1. Introduction

Filtration is the operation of separating a dispersed phase of solid particles from a fluid by means of a filter medium which permits the passage of the fluid but retains the particles. Filtration is probably one of the oldest unit operations. The old forms of filtration by straining through porous materials were described by the earliest Chinese writers. A gravity filter used in a chemical process industry was described in an Egyptian papyrus which has its origin in about the third century A.D.

In recent years, many developments have increased the application of filtration. Filtration steps are required in many important processes and in widely divergent industries. The importance of filtration techniques has been emphasized by the increased need for protection of the environment and by the increasingly critical need for larger supplies of energy. Recently, membrane filtration of colloids has become increasingly important in widely diversified fields.

The appropriate control of both the filtration rate and the rejection (or the transmission) of the particles and/or the macromolecules in the filtration process is of great interest in both industry and academia. Such filtration behaviors are strongly affected by the properties of the filter cake formed by the accumulation of the particles and/or the macromolecules on the surface of the filter medium or the membrane. This paper overviewed the author's own contributions on the recent developments on the behaviors of the filter cake in cake filtration and membrane filtration. The paper will mainly deal with measurements and analysis of the internal structures of the filter cake, the role of the solution environment in the properties of the filter cake, and filtration and fractionation mechanism of mixtures.
sured by conducting vacuum or pressure filtration experiments. In general, \( \alpha_{av} \) is calculated by using both the constant pressure filtration coefficient \( K_v \) determined from the slope of the Ruth plots [3] and the \( m \)-value. The value of \( m \) has almost invariably been determined by weighing the filter cake before and after the cake dries. However, visual determination of the end of filtration often leads to erroneous values for \( m \) and also \( \alpha_{av} \) because of the indistinct interface between the cake surface and the slurry.

The compression-permeability cell [4-6] is a device in which a mechanical load is applied through a piston to a cake resting on a filter medium. Both overall filtration characteristics and internal structures of the filter cake are analyzed on the basis of experimental data of the equilibrium porosity and the specific flow resistance of the compressed cake in the compression-permeability cell. However, this technique may be rather tedious and time-consuming for industrial practice.

A simple alternative procedure recently developed depends on measurement of the capillary suction time (CST), utilizing the very small suction pressure applied to the slurry by the capillary action of an absorbent filter paper [7]. However, little attempt has been made to determine filtration characteristics precisely and easily under relatively high filtration pressure conditions.

A method has been developed for evaluating rigorously the properties of the filter cake, such as the average porosity \( \varepsilon_{av} \) and the average specific filtration resistance \( \alpha_{av} \) [8]. It utilizes the sudden reduction in filtration area of the cake surface. The specially designed apparatus is schematically shown in Fig. 1.

A close-fitting cylinder with an inner diameter \( D \) of 4 cm is inserted in the cylindrical brass filter. Inserted cylinders having heights \( h \) of 5, 10 and 15 mm are used. A disk with a hole having a diameter \( D_h \) of 6 mm is placed on top of the inserted cylinder, and the part below this constitutes the filter chamber.

Filter cake steadily builds up on the filter medium as soon as the filtration process starts. The surface area of the growing filter cake equals exactly the area of the filter medium. At this first stage, on the assumption of negligible medium resistance, the reciprocal filtration rate \( (d\theta/dv) \) is represented by the well-known Ruth equation for constant pressure filtration in the form [3]

\[
\frac{dq}{dv} = \frac{2}{K_v} v
\]

(1)

where \( \theta \) is the filtration time, \( v \) is the cumulative filtrate volume collected per unit effective medium area, and \( K_v \) is the Ruth coefficient of constant pressure filtration defined by

\[
K_v = \frac{2p(1-ms)}{mfsa_{av}}
\]

(2)

where \( p \) is the applied filtration pressure, \( s \) is the mass fraction of solids in the slurry, \( \mu \) is the viscosity of the filtrate, and \( \rho \) is the density of the filtrate.

Once the filter cake builds up to the underside of the disk, the subsequent filter cake can form only inside the hole in the disk. Consequently, the filtration area of the cake surface is reduced suddenly, and the filtration rate decreases markedly in accord with the decrease in formation rate of the filter cake. After the filtrate volume \( v \) is beyond the critical volume \( v_t \) at the transition point, the reciprocal filtration rate \( (d\theta/dv) \) vs. \( v \) deviates remarkably from the relation represented by Eq. (1).

From the value of \( v_t \), the average porosity \( \varepsilon_{av} \) of the filter cake can be calculated using an overall mass balance of dead-end filtration, to give

\[
\varepsilon_{av} = \frac{r_h(1-s) - rs v_t}{r_h(1-s) + rsh}
\]

(3)

where \( \rho_s \) is the true density of solids. The ratio \( m \) of wet to dry cake mass in Eq. (2) is related to the average porosity \( \varepsilon_{av} \) of the filter cake by
Thus, the average specific filtration resistance $\alpha_{av}$ can be evaluated accurately from Eqs. (1), (2), and (4) by using the slope of the plot of $d\theta/dv$ against $v$ and the value of $m$.

The values of $m$ and $\alpha_{av}$ obtained by this method can be predicted by using the compression-permeability cell data. On the basis of the compressible cake filtration model [9, 10], the distributions of the local porosity $\varepsilon$ and the local specific filtration resistance $a$ and the apparent liquid velocity $u$ relative to solids in the filter cake can be estimated by using the compression-permeability cell data in the form of $e$ and $a$ as functions of the local solid compressive pressure $p$. Consequently, the values of $m$ and $\alpha_{av}$ can be calculated from Eqs. (5) and (6), respectively.

\[
m = 1 + \frac{r f_1^1 \varepsilon \frac{d}{dW} \left( \frac{W}{W_0} \right)}{r_s f_0^1 (1-\varepsilon) \frac{d}{dW} \left( \frac{W}{W_0} \right)}
\]

\[
a_{av} = \int_0^1 \left( \frac{u}{u_1} \right) \frac{d}{dW} \left( \frac{W}{W_0} \right) \left( \frac{p-p_m}{\alpha} dp \right)
\]

where $W$ is the net solid volume per unit medium area lying from the medium up to an arbitrary position in the cake, $W_0$ is the net solid volume of the entire cake per unit medium area, and $u_1$ is the filtration velocity. The quantity $p_m$ is the pressure loss through the filter medium, and can be neglected in this article.

