The Experimental Stress – Strain Rate Relationship of Granular Materials †

Shinichi Yuu and Shigenori Kamimura

Dep. of Mech. Eng., Kyushu Institute of Technology*

Abstract

The flow of granular materials is important in the field of material handling. However, there are many unknown factors associated with the analysis of these flows. Constitutive relations (stress and strain rate relations) are needed to describe the flow dynamics of granular materials. We experimentally obtained the constitutive relations of granular materials by using a chute-type shear apparatus. The chute in which the granular material is contained and covered by a top board, can be moved on straight rails, while the top board is loaded vertically and kept immobile. The shear stresses, normal stresses and strain rates measured by using the chute-type shear apparatus give the constitutive relations of the granular materials. The experimental results describe well the flow characteristics (e.g. the stress ratio of shear to normal stress is proportional to the strain rate, etc.) under the condition in which the friction forces between granular materials dominate.

1. Introduction

In the handling of granular materials, flow is a phenomenon of great importance. However, compared with continuum such as water or air, very little is known at present about the flow mechanism of granular materials. This is, of course, because the handling of granular materials is much more complex compared with that of continuum but it is believed that one of the reasons for this is that flow dynamics have not been applied to granular materials as liquids. This is to say that the velocity and strain rate of granular materials, and also the relation between the stress and strain rate and such have not been correctly introduced into the dynamics of granular materials. Especially, in clarifying the flow of granular materials through flow dynamics, the constitutive relations (relation between stress and strain ratio etc.) must be clarified.

Experimental research in this connection has been conducted in the past but it is believed that a constitutive relation which properly expresses the flow of granular materials and which is simple and can be applied in practical uses has not yet been obtained. For example, the experiment by Savage et al¹⁾ in which shear flow is applied to a granular material

inside an annular cell and the constitutive relation obtained involves the following problem. The annular cell used in their experiment is divided into a lower disk which rotates and an upper disk which is kept at rest to cause shearing while the range of the granular material layer moving at the same speed as the lower disk, namely the range which is not affected by shearing is defined by the condition $\tau/\sigma < \tan \phi$ (τ : shear stress, σ : vertical stress, ϕ : internal frictional angle) and it is assumed that the velocity gradient in the area above this becomes linear. However it is difficult to determine the area of $\tau/\sigma < \phi$ accurately since it is impossible to measure τ/σ inside the apparatus. Moreover, with the annular cell continuously rotating and the granular material sealed therein, it is impossible to experimentally define the area which is not affected by the shearing effect. Since the existence of this area greatly affects the velocity gradient value inside the annular cell, it is difficult to obtain an accurate velocity gradient. Therefore, we prepared an apparatus consisting of a cell 3000 mm long and having a rectangular cross section (60 mm × 80 mm). This was packed with granular material and driven linearly and a shearing flow was induced in the material. The stress applied to the granular material and the velocity gradient were measured and from these results, the constitutive relation of the granular material was obtained. These show a non-linearity of the granular

* Sensui-cho 1-1, Kita-Kyushu 804, Japan

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material flow characteristics. Furthermore, the various characteristics of the granular material would be clearly expressed.

2. Experiment apparatus and methods

Fig. 1 shows the apparatus that induces shear flow in a granular material and measures the shearing stress and strain rates. The container which is packed with granular material is 3000 mm long, 60 mm high, and 80 mm wide with a dolly provided under the container for free movement on rails in the lengthwise direction. A wire was tied to this apparatus and pulled by means of a motor which provides traction. For granular material specimens, circular pellets ($D_p = 2.82$ mm), colored pellets ($D_p = 2.87$ mm), and ceramic balls ($D_p = 3.42$ mm) were used. The container was packed with granular material as shown in Fig. 1 then a top press board and weights (vertical load) were installed. The top press board of 2000 mm in length, 5 mm in thickness and 60 mm in width was provided with a support plate on its top surface to form a concave shape and prevent distortion of the press board. Also to prevent the press board from moving widthwise, smooth moving

wheels were attached at a height precluding contact with the particles to control the movement of the board. In addition, a top press board with a width of 78 mm was provided for use in experiments requiring observation from the side. The top press board was fastened to a load cell to prevent motion in the direction of overall movement of the container. The shear strength generated at the time of shear flow was measured and the shear stress was obtained. The measured results were recorded using a pen recorder. An example is shown in Fig. 2. Point (a) is the starting point and when the top press is set and the wire pulled, the load increases up to (b) due to the friction force between the granular materials. Feeding of the recording paper starts at (b) and the motor is started at (c). The load increases, yielding occurs at point (d), the container starts moving and shearing force is generated by the flow of granular material. When the motor is stopped at (e), the container stops, the shearing force due to the flow of granular material is relieved and the load decreases up to (f). When the top press board is removed at (f), the yield stress disappears and the force returns to (g). The value of the straight line (h) – (i) averaged over the loads between

