Modelling of the gas fluidization of a mixture of cohesive and cohesionless particles by a combined continuum and discrete model[†]

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Abstract

This paper presents a numerical study of the gas fluidization of a mixture of 45,000 cohesive and cohesionless particles ($D=100\ \mu\text{m}$ and $\rho=1,440\ \text{kgm}^{-3}$) using a Combined Continuum and Discrete Model (CCDM). In such a model, the motion of individual particles is obtained by solving Newton's second law of motion and flow of continuum fluid by the local averaged Navier-Stokes equations. In this work, the cohesion among particles is caused by van der Waals interactions. The Hamaker constants are used to distinguish the cohesivity among particles so that finite values are assigned to cohesive particles and zero values to cohesionless particles. It is shown that the presence of cohesionless particles in an assembly of cohesive particles can improve their flowability and that sustainable fluidization can be achieved if the amount of cohesionless particles is sufficient.

1. Introduction

It has long been recognised that when fluidized by gas, particles show four distinct types of behaviour, i.e., cohesive, aeratable, sand-like and spoutable [1]. These effects are believed to be due to the variation in the relative importance of interparticle cohesive forces, such as van der Waals, capillary and electrostatic forces, compared with the fluid drag force exerted on particles by a fluidizing gas. However, such effects are difficult to quantify both analytically and experimentally due to difficulties in describing multibody interactions and in measuring interparticle forces in such a dynamic environment. With the recent development in numerical methods and computer technology, the effects of interparticle cohesive forces on gas fluidization can be evaluated by means of computer simulations.

Together with appropriate boundary and initial conditions, the solutions to Newton's second law of motion for discrete particles and the Navier-Stokes equations for continuum fluid will theoretically deter-

mine the solids and fluid mechanics in a gas-fluidized bed. In practice, however, there are usually a very large number of particles in such a bed. Consequently, this requires a very large number of governing equations to be solved for the motion of each particle, and the resolution of the fluid field has to be fine enough to resolve the flow of fluid through the gaps among closely spaced particles. The task is almost prohibitive with the current computing capacity. As a result, depending on the time and length scales of interest, simplifications have to be made when this theoretical approach is followed. This is reflected in the three models applied to the modelling of particle-fluid flow systems, i.e. Two-Fluid Model (TFM), Direct Numerical Simulation (DNS) and Combined Continuum and Discrete Model (CCDM).

In TFM, both solid and gas phases are treated as interpenetrating continuum media in a computational cell which is much larger than the individual particles but still small compared to the size of the process equipment, so that the number of governing equations is reduced significantly [2]. Two sets of local-averaged Navier-Stokes equations can be derived for both solid and fluid phases, which are then solved numerically. Since the first numerical simulation showing realistic bubbling in a gas-fluidized bed by Pritchett et al. [3], TFM has dominated the modelling

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of gas fluidization for decades, as summarized by Gidaspow [4]. However, successful applications of TFM depend on the establishment of constitutive laws governing the inter-phase transfer of momentum and energy for a given system, which cannot be derived from the continuum framework employed in TFM.

In DNS, the fluid field is resolved at a scale comparable with particle spacings while particles are treated as discrete moving boundaries [5]. DNS has great potential to produce detailed results of the hydrodynamic interactions between particles and fluid in a system [5, 6]. However, a major weakness of this model is its capacity in handling particle-particle interactions. In the earlier development of DNS [5], interactions among particles were not modelled at all: if the gap between two approaching particles was less than a preset small value, the simulation had to stop. In a recent development [7, 8], an arbitrary repulsive body force was introduced into the momentum equation to prevent possible collisions between particles. Therefore, DNS has mainly been applied to particle-liquid flow systems where either liquid is sufficiently viscous or the density difference between liquid and particles is small, so that the hydrodynamic interactions are dominant and particle-particle interactions are non-violent if they occur. This limits its applicability to gas fluidization where interactions among particles and hence interparticle forces are significant.

In CCDM, the motion of individual particles is determined by solving Newton's second law of motion, while the flow of continuum fluid is determined in the same way as TFM. As a result, two phases are modelled at two different length scales: with the solid phase at an individual particle level and the fluid phase at a computational cell level. Therefore, correct coupling between these two scales is important, and can be achieved by the principle of Newton's third law of motion as suggested by Xu and Yu [9, 10]. The major advantage of this model is its ability in handling detailed particle-particle and particle-wall interactions based on the Distinct Element Method (DEM) [11]. CCDM has been successfully applied to various particle-fluid flow systems [12-18]. Among those works, Horio and his co-workers [12] studied the effects of capillary forces on gas fluidization for a type D powder [1] whilst Rhodes and his co-workers [18] demonstrated the influence of an arbitrary attraction force, which is scalable to particle weight, on the fluidization of type B and D powders. Recently, Xu et al. [19, 20] studied the fluidization behaviour of a type A powder with van der Waals

interactions. In this work, the gas fluidization of a mixture of 45,000 cohesive and cohesionless particles ($D=100\ \mu\text{m}$ and $\rho=1,440\ \text{kgm}^{-3}$) is investigated. The aim of this study is to exploit possible ways of improving the flowability of cohesive powders in a gas-fluidized bed by adding cohesionless particles.