In Fig. 2, the results obtained with the setup shown in Fig. 1 are plotted in the form of the reciprocal flow rate $(dq/dv)$ vs. the filtrate volume $v$ per unit medium area. In the first stage of the operation, each curve yields a straight line in accordance with Eq. (1). As soon as the cake builds up to the underside of the disk with the hole, the value of $dq/dv$ increases remarkably.

The value of $m$ can be calculated from Eqs. (3) and (4) by using the values of the thickness $h$ of the filter chamber and the filtrate volume $v_t$ at the transition point determined from Fig. 2. In Fig. 3, $m$ is plotted with respect to the filtration pressure $p$. The value of $m$ decreases with pressure. The experimental results are fairly consistent with the calculations based on the compression-permeability cell data. The discrepancy in the low-pressure region may be due to the influence of sedimentation, which becomes important in the case of small formation rate of the cake. For comparison, the values of $m$ determined by the conventional method of weighing the filter cake before and after being dried are also included in the same figure. The values thus obtained are rather large compared with the calculations because of the indistinct interface between the cake surface and the slurry. In Fig. 4, $a_{av}$ is plotted against pressure $p$. The resis-
tance \( a_{av} \) increases with pressure. The experimental results obtained by this method are fairly consistent with the calculations, whereas the results obtained by the conventional method show poor agreement because of incorrect values of \( m \). It is demonstrated from the results of Figs. 3 and 4 that this method is more accurate compared with the conventional one.

### 3. Compressible Cake Filtration Model

The compressible cake filtration model [9, 10] is used to evaluate such internal quantities as the solid concentration, the compressive pressure acting on the solids and the local specific filtration resistance within the filter cake exhibiting compressible behavior. The build-up of the filter cake increases the hydraulic resistance to flow, thereby reducing the filtration rate. Basically, the compressible cake filtration model can evaluate the reduction of the filtration rate due to the increase in the hydraulic resistance caused by the filter cake.

According to the compressible cake filtration model, the filtration rate \( J (= dv / dq) \) in dead-end filtration with negligible medium resistance compared with the resistance of the filter cake is represented by [3]

\[
J = \frac{-p}{m a_{av} w}
\]  
(7)

where \( w_0 \) is the net mass of deposited solids per unit effective medium area. By accounting for the effects of non-homogeneity and compressibility of the filter cake, the apparent solvent velocity \( J_w \) relative to solids at an arbitrary position \( w \) in the filter cake can be described by [11]

\[
J_w = \frac{1}{m a_{av}} \frac{\partial p_s}{\partial w}
\]  
(8)

On the assumption that the solvent velocity \( J_w \) is constant throughout the entire cake, on the basis of Eqs. (7) and (8), one obtains [11]

\[
\frac{w}{w_0} = 1 - \int_0^p \frac{dp_s}{a}
\]  
(9)

\[
a = \frac{a_{av}}{1 - \frac{d(\ln a_{av})}{d(\ln p)}}
\]  
(10)

\[
e = e_{av} \left\{ 1 + \frac{\frac{d(\ln e_{av})}{d(\ln p)}}{\frac{d(\ln a_{av})}{d(\ln p)}} \right\}
\]  
(11)

where \( w_0 (= w_0 / r_s) \) is the net solid volume of the entire filter cake per unit effective medium area, and \( e (= e / (1 - e)) \) is the local void ratio. Therefore, if the average specific filtration resistance \( a_{av} \) and the average void ratio \( e_{av} (= e_{av} / (1 - e_{av})) \) of the filter cake are represented as functions of the applied filtration pressure \( p \), then the variations of the local solid compressive pressure \( p_s \), the local specific filtration resistance \( a \) and the local void ratio \( e \) with \( w \) can be evaluated from Eqs. (9) – (11). Thus, the variation of the mass fraction \( c \) of the solid in the filter cake with the distance \( x \) from the medium surface can be also calculated from the result of \( e \) versus \( w \).

For a practical standpoint, the following empirical functions of \( p_s \) may be convenient for simplified evaluations of filtration characteristic values [12, 13].

\[
a = a_1 p_s^n
\]  
(12)

\[
e = E_0 - C_c \ln p_s
\]  
(13)

where \( a_1, n, E_0 \), and \( C_c \) are the empirical constants. With the aid of Eqs. (12) and (13), Eqs. (9) – (11) can be written as [11]

\[
\frac{p_s}{p} = \left( 1 - \frac{w}{w_0} \right)^{\frac{1}{1-n}}
\]  
(14)

\[
a_{av} = a_1 (1-n) p^n
\]  
(15)

\[
e_{av} = E_0 + \frac{C_c}{1-n} - C_c \ln p
\]  
(16)

On the basis of the model of solutions in which the
macromolecules appear as solid particles [14], it is assumed that the model which has been employed in filtration of particulate suspensions can be applied to the flow of solvent through the filter cake in ultrafiltration of the solution. In Fig. 5, the average specific filtration resistance \(a_{av}\) and the average void ratio \(e_{av}\) of the filter cake in protein ultrafiltration measured with a filter which has a sudden reduction in its filtration area are plotted as functions of the filtration pressure \(p\) [15]. The macromolecule used in the experiments was bovine serum albumin (BSA) with a molecular weight of 67,000 Da and with an isoelectric point of 4.9. Increasing the filtration pressure causes a reduction in the void ratio of the filter cake, leading to an increase of the specific filtration resistance. It is apparent that \(e_{av}\) around the isoelectric point (pH 4.9) is much smaller than that at pH 7.0. The two types of plots show a linear relationship over the entire range of data in accordance with Eqs. (15) and (16), respectively. The parameters \(a_1\) and \(n\) in Eq. (12) can be calculated by fitting Eq. (15) to the logarithmic plot of \(a_{av}\) vs. \(p\). The value of \(E_0\) and \(C_c\) in Eq. (13) can be obtained from a plot of \(e_{av}\) vs. \(p\) in accordance with Eq. (16), using the predetermined \(n\) value. The distributions of \(p_s\) within the filter cake can be obtained from Eq. (14). With the aid of Eq. (14), Eqs. (12) and (13) provide the distributions of \(a\) and \(e\) within the filter cake, respectively.

The variations of the local solute compressive pressure \(p_s\) and the local specific filtration resistance \(a\) across the filter cake for different values of the filtrate volume \(v\) calculated from Eqs. (14) and (12) are illustrated in Figs. 6 and 7, respectively [16]. The distance \(x\) from the membrane surface in the figures may be calculated from \(w\) by

\[
\frac{x}{L} = \frac{1 + e_{av,w}}{1 + e_{av}} \cdot \frac{W}{W_0}
\]

Fig. 5 Effect of applied filtration pressure on average specific filtration resistance and average void ratio of filter cake for different values of pH.