Fig. 1 Chute type shear apparatus

Fig. 2 Typical chart recording of output from load cell used to determine shear stress generated in sheared granular material

(d) – (e) was used as the shearing force. Also the stress in the vertical direction applied on the shearing surface by the vertical load and weight of the press board was obtained. To measure the relation between stress and strain rates in the inner part of the granular material, a granular material similar to the specimen was attached to the upper and lower surfaces of the container and the press board which comes into contact with the granular material at a ratio approximating the packing ratio of the granular material. For the particles of the uppermost surface of the granular material layers packed in the container, namely for the granular material on the surface which generates shearing by contact with particles fixed to the press board, colored materials were arranged partially, the distance travelled and the travel time when the shearing force was generated were measured, and the strain rate in the fixed granular material was obtained. Fig. 3 shows

Fig. 3 Side view of granular material in chute

a schematic drawing of the condition seen from the side. The colored particles were arranged on one line

in the widthwise direction at a distance 1000 mm from the end of the container.

The distance through which the colored particles move varies depending on the position, between the vicinity of the side wall surface and near the center, and therefore the average was taken as the travelled distance. Fig. 4 shows an example of measured

Colored Pellet P = 0.329kPa

Fig. 4 Measured velocities of colored particles, used to determine the strain rate of a granular material

results of the velocity of colored particles when the container was driven at a velocity u_0 . These were averaged and given as u to express the distance travelled by the colored particles.

The velocity of motion of the container from the start of the movement up to just before stopping measured by VTR is shown in Fig. 5. It can be seen that the time required for the velocity of the container to reach a value close to the steady-state value from 0 is very short compared with the whole time of the motion. Therefore, the error in the non-steady state container velocity before the steady container velocity is reached can be ignored. After reaching the steady state velocity, a fluctuation of about 10% in velocity can be seen but by averaging the total, u_0 was obtained. An example is shown in Fig. 5 by the dashed line.

When the motion velocity of the container is changed, the strain rate changes. Also the vertical stress is altered when different weights are placed on the press board. As the container of the granular material layer is transparent, the movement of the press board in the vertical direction was magnified through

Fig. 5 Measured velocity of chute

a VTR and observed from the side to see the changes occurring in the packed condition of the inner part of the granular material; The video camera was set up at a distance 1000 mm from the end of the container, in the same way as the arranged position of the colored particles.

3. Experimental results and observations

Experimental results for the relation between shearing stress and strain rate of circular pellets are shown in Fig. 6. The shearing stress γ is divided by the vertical stress P due to the vertical load to obtain a non-dimensional value. As shown in Fig. 6, the P value was changed 3 times and experiments were carried out. The strain rate $\partial u / \partial y$ along the x axis is made non-dimensional using the particle diameter D_p and the gravitational acceleration g .

Fig. 6 can be considered as a flow curve of the granular material (circular pellet) but the existence of the yield value is a characteristic differing from that of a Newtonian fluid. This is to say the flow curve of a granular material resembles that of a Bingham fluid. However it differs from a Bingham fluid in that γ/P is derived for the Y axis. If only γ is derived for the Y axis as in a Bingham fluid, the value of γ will depend greatly on the value of P but when combined as γ/P , data concentrates roughly in one place. Therefore, considering the results, it is thought that the combination γ/P is more reason-

able. However, as is obvious from Fig. 6, the γ/P value changes with the value P . Namely, with the increase of P , the γ/P value including the yield value increases greatly. As shown later in Fig. 8, along with the increase of P , the packing rate has increased and therefore it is believed that the packed condition in the inner part of the granular material becomes dense and that the γ/P value is increased. When γ is expressed by strain rate, $\partial u / \partial y$ is affected by P and it is seen that the constitutive equation becomes non-linear.

Fig. 7 shows the experimental results of γ/P only in the case of $P = 0.455$ kPa. With the increase of strain rate, γ/P becomes non-linear. In granular materials, while the strain rate is low, the generation of shearing force is dominated by the friction force generated between the particles. When the strain rate becomes large, the force due to the collision of particles is added to this friction force, shearing force is generated and therefore the slope of the flow curve becomes steeper. However this phenomenon requires the condition that the packing rate is constant. Fig. 6 shows the results where all data including those of Fig. 7 are summarized but in Fig. 6, the change in the slope of the curve is not conspicuous. Consequently, it is thought that the packing rate inside the layer during flow has changed (decreased).

Therefore, the packing rate was obtained from the change in the layer height at rest, under no-load

Fig. 6 Ratio of shear to normal stresses as a function of strain rate for circular pellet

and under flow conditions. The results are shown in **Fig. 8**. In making this measurement, a 78 mm top press board which readily enabled observation of the changes in layer height from the side was used. Accompanying the increase in strain rate, the packing rate decreased slightly. This is because a dilatancy phenomenon occurs by including a shearing flow in the granular material layer and that the packing condition of granular material becomes coarse. Also when P becomes large, in some cases the packing rate exceeds 1 but this indicates that the packing rate has become dense due to the increase in load.

