal subgroup M of index P, in H such that the intersection M \cap H is not maximal in M or M
 is a suitable element h of H. Now we may assume that M contains no minimal normal subgroup which is contained in G or G2. For if not, induction can be applied and we see that M\cap H is maximal in M and M
 which is a contradiction. In particular, M does not contain P ,. Then G2M=G2P, M=G2H=G . Similarly G2M=G . Now consider G2 \cap M, then T (G2 \cap M) \not= M and G2 \cap T (G2 \cap M) \not= G2. Then as is easily seen that H \cap T (G2 \cap M) = P and H \cap G2 = P2 where P2 is a minimal normal subgroup which is contained in G2 of order P . Therefore H = P2M = P2 \cdot M , Since H \not= P2H, (M) , P2H, (M) is an (LM)-group. Then S2 = (H, (M)) \cap P2 except at most one i by PROPOSITION 10. Then as in the proof of PROPOSITION 11 it is easily seen that M \cap P2 is maximal in M which is a contradiction. Therefore induction completes the proof of PROPOSITION 15.

Lastly we shall analyse a structure of fully irreducible (LM)-groups. Since it is evident that a p-group belongs to this class if and only if its centrum is cyclic, we shall treat in the following only non-p-groups. We however, contrary to Hall's case, have not succeeded in writing out a structure of such groups.

Let G be a fully irreducible (LM)-group. Then since G is a (C)-group, \( \Theta \) is nilpotent by a theorem of ORS[11]. Therefore \( \Theta \subseteq S1 \) by our assumption where P1 is the largest prime factor of the order of G and hence H1 is abelian. Let \( \Omega(C(S1)) \) be a subgroup of C(S1) which is consisted by all elements of order P1 of C(S1) and consider \( \Omega(C(S1)).H1 = \cdot \cdot \cdot . \) Then it is easily verified that \( \Omega(C(S1)) \) is of order \( \Theta \) since if not G has at least two minimal normal subgroups. Therefore C(S1) is cyclic and S1 is fully irreducible. Further it is evident that H1 is considered as a group of some automorphisms of S1 and that every prime factor of the order of H1 satisfies the condition \( p1 \equiv 1 \pmod q \). We have used no fact that G is an (LM)-group in above observation.

PROPOSITION 15. Let G be an (LM)-group and H be its proper subgroup. If M is a maximal subgroup of G, then \( M \not\subseteq H \) or \( H \cap M \) is maximal in H . In particular \( \Phi(H) \subseteq \Phi(G) \).

PROOF. Let N be a maximal subgroup of G which contains H. If N = M then M \not\subseteq H . If N \not\subseteq M , then N \cap M is maximal in M and N since G is an (LM)-group. Now induction can be applied to N , H and N \cap M and we can see that N \cap M \subseteq H . Thus induction proves PROPOSITION 15.

Again let G be a fully irreducible (LM)-group. Since \( \Phi(G) \) is nilpotent, \( \Phi(G) \subseteq S1 \) from our assumption. Therefore \( \Phi(H) \subseteq S1 \) and H1 is a direct product of elementary abelian \( \pi \)-groups where \( \pi \) runs all the prime factors of the order of H. Finally let \( S = T0 \rightarrow T1 \rightarrow \cdot \cdot \cdot \rightarrow Tn = e \) be a part of principal series of G . Then it is easily verified that H1 induces a group of automorphism of at most prime order into each \( T \), \( (i = 0, \cdot \cdot \cdot , \pi -1) \). Conversely such a group is evidently a fully irreducible (LM)-group. Such a characterization, however, is not constructive at all, we think.

EXAMPLE. Let \( p \) be a prime such that \( P - 1 = \pi, \cdot \cdot \cdot , \pi, \cdot \cdot \cdot , \pi \) where \( \pi, \cdot \cdot \cdot , \pi \) are primes and \( r \) is a positive integer. Let S be a \( P \)-group of order \( p^{2rn} \) defined by following relations:

\[ [A_{i-1}, A_{i}] = A_{i} \text{ for } i = 0, \cdot \cdot \cdot , n, \]

\[ [A_{i+1}, A_{i}] = e \text{ for } (k_i) \equiv (2i - 1, 2i - 1) \text{ or } (2i, 2i - 1) \] and \( A_{j} = e \). Then, as is easily verified, S is fully irreducible and of class 2. Denote by \( T \) the subgroup which is generated by \( A_{2i-1} \) and \( A_{2i} \) for \( i = 1, \cdot \cdot \cdot , n \). Then, as is easily verified, \( T \) has a cyclic group \( Q = \{ \Theta \} \) of prime order \( \pi \) as a group of automorphisms such that \( A_{2i-1} = A_{2i} \), \( A_{2i} = A_{2i} \), and \( A_{2i} \).
and $x_1 x_p^4 \equiv x_1 x_p (\mod p)$. Let $H = Q_1 \times Q_2 \times \cdots \times Q_n$ be a group of automorphisms of $S$ where $Q_k$ induces an automorphism of order $q_k$ into $T_k$ in the same manner as above and does an identical automorphism into $T_k$ with $l \neq k$.

Let $G = S \cdot H$ be a holomorph of $S$ by $H$. Then, as is easily verified, $G$ of order $p^{m+1} \cdot \cdots \cdot q_n$ is a fully irreducible $(LM)$-group.

(*) Received December 29, 1950.

(**) I have obtained the following results during 1947-1948. After I had accomplished this work, it was reported that Mr. A. JOHNES had studied groups of similar type. Yet his proof is not communicated to me.

2) l.c. (1).
3) l.c. (1).
4) l.c. (1).
11) l.c. (10).
15) l.c. (1).
16) l.c. (12).
18) l.c. (12).
19) l.c. (12).
20) l.c. (12).
22) l.c. (12).
23) l.c. (12).