Fig. 6 Distributions of local solute compressive pressure in filter cake for different filtrate volumes.

Fig. 7 Distributions of local specific filtration resistance in filter cake for different filtrate volumes.
The thickness demonstrated that both 
As shown in the figure, high specific filtration resistance to flow. At the interface of the filter 
the form cake and the membrane, 
the net compressive pressure increases as the mem-
brane surface is approached, resulting in the increas-
in the solute compressive pressure at the sur-
face of the protein filter cake is essentially zero as no 
drag on the solutes has developed. The frictional drag 
on each solute adds to the drag on the previous 
drag on the solutes has developed. The frictional drag 
face of the protein filter cake is essentially zero as no 
increase. The solute compressive pressure at the sur-
face of the protein filter cake is expressed 
based on the material balance as 
\[ L = \frac{r_s(1 + e_m)}{r_s(1 - s) - r_m e_m} - p \] (18)
As shown in the figure, high specific filtration resis-
tance can be obtained although the porosity in the 
cake is not so small [2, 17]. The figure obviously 
demonstrates that both \( p_s \) and \( \alpha \) decrease with the distance \( x \). As the filtrate volume \( v \) increases, \( p_s \) and \( \alpha \) increase. The solute compressive pressure at the surface of the protein filter cake is essentially zero as no drag on the solutes has developed. The frictional drag on each solute adds to the drag on the previous solutes as the solvent passes frictionally along the solutes in the compressible filter cake. Consequently, the net compressive pressure increases as the membrane surface is approached, resulting in the increasing resistance to flow. At the interface of the filter cake and the membrane, \( p_s \) has risen to its maximum value and is equal to the applied filtration pressure \( p \) for the negligible membrane resistance. Therefore, the protein filter cake tends to have a much more compact structure at the membrane exhibiting a large resistance to flow in comparison to a relatively loose condition at the surface because the cake is compressible. In the compressible filter cake formed by ultrafiltration of BSA solutions, it is demonstrated that a pronounced variation of characteristic values of the filter cake can be seen. This is in agreement with the results obtained for filtration of particulate suspensions [9, 18].

The local mass fraction \( c \) of the solute in the filter cake may be calculated from the local void ratio \( e \) in the form 
\[ c = \frac{r_s}{r_s + r_m e} \] (19)
Thus, the concentration distributions in the protein filter cake may be predicted from Eqs. (13), (14), (17)–(19).

4. Analysis ofUltrafiltration Behaviors Based on Ultracentrifugation Experiment

The flux decline and the cake structure in ultrafiltration of protein solutions can be evaluated from analytical ultracentrifugation experiments [15, 19, 20]. The experiments were performed in a Hitachi Model 282 ultracentrifuge equipped with an optical system. The rotor speed ranged from 45,000 to 60,000 min\(^{-1}\). The distributions of the concentration, the concentration gradient, and the refractive index gradient of the solutions in a 1.5 mm double-sector centerpiece were measured over time using both the ultraviolet scanner absorption system and Schlieren optics. In the sedimentation velocity experiment, the sedimentation velocity was determined from the displacement of the sedimentation boundary. In the high-speed sedimentation equilibrium experiment, the equilibrium thickness of the sediment of macromolecular solutes was measured after equilibrium was reached at a constant rotor speed.

At relatively high solution concentrations, there is an analogy between the sedimentation of a macromolecule in a solvent and the permeation of a solvent through the filter cake of macromolecules. The local specific flow resistance \( \alpha \) can be calculated as [15, 19]
\[ \alpha = \frac{(r_s - r) r_i \Omega^2}{m_r v_0} = \frac{r_s - r}{m_r S} \] (20)
where \( r_i \) is the radial distance of the sedimentation boundary from the center of rotation, \( W \) is the angular velocity of the rotor, \( v_0 \) is the sedimentation velocity, and \( S \) is the sedimentation coefficient.

Thus, the relation between \( \alpha \) and the volume fraction \((1 - e)\) of the solute may be written as [15, 19, 21]
\[ \frac{1}{\alpha}^{1/4.65} = \frac{r_s d_i^2}{18 C_h} \left\{ 1 - C_h (1 - e) \right\} \] (21)
where \( d_i \) is the average equivalent spherical diameter of solutes, \( C_h \) is the ratio of the volume of the hydrous protein molecule to volume of anhydrous protein molecule, and \( e \) is the porosity of the solution.

Figure 8 shows the permeability data obtained by measuring the sedimentation velocities for a number of different solution concentrations. The plots are virtually linear as would be expected from Eq. (21). It is apparent that \( \alpha \) around the isoelectric point is much smaller than that at pH 7 with the same solution concentration. On the other hand, the average specific filtration resistance \( a \) of the cake in the ultrafiltration of BSA solutions reaches a definite maximum around the isoelectric point [2]. In order to account for this discrepancy, a high-speed sedimentation equilibrium experiment was conducted.

If the local solute concentration of the cake formed in ultrafiltration is known, then the specific flow resistance \( \alpha \) of the cake can be evaluated by using the permeability data. Therefore, it is necessary to evaluate the local solute concentration of the cake. We thus determined the compression data representing the
relation between the local porosity $\varepsilon$ and the local solute compressive pressure $p_s$ from high-speed sedimentation equilibrium experiments. If $\varepsilon$ is related to $p_s$ by Eq. (22), the equilibrium thickness $(R-r_s)$ of the sediment is related to the centrifugal acceleration $RW^2$ by Eq. (23) [15, 19, 22].

$$1 - \varepsilon = E p_s^b$$

$$p_s \geq p_{si}$$

$$R - r_s = \frac{w_0^{1-b}}{E (1-b)} \left( (r_s - r) R \Omega^2 \right)^b$$

where $R$ is the distance from the center of rotation to the bottom of the sediment, $r_s$ is the distance from the center of rotation to the surface of the sediment, $w_0$ is the net solute volume of the entire sediment per unit cross-sectional area, and $E$ and $b$ are the empirical constants. Below $p_{si}$, $\varepsilon$ is assumed to be constant, and is equal to the porosity at pressure $p_{si}$.