Fig. 9 shows the experimental results for the relation between shearing force and strain rate when colored pellets are used as specimen. Including the

Fig. 7 Ratio of shear to normal stresses as a function of strain rate for circular pellet ($P = 0.455\text{kPa}$)

Fig. 8 C/C_0 as a function of strain rate for circular pellet

yield value, the γ/P values of the colored pellets are greater than those of circular pellets in their respective P values. Since the colored pellets are cylindrical in shape as opposed to the disk shape of circular pellets, the effect due to this difference in shape may be considered. Also in **Fig. 9**, the γ/P value is larger with a larger P value. This is perhaps caused by a dense packing condition resulting from the increase of P , in the same way as with circular pellets.

The results of γ/P where $P = 0.455\text{kPa}$ are shown in **Fig. 10** and unlike **Fig. 9** which summarizes data of various P values, the linearity of γ/P is weak. It is thought that a decrease in the packing rate during flow is the cause.

Fig. 9 Ratio of shear to normal stresses as a function of strain rate for colored pellet

Fig. 11 shows measurement results of colored pellet packing rate, made in the same manner as those of **Fig. 8**. The results approximate those of **Fig. 8** (results for circular pellets) with the packing condition becoming coarse with the increase of the strain rate. Also, a dense packing rate is seen with the increase of P .

Fig. 12 shows the experimental results for the relation between shearing force and strain rate of ceramic balls. Regardless of the changes in P , the changes in γ/P are smaller compared with the other two types of specimens. As can be seen from **Fig. 14**, this is because the change in the initial packing rate due to the increase of P is smaller. Also, the yield value is the smallest among the three types of granular materials. As the ceramic balls are

Fig. 10 Ratio of shear to normal stresses as a function of strain rate for colored pellet ($P = 0.455 \text{ kPa}$)

Fig. 11 C/C_0 as a function of strain rate for colored pellet

almost perfect spheres, it is thought that compared with disk shaped or cylindrical shaped particles, they tend to move more easily and therefore have a smaller yield value.

Fig. 13 shows results of γ/P where $P = 0.455 \text{ kPa}$. Non-linearity of γ/P can hardly be seen. Also in **Fig. 12**, rather than the slope of the curve becoming steeper due to the increase in strain rate, the data shows that it becomes gentler. As can be seen from the measurement results of the packing rate given in **Fig. 14**, the reduction in the packing rate due to the increase in strain rate is more significant than for the other granular materials and this can be considered as the cause for the above. It is believed that since the ceramic balls are nearly spherical and have approximately the same diameter,

Fig. 12 Ratio of shear to normal stresses as a function of strain rate for ceramic ball

the dilatancy phenomenon becomes conspicuous at the time of flow induction and the reduction of the packing rate becomes greater compared with other specimens of granular materials. Consequently, it is thought that if the packing rate is perfectly constant, the non-linearity (the tendency of the slope of the flow curve to become greater) of γ/P in Fig. 12 will also become obvious.

4. Conclusion

A shearing force was applied to a granular material by a linear movement and, shearing stress τ , strain rate $\partial u/\partial y$, vertical stress P and packing ratio C/C_0 were measured. From the results of measurements for three types of granular materials, their constitutive relations were experimentally obtained. Consequently, it was found that generally, a primary relation exists between τ/P and $\partial u/\partial y$. When P increases and packing becomes dense, the γ/P value including the yield value becomes large. Also it is thought that when the packing ratio is constant, the

Fig. 13 Ratio of shear to normal stresses as a function of strain rate for ceramic ball ($P = 0.455 \text{ kPa}$)

Fig. 14 C/C_0 as a function of strain rate for ceramic ball

τ/P value as opposed to the $\partial u/\partial y$ value becomes larger in a non-linear way but this tendency disappears when the packing condition becomes coarse due to the increase of $\partial u/\partial y$. Consequently, it was seen that the relation between τ/P and $\partial u/\partial y$ was extremely sensitive to small increases or decreases in the packing ratio. Also it was found that the yield value was smaller for spherical particles than for disk or cylindrically shaped particles. If an experimental constitutive equation can be obtained from these experiment results, and provided that relations between other vertical stress and strain rates can be obtained, then using these, it is possible to calculate the velocity distribution, stress distribution etc. over the entire area of hoppers from which granular materials flow out.

Nomenclature

- C : packing ratio at an arbitrary strain rate of granular material [—]
 C_0 : packing ratio at $P = 0$ and $U_C = 0$ [—]

D_p	: mean diameter of granular material [mm]
g	: gravitational acceleration [mm/s ²]
P	: normal stress [kPa]
u_p	: velocity of the colored particle [mm/s]
u_c	: velocity of the chute [mm/s]
U_0	: mean velocity of the chute [mm/s]
x, y	: cartesian coordinates [–]
τ	: shear stress [kPa]

References

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