**Figure 9** represents the logarithmic plot of the equilibrium thickness $(R-r_s)$ of the sediment against the centrifugal acceleration $RW^2$. The plot shows a linear relationship in accordance with Eq. (23). It is interesting to note that the sediment at the isoelectric point is much more compact than that at pH 7 because the BSA molecule carries no net charge at the isoelectric point. Therefore, it would be expected that in protein ultrafiltration a compact filter cake forms around the isoelectric point. The relation between $\varepsilon$ and $p_s$ can be determined from the plot in the figure using Eqs. (22) and (23). On the basis of these relations, the variations of the filtration rate in ultrafiltration of protein solutions can be determined from the compressible cake filtration model [2, 9, 15].

In **Fig. 10**, the results of unstirred dead-end ultrafiltration are plotted in the form of the reciprocal filtration rate ($\frac{dq}{dv}$) versus the filtrate volume $v$ per unit membrane area. The filtration rate around the isoelectric point is much smaller than that at pH 7. The solid lines indicate the theoretical predictions based on the compression-permeability data obtained in analytical ultracentrifugation. The experimental data are in relatively good agreement with the theory, indicating that the compressible cake filtration model accurately describes the ultrafiltration behavior.

**Figure 11** shows the variations of the mass fraction $c$ of the solute across the cake calculated on the basis of the ultracentrifugation data. In the compressible cake resistance model, the solutes deposited on the membrane are treated as the cake, and it is attributed to the hydraulic barrier. The cake tends to have a much more compact structure at the membrane in comparison to a relatively loose condition at the surface because the cake is compressible. Of considerable practical interest is that this result is in agreement with that obtained for cake filtration of par-
ticulate suspensions [23]. A much more compact filter cake exhibiting a large resistance to flow may form around the isoelectric point than that which forms at pH 7. Consequently, the filtration rate at the isoelectric point becomes lower than that at pH 7.

5. Measurement of Concentration Distribution in Filter Cake of Ultrafiltration

A variety of theoretical models describing the fouling phenomenon during ultrafiltration quantitatively have been developed: the gel polarization model [24-26]; the osmotic pressure model [27-29]; the boundary layer model [30, 31]; and the cake filtration model [32-34]. Although a significant amount of research has been conducted to better understand the dynamics of ultrafiltration, the real mechanism of separation remains incompletely understood.

The compressible cake filtration model has been widely used to describe filtration behaviors of particulate suspensions [9]. Recently, some authors prefer to use the compressible cake filtration model to explain the mechanism of ultrafiltration [2, 15, 35], and it has the potential for analyzing the membrane fouling during ultrafiltration. The filtration flux rate during dead-end ultrafiltration of bovine serum albumin (BSA) solutions could be well evaluated by use of the ultracentrifugation data on the basis of the compressible cake filtration model [19]. The model regards a stagnant filter cake on the membrane as the structural assemblage of “particulate” solutes. In this model, highly resistant filter cake of excluded protein molecules accumulated in front of the membrane surface forms, and the filter cake acts as an additional hydraulic resistance to flow in conjunction with that provided by the membrane, thereby reducing the filtration flux rate. The model also takes the compressibility of the filter cake into account. Therefore, an understanding of the properties of the filter cake formed by the accumulation of the protein molecules on the membrane can serve as a basis for clarifying the real mechanism of ultrafiltration.

Several approaches have been developed for measuring the average value of the porosity in the filter cake formed on the membrane surface in ultrafiltration of protein solutions. Nakao et al. [36] measured the gel concentration by scraping a thin layer of deposited material from the surface of a tubular membrane in ovalbumin ultrafiltration. Iritani et al. [2] used a batchwise filter device which had a sudden reduction in its filtration area to determine the average porosity of the BSA cake. Also, in more recent work by Nakakura et al. [35], the porosity was obtained from the measurement of the electric conductivity within the filter cake during ultrafiltration of BSA solutions. It is, however, crucial to measure the porosity distribution since the filter cake formed on the membrane during ultrafiltration is extremely thin.
in general.

Relatively few investigators have measured concentration distributions within the protein layer accumulated on the membrane during ultrafiltration. Vilker et al. [29] used an optical shadowgraph technique to measure the concentration gradient within the polarized layer on the membrane during unstirred dead-end ultrafiltration of BSA solutions. McDonogh et al. [37] reported the variation with time of concentration in the polarized layer on the membrane during ultrafiltration of BSA and Dextran Blue solutions using the electronic diode array microscope. Gowman and Ethier [38] used an automated, laser-based refractometric experimental technique for the measurement of concentration and concentration gradient in the concentration polarization layer during dead-end ultrafiltration of the biopolymer hyaluronan. Fernández-Torres et al. [39] obtained the evolution of concentration profiles during BSA ultrafiltration in an unstirred cell using the technique of holographic interferometry. Their results demonstrated that a filter cake was actually formed during BSA ultrafiltration. Although these non-destructive methods have the enormous advantage, they are technically complex and expensive.

A potential method has been explored for measuring the concentration distributions in the filter cake using the principle of the inclined ultrafiltration [40, 41], where the membrane was inclined and a large amount of filter cake was formed. The concentration distributions in the filter cake were measured after downward ultrafiltration was performed in the wake of inclined ultrafiltration. This destructive method is simple, and inexpensive.

The flowing behaviors of filter cake have been observed, as depicted in Fig. 12 schematically. It was clarified that the filter cake formed mainly on the lower part of the membrane surface due to the effect of gravity during inclined ultrafiltration, as shown in Fig. 12(a) [40]. The same phenomenon cannot be observed in downward dead-end ultrafiltration. Once downward ultrafiltration was conducted after inclined ultrafiltration, the filter cake was spread over the membrane surface, as shown in Fig. 12(b) [41].

In Fig. 13, the flux decline behaviors in various filtration modes are plotted in the form of the reciprocal filtration rate $1/J (=dq/dv)$ versus the cumulative filtrate volume $v$ collected per unit effective membrane area, which is well known as the Ruth plot [3] in the ordinary cake filtration. For conventional downward dead-end ultrafiltration ($y=0$), the filtration rate declines gradually due to the build-up of a BSA cake as filtration proceeds. The angle $y$ represents the angle between the filtrate flow and the direction of gravity. The plot is virtually linear throughout the course of filtration in accordance with the Ruth filtration rate equation (1) [3] with negligible membrane resistance compared with the filter cake resistance. In contrast, in inclined ultrafiltration ($y=\pi/3$ rad), the filtration rate becomes remarkably high because the filtrate passes through the upper part of the membrane which is less resistant. Furthermore, once downward ultrafiltration was conducted after inclined ultrafiltration, the filtration rate dramatically decreased because of the remarkable increase of the total filtration resistance. Surprisingly, the value of $1/J$ in this case lies on the extension line in ordinary downward ultrafiltration, as shown in the figure. This

![Fig. 12](image_url) Schematic diagram of flowing filter cake: (a) inclined ultrafiltration, and (b) downward ultrafiltration.
result supports that the structure of the filter cake formed on the membrane such as the concentration distributions is almost the same between both operations.

In Fig. 14, the mass fraction $c$ of BSA on the membrane measured after changing the angle $y$ from $\pi/3$ rad to zero is plotted against distance $x$ from the membrane surface for the different values of the filtrate volume $v$ collected until inclined ultrafiltration was switched to the downward mode. The profiles in this figure may be considered to be similar to those in ordinary downward ultrafiltration, as expected from the results of flux decline behaviors shown in Fig. 13. The filter cake which was thicker than 1 cm was formed at the filtrate volume of 20 cm. It is very difficult to obtain such a thick cake as this from downward dead-end ultrafiltration. The solid lines on the figure indicate the theoretical predictions based on a compressible cake filtration model. In the compressible cake filtration model, the concentration distributions of the solutes deposited on the membrane treated as the filter cake act as the hydraulic barrier. The pressure gradient on the liquid passing through the solute varies, and the porosity in the cake is not constant [42]. The figure distinctly demonstrates that there exists a concentration distribution within the filter cake formed on the membrane. The concentration $c$ has a maximum at the membrane surface ($x=0$) and decreases markedly with the increase of $x$, which shows the same trend as that observed for the cake filtration of particulate suspensions [9, 18]. It was verified that the solutes forming the cake are compact and dry at the membrane whereas the surface layer is in a wet and soupy condition. It was also found that the thickness of the filter cake formed on the membrane increases as filtration proceeds.

6. Role of Solution Environment in Properties of Filter Cake

Cake filtration of suspensions of solid particles in aqueous solutions is frequently encountered in widely diversified fields. In cake filtration, the nature of the cake formed upon the medium surface during filtration will govern the filtration rate of the remaining suspension. For fine particle suspensions, colloidal forces control the nature of the filter cake. Colloidal forces arise from interaction between the suspended particles. The two main colloidal forces are the attractive van der Waals forces which originate from fluctuating dipoles as a result of the motions of outer electrons on the interacting particles and the repulsive electrostatic forces due to the presence of like charges on the particles and a dielectric medium. Whilst for a given system the van der Waals forces are essentially constant, the electrostatic forces will vary with the surface charge of the suspended particles, which varies with the solution environment. Therefore, the filtration behaviors of the colloids are
affected significantly by the solution properties, including pH and electrolyte strength. Considerable work has been published on the effects of the solution environment on the filtration behaviors [43-52].

Figure 15 shows the average specific filtration resistance \( \alpha_{av} \) and the average porosity \( \varepsilon_{av} \) of the filter cake formed in particulate microfiltration at different values of pH under otherwise identical filtration conditions [53]. The particles used in the experiments was titanium dioxide of the rutile form with an original mean specific surface area size of 0.47 \( \mu m \) and with an isoelectric point of 8.1. It should be noted that the resistance \( \alpha_{av} \) is a measure of the filtrability of the suspensions. A larger \( \alpha_{av} \) causes a smaller filtration rate. It can be seen that \( \alpha_{av} \) goes through a minimum around the isoelectric point. Thus, the largest filtration rate can be obtained when the particle carries no charge. Tarleton and Wakeman [49] reported similar results for crossflow microfiltration of anatase suspension. However, this result is in sharp contrast to that obtained in protein ultrafiltration [2, 54]. It is clear that \( \varepsilon_{av} \) is much larger near the isoelectric pH. Since the titanium dioxide particles are hydrophobic colloids, they are destabilized around the isoelectric point where the van der Waals attraction is more dominant. Consequently, the particles will tend to come together, i.e. to flocculate, and the very porous flocs are then formed. Thus, it is speculated that the filter cake formed from such porous flocs has often loose and wet structures. On the other hand, the filter cake becomes compact and dry when the particle carries the charge. Since the most loose filter cake forms around the isoelectric pH, the filter cake is most permeable. Thus \( \alpha_{av} \) is much smaller near the isoelectric point.

It is interesting to note that the results in protein ultrafiltration had a distinctly different behavior. In protein ultrafiltration of BSA solution, the filter cake is in its most compact state around the isoelectric point [2, 54], as shown in Fig. 16. Since the BSA molecules are hydrophilic colloids, their stability in the solution would appear to be influenced not only by the presence of a surface charge on the protein but also by hydration of the surface layers of the protein. The BSA molecules, because of hydrated layers surrounding them, are not destabilized by such considerations as depression of the electrical double layer. Thus, the BSA molecules have water bound to them even around the isoelectric point. The hydrophilic BSA molecules maintain a dispersed state in the solution due to hydration of the surface layers of the protein even around the isoelectric point. When a BSA molecule acquires a charge, the filter cake becomes loose and wet due to electrostatic repulsion between the charged BSA molecules. This contrasts to the compact filter cake around the isoelectric point. The average specific filtration resistance \( \alpha_{av} \) has a definite maximum around the isoelectric point since a compact filter cake provides a large hydraulic flow resistance.

Charge effects are weakened in the presence of solids. In Fig. 17, the average porosity \( \varepsilon_{av} \) of the filter
cake formed in particulate microfiltration is plotted against absolute values of zeta potentials. This figure shows a close relation between $\varepsilon_{av}$ and the zeta potential. It is surprising that the plots can be represented by a unique curve irrespective of the sign of the zeta potential. The porosity remains constant in the range of the zeta potential below ca. 20 mV. As the zeta potential decreases, the electrostatic repulsion decreases and thus the porous floc forms. Therefore, the average porosity of the filter cake increases. The repulsive force that stems from the overlapping of the electric double layers, depends on the presence or absence of ionized salt in suspension. The repulsive force is less effective at higher ion concentrations and cannot counteract the van der Waals attraction, and so the solution flocculates. Therefore, it is conceivable that the filtration rate is markedly influenced not only by the solution pH but also by the solution ionic strength. The result for the NaCl concentration of 200 mol/m$^3$ at pH 10.5 is shown in the figure. The magnitude of the zeta potential decreased remarkably as a result of the addition of NaCl compared with that for the NaCl concentration of 0.2 mol/m$^3$ at the same pH. The average porosity $\varepsilon_{av}$ of the filter cake is augmented markedly by the addition of salt. This is because the addition of salt destabilizes the suspension by reducing the double layer repulsion between rutile particles. It should be emphasized that the plot for the NaCl concentration of 200 mol/m$^3$ lies on the unique curve.

7. Properties of Filter Cake Composed of Mixtures

Especially when two types of particles which may be membrane foulants in combination are present in the feed fluid, the situation becomes more complex. The filtration behaviors can be strongly affected by the nature of the interaction between the dissimilar components.

The variations of the filtration rate with time and the properties of the filter cake were examined for microfiltration of binary mixtures of fine particles [55]. The average porosity in the filter cake of the binary particulate mixtures was explained well by the mixture packing model in which small particles fill the voids between large particles. Further, the average specific filtration resistance of the binary mixtures was well evaluated on the basis of the additive law of the effective specific surface area. However, in most of published works on filtration of binary particulate mixtures [56, 57], the effects of the physicochemical factors such as the pH and salt concentration on the filtration performance have received little or no attention.

Properties of a filter cake formed in dead-end microfiltration were examined using binary particulate mixtures of titanium dioxide and silicon dioxide having the different values of the isoelectric point for each material [58]. The original surface mean diameter of silicon dioxide is 0.92 µm. The silicon dioxide particle has negative charge because it contains only dissociable group of one type, the silanol groups SiOH. In Fig. 18, the average specific filtration resistance $\alpha_{av}$ and the average porosity $\varepsilon_{av}$ of the filter cake at pH 4.5 are shown against the mixing ratio $(s_t/s)$ of particles, where $s_t$ is the mass fraction of titanium dioxide particles in the bulk suspension, and the total mass fraction $s$ of particles is kept constant. It is immediately obvious that $\alpha_{av}$ tends to become smaller in a mixed system of two components in comparison with a single component system. A smaller specific filtration resistance was generally caused by a larger cake porosity. At this pH value, a titanium dioxide has a positive electrical charge, whilst a silicon dioxide is electronegative. For the single component suspension, the particles are likely to be well dispersed by repulsive electrostatic forces, and form cakes of higher resistance during filtration. However, the particles in mixed suspension come together due to heterocoagulation associated with a coulombic attractive force and the London-van der Waals force, and the very porous flocks are formed prior to deposi-
tion. Thus, the coagulated colloids form highly permeable filter cakes, minimizing decreases in the filtration rate. For reference, the calculations based on the additive law on the specific volume and the effective specific surface area are shown as the dotted lines [56]. It is obvious that the effect of the surface charge of the particles on the cake properties should be considered.

It is of significance to compare the results with those obtained in dead-end ultrafiltration of the mixed protein solutions. It was observed that the results obtained with protein ultrafiltration were in sharp contrast to those with particulate microfiltration [17]. The properties of the cake formed on the retentive membranes in dead-end ultrafiltration of binary protein mixtures have been studied by using a mixture of the two proteins BSA and egg white lysozyme. The molecular weight and the isoelectric point of lysozyme are 14,300 Da and 11.0, respectively. In Fig. 19, the properties of the filter cake at pH 6.9 are shown against the mixture ratio of the proteins, where $s_b$ is the mass fraction of BSA in the solution, and the total mass fraction $s$ of solutes is kept constant. Of particular importance is the surprising result that the porosity $e_{av}$ shows a distinct minimum. In the case of the single solute solutions, the average porosity is relatively large, since only an electrostatic repulsive force acts between the solutes. In contrast, in the mixed protein solutions, the BSA and lysozyme molecules are oppositely charged at pH 6.9. Therefore, the average porosity of the cake decreases markedly by adding the other protein to the single protein solution because of a higher attraction between solutes. The decrease of the porosity $e_{av}$ roughly corresponds to the increase of the resistance $a_{av}$. The resistance shows a definite maximum at the mixing ratio of about 0.6 because the most compact cake forms at that mixing ratio. Since the protein molecules are hydrophilic colloids, their stability in the solution is influenced not only by the presence of a surface charge on the protein but also by hydration of the surface layers of the protein. Even in the pH range where the two protein molecules have opposite electrical charges, protein molecules are well dispersed in the solution due to hydration of the protein. Consequently, the mixed protein cake becomes compact because of the attractive force associated with oppositely charged solutes.

In microfiltration of binary particulate mixtures, the effects of adding electrolyte on $a_{av}$ and $e_{av}$ at two different pH values are shown in Fig. 20. The addition of salts leads to a decrease of the zeta potential because of a less extensive electrical double layer. It is observed that at pH 4.5 $a_{av}$ increases and $e_{av}$ decreases slightly, but discernibly with the increase of the salt concentration. The result is explained by assuming that the coulombic attractive force between the particles decreases due to a decrease of the zeta potential, and as a result the effect of heterocoagulation is weakened. At pH 9.6, it is found that $a_{av}$
decreases and $e_{av}$ increases markedly with increasing NaCl concentration. At this pH, the repulsive force that stems from the overlapping of the electric double layers, is reduced because of the charge-shielding due to the presence of salts, and consequently particles tend to aggregate [59]. Thus, the cakes formed are more open and have a lower resistance to flow.

In Fig. 21, the profiles of the average specific filtration resistance $a_{av}$ obtained in dead-end ultrafiltration of titanium dioxide suspensions containing BSA molecules using 10 kDa membranes are illustrated for various pH values against the mass ratio ($s_b/s$) of BSA to the net colloids. At pH 4.2 and 5.1, the value of $a_{av}$ increases with an increase of the mass fraction of BSA. This is because the specific resistance $a_{av}$ of BSA is significantly larger than that of titanium dioxide. At pH 6.0, however, in the range of quite small BSA fractions $a_{av}$ tends to become smaller than that of the cake composed of titanium dioxide alone ($s_b/s = 0$). The result is perhaps surprising when one considers the high specific resistance of BSA. At pH 6.0 the positive charge of the titanium dioxide particle is cancelled by the adsorption of negatively charged BSA molecules. The flocculation of titanium dioxide therefore occurs due to a marked decrease of the zeta potential of the complex of titanium dioxide particles and BSA molecules. This brings about the specific behavior of $a_{av}$ at pH 6.0.

The average specific filtration resistance $a_{av}$ for different concentrations of sodium chloride is plotted in Fig. 22 against the mass ratio of BSA to the net colloids. The value of $a_{av}$ decreases markedly by the addition of an appropriate volume of sodium chloride and BSA. This is due to charge-shielding of both particles and proteins caused by the presence of the salts, thus reducing the electrostatic repulsion between titanium dioxide and BSA.

![Fig. 20](image1.png)

**Fig. 20** Effect of NaCl concentration on average specific filtration resistance and average porosity.

![Fig. 21](image2.png)

**Fig. 21** Effect of mixing ratio of proteins on average specific filtration resistance in ultrafiltration of particle/protein mixtures.

![Fig. 22](image3.png)

**Fig. 22** Effect of salt addition on average specific filtration resistance in ultrafiltration of particle/protein mixtures.
Fractionation of Mixtures by Filtration

Solute/solute interaction plays an important role in both the filtration rate and the solute rejection when two or more proteins are ultrafiltered [60-62]. It is generally accepted that solutes whose molecular dimensions are clearly small enough to permeate the membrane are substantially retained by that same membrane when larger solutes are present [63].

In Fig. 23, the solute rejection behavior following ultrafiltration of binary BSA/lysozyme mixtures containing equal amounts of each protein is shown in the form of the apparent rejection $R_{obs,l}$ of lysozyme versus the cumulative filtrate volume $v$ per unit effective membrane area collected in the filtration time $q$ [64]. Ultrafiltration experiments were carried out with the upward filtration mode, in which the filtrate flow was opposite to the direction of gravity, using hydrophobic, sorptive polysulfone membranes with a molecular weight cut off of 30,000 Da, making it essentially impermeable to BSA, but permeable to lysozyme. Solution pH was adjusted to 7, which is pH between the isoelectric points of both proteins, and the NaCl concentration $c_s$ was varied. The apparent rejection $R_{obs,l}$ of lysozyme can be defined by

$$R_{obs,l} = 1 - \frac{c_l}{s_l}$$

where $c_l$ and $s_l$ are the mass fractions of lysozyme in the filtrate and bulk feed solution, respectively. Filtration of binary protein mixtures with this membrane resulted in nearly complete retention of BSA, but the variation in the lysozyme rejection with the filtrate volume. In the incipient stages of filtration, the membrane exhibits a high lysozyme rejection which may be attributed to adsorption of the lysozyme solutes on the hydrophobic surface of the membrane [65]. The rejection $R_{obs,l}$ is high during ultrafiltration in the absence of NaCl ($c_s=0$). The BSA molecule is negatively charged at pH 7, while the lysozyme molecule has a net positive charge at this pH value. The filter cake formed on the membrane surface consists of the admixture of two proteins very well packed to form a compact layer because the coulombic, attractive force associated with oppositely charged solutes, resulting in high rejection of lysozyme. It can be seen that the lysozyme rejection decreases with the increase of the NaCl concentration. In the case of a charge-stabilized colloid such as protein, solute interactions depend on the magnitude of the surface charge and/or on the extent of the electrical double layer, and this depends on the total electrolyte concentration. When salts are added, this leads to a less extensive electrical double layer. Such charge-shielding between the protein molecules resulting from the existence of salts would reduce electrostatic attraction between BSA and lysozyme molecules. Thus, the solute rejection at first declines to a minimum value. However, as filtration proceeds, the compressible filter cake of retained BSA solutes provides a barrier to transport of the smaller lysozyme solutes [66], and hence the rejection of the lysozyme solutes rises dramatically.

In Fig. 24, the flux behavior of the experiment described in Fig. 23 is shown in the form of the reciprocal filtration rate ($dq/dv$) versus $v$. This plot is well known as the Ruth plot [3] in the cake filtration of particulate suspensions. In principle, the value of $dq/dv$ is directly proportional to the hydraulic resistance due to solute accumulation on or in the membrane. It would be expected that a rather dense, finely porous filter cake of retained BSA solutes provides a barrier to transport of the smaller lysozyme solutes [66], and hence the rejection of the lysozyme solutes rises dramatically.

![Fig. 23](image-url)  
**Fig. 23** Effect of salt addition on apparent lysozyme rejection at pH 7.
protein solutions [40, 54]. The filtration rate increases markedly on addition of NaCl because of the formation of a filter cake which is substantially free from lysozyme molecules. However, as filtration proceeds, the filtration rate decreases gradually because the lysozyme molecules are trapped in the pores of the filter cake.

In Fig. 25, the reciprocal filtration rate \((dq/dv)\) in the unstirred downward and upward, and the stirred downward ultrafiltration of binary BSA/lysozyme mixtures is shown as a function of \(v\) [67]. For these experiments, ultrafiltration membranes with very low adsorptivity for proteins were used. Solution environment was set at pH 7 and the NaCl concentration of 300 mol/m\(^3\) where electrostatic interactions between BSA and lysozyme weakened comparatively as implied from Figs. 23 and 24. For conventional unstirred downward ultrafiltration, the plot is virtually linear throughout the course of filtration because of the continuous formation of the filter cake on the membrane in accordance with the compressible cake filtration model [2, 9, 15]. For unstirred upward ultrafiltration, the flux decline is suppressed, similar to the result of Fig. 24. The increase in the shear stress acting on the membrane surface by stirring leads to the suppression of the cake deposition, and consequently the marked increase in the filtration rate. Since the filtration rate in upward ultrafiltration is slightly lower than that in stirred ultrafiltration with stirring of 5.2 rad/s, an effect of gravity acting on the filter cake in upward ultrafiltration corresponds to that of very low shear stress.

Figure 26 shows the lysozyme rejection behavior of the experiment described in Fig. 25. The figure indicates that the lysozyme solutes pass almost completely through the membrane in unstirred downward
ultrafiltration and are only a little retained by the membrane in upward ultrafiltration except that the high rejection appears in the incipient stages of filtration. It has been reported that in single protein solutions the stirring resulted in increasing the solute rejection [63]. It should be emphasized that also in binary protein mixtures the higher stirring speed causes the higher rejection of the solutes. This is because the concentration of lysozyme near the membrane decreases with increasing shear stress acting on the membrane.

Figures 25 and 26 suggest that it is necessary to control the hydrodynamics above the membrane in order to obtain high lysozyme transmission simultaneously with high filtration rate. While hard stirred ultrafiltration, which produces the high filtration rate and solute rejection, is very effective for concentrating protein solutions, it is not suitable for fractionation of binary protein mixtures. It can be concluded from the figures that unstirred upward or mildly stirred ultrafiltration is more advantageous for the efficient fractionation of binary protein mixtures.

The effects of ultrasonic irradiation on the filtration rate and lysozyme rejection in upward ultrafiltration of binary BSA/lysozyme mixtures using the ultrafiltration membrane with very low adsorptivity for proteins at pH 7 and high salt concentration are shown in Fig. 27 [67]. The whole of the filter was kept immersed in the ultrasonic cleaning bath full of water during the course of filtration. The filter was set up so that the membrane surface was perpendicular to the propagation direction of the ultrasonic wave. The oscillating frequency and the output power of the ultrasonic irradiation were 25 kHz and 180 W, respectively. It is evident that the ultrasonic fields can contribute to the remarkable improvement in the filtration rate. As was shown in Figs. 25 and 26, although the observed filtration rate becomes high by increasing the shear stress on the membrane, the transmission of the lysozyme molecules becomes less. In contrast, of particular importance is the surprising observation that the ultrasonic irradiation does not lower the lysozyme transmission regardless of involving a marked increase in the filtration rate. One possible explanation for this result is that ultrasonic irradiation supplies vibrational energy to the filter cake to keep the solutes partly suspended, and therefore leaves more free channels for the filtrate flow.

To clarify the effects of the ultrasonic irradiation, the lysozyme rejections observed in the cases where the filtration rates with and without ultrasonic irradiation are very similar are compared in Fig. 28. The figure shows that the filtration rate in the ultrasonic upward ultrafiltration irradiated under output power of 180 W is substantially similar to that in downward ultrafiltration stirred with a rotational speed of 20.9 rad/s without ultrasonic irradiation. However, it is of importance to note that there is a marked difference in the lysozyme rejection. The lysozyme transmission in ultrasonic upward ultrafiltration is much higher than that in stirred ultrafiltration. Therefore, it is
believed that the ultrasonic irradiation is quite effective not only for concentration of proteins by ultrafiltration [68] but also for fractionation of the binary protein mixtures by ultrafiltration.

Conclusions

As was seen from this review, an understanding of the properties of the filter cake formed on the filter medium or the membrane can serve as a basis for clarifying the real mechanism of cake filtration and membrane filtration. In recent years, experimental testing procedures have been newly developed such as filtration experiments in which a filter was subjected to a sudden reduction in its filtration area and ultracentrifugation experiments. The compressible cake filtration model which explicitly took the non-homogeneity and the compressibility of the filter cake into consideration was used to describe the properties of the filter cake and the filtration behaviors. In protein ultrafiltration, the validity of the compressible cake filtration model was verified by the measurements of the concentration distributions in the filter cake using the principle of inclined filtration where a large amount of filter cake is formed. Moreover, it was shown that the solution properties (in particular, the solution pH and the electrolyte strength) play an important role in the filtration behaviors of colloids. The filtration behaviors of mixtures can be strongly affected by the nature of the interaction between the dissimilar components. Factors influencing fractionation behaviors by filtration were clarified. The author believes that many existing separation problems would have been avoided by the application of available scientific data although the random nature of most particulate dispersions has resulted in a difficult process problem.

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Nomenclature

- $c_i$: mass fraction of lysozyme in filtrate
- $c_s$: NaCl concentration of solution (mol/m$^3$)
- $D$: inner diameter of inserted cylinder (m)
- $D_h$: diameter of hole (m)
- $d_i$: average equivalent spherical diameter of solute (m)
- $E$: empirical constant defined by Eq. (22) (Pa$^{-9}$)
- $E_0$: empirical constant defined by Eq. (13)
- $e$: local void ratio
- $e_{av}$: average void ratio
- $e_{av,\omega}$: average void ratio from filter medium surface to distance $\omega$ from filter medium
- $h$: height of filter chamber (m)
- $J$: filtration rate (m/s)
- $J_0$: apparent solvent velocity relative to solid at arbitrary position $\omega$ (m/s)
- $K_v$: Ruth coefficient of constant pressure filtration defined by Eq. (2) (m$^2$/s)
- $L$: thickness of filter cake (m)
- $m$: ratio of wet to dry cake mass
- $n$: empirical constant defined by Eq. (12)
- $p$: applied filtration pressure (Pa)
- $p_m$: pressure loss through filter medium (Pa)
- $p_s$: local solid compressive pressure (Pa)
- $p_{si}$: pressure below which $e$ remains constant (Pa)
- $R$: distance from center of rotation to bottom of sedimentation (m)
- $R_{obs,\omega}$: apparent rejection of lysozyme defined by Eq. (24)
- $r_1$: radial distance of sedimentation boundary from center of rotation (m)
- $r_s$: distance from center of rotation to surface of sediment (m)
- $S$: sedimentation coefficient (s/rad$^2$)
- $s$: mass fraction of solids in slurry, or mass fraction of solutes in bulk feed solution
- $s_0$: mass fraction of BSA in bulk feed solution
- $s_1$: mass fraction of lysozyme in bulk feed solution
- $s_t$: mass fraction of titanium dioxide particles in slurry
- $u$: apparent liquid velocity relative to solid (m/s)
- $u_1$: filtration rate (m/s)
- $v$: cumulative filtrate volume collected per unit effective medium area (m$^3$/m$^2$)
- $v_1$: cumulative filtrate volume collected per unit effective medium area until cake surface reaches disk with hole (m$^3$/m$^2$)
- $v_0$: sedimentation velocity (m/s)
- $w_0$: net mass of solid per unit effective medium area (kg/m$^2$)
- $x$: distance from medium surface (m)
- $\alpha$: local specific filtration resistance (m/kg)
Average specific filtration resistance \( (m/kg) \)

\( a \) empirical constant defined by Eq. (12) \( (m^{a_1}s^{2n}/kg^{a_1}) \)

\( b \) empirical constant defined by Eq. (22)

\( e \) local porosity

\( e_{ar} \) average porosity

\( z \) zeta potential \( (V) \)

\( q \) filtration time \( (s) \)

\( m \) viscosity of filtrate \( (Pa\cdot s) \)

\( r \) density of filtrate \( (kg/m^3) \)

\( r_s \) true density of solid \( (kg/m^3) \)

\( y \) angle between filtrate flow and direction of gravity \( (rad) \)

\( W \) angular velocity \( (rad/s) \)

\( w \) net solid volume per unit medium area lying from medium up to an arbitrary position in cake \( (m) \)

\( w_0 \) net solid volume of entire cake per unit medium area \( (m) \)

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**Author’s short biography**

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Dr. Eiji Iritani is Professor of the Department of Chemical Engineering at Nagoya University. He received his PhD from Nagoya University in 1981. He has worked extensively on most aspects of solid-liquid separation. These works now also include membrane separation, water treatment, and colloid engineering.