2. Combined continuum and discrete model

2.1. Discrete model

The translational and rotational motions of particle i at any time t in a bed are determined by Newton's second law of motion which can be written as

$$m_i \frac{d\mathbf{v}_i}{dt} = \mathbf{f}_{pf,i} + m_i \mathbf{g} + \sum_{j=1}^{k_i} \mathbf{f}_{pp,ij} \quad (1)$$

and

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = \sum_{j=1}^{k_i} \mathbf{T}_{pp,ij} \quad (2)$$

where m_i , I_i , $\mathbf{v}_i$ and $\boldsymbol{\omega}_i$ are, respectively, the mass, moment of inertia, translational and rotational velocities of particle i . The forces involved are: the particle-fluid interaction force, $\mathbf{f}_{pf,i}$, gravitational force, $m_i \mathbf{g}$, and interparticle forces, $\mathbf{f}_{pp,ij}$, between particles i and j . $\mathbf{T}_{pp,ij}$ represents the interparticle torques. For multiple interactions, the interparticle forces and torques are summed for k_i particles interacting with particle i . Here, interparticle forces and torques also include possible contributions from particle-wall interactions.

2.1.1. Interparticle forces

The interparticle forces result from particle-particle interactions. Generally speaking, these interactions include the forces due to direct or non-direct contacts between particles. In this work, the direct contact forces include the contact force and the viscous contact damping force, which are calculated based on the linear model given by Cundall and Strack [11]:

$$\mathbf{f}_{c,ij} = -\kappa_i \boldsymbol{\delta}_{ij} \quad (3)$$

and

$$\mathbf{f}_{d,ij} = -\eta_i \mathbf{v}_{ij} \quad (4)$$

where κ_i and η_i are, respectively, the spring constant and viscous contact damping coefficient of particle i ; $\boldsymbol{\delta}_{ij}$ is the displacement vector between particles i and j , and $\mathbf{v}_{ij}$ is the velocity vector of particle i relative to particle j at the point of contact, defined as $\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j + (\boldsymbol{\omega}_i \times \mathbf{R}_i - \boldsymbol{\omega}_j \times \mathbf{R}_j)$. $\mathbf{R}_i$ is a vector running from the particle centre to the contact point with its magnitude equal to the radius of particle i , R_i .

The increment of the displacement, $\Delta\delta_{ij}$, can be determined from the motion history of particles i and j , given by

$$\Delta\delta_{ij} = v_{ij}\Delta t_{c,ij} \quad (5)$$

where $\Delta t_{c,ij}$ represents the actual contact time between two colliding particles that can be determined based on the collision dynamics model developed by Xu and Yu [9]:

$$\Delta t_{c,ij} = \frac{-(\mathbf{r}_{ij} \cdot \mathbf{v}_r) + \sqrt{(\mathbf{r}_{ij} \cdot \mathbf{v}_r)^2 + |\mathbf{v}_r|^2(R_{ij}^2 - |\mathbf{r}_{ij}|^2)}}{|\mathbf{v}_r|^2} \quad (6)$$

where $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$, $\mathbf{v}_r = \mathbf{v}_i - \mathbf{v}_j$, $R_{ij} = R_i + R_j$, and $\mathbf{r}$ is the position vector of a particle at its mass centre.

Equations (3) and (4) are applicable to both normal and tangential directions. However, if $|\mathbf{f}_{ct,ij}| > \gamma_{s,ij}|\mathbf{f}_{cn,ij}|$, then sliding occurs, and $|\mathbf{f}_{ct,ij}| = \gamma_{s,ij}|\mathbf{f}_{cn,ij}|$. $\gamma_{s,ij}$ is the sliding friction coefficient between particles i and j . The subscripts n and t represent, respectively, the normal and tangential components.

The non-direct contact forces that affect the gas fluidization comprise interparticle cohesive forces such as van der Waals, capillary and electrostatic forces. There is a general agreement in the literature that the interparticle cohesive forces encountered in the gas fluidization of fine powders are mainly attributed to van der Waals attractions [21, 22]. According to Israelachvili [23], the van der Waals force between two closely spaced spheres i and j is given by

$$\mathbf{f}_{v,ij} = \frac{H_{ij}d_{ij}}{6z^2} \frac{\mathbf{R}_i}{R_i} \quad (7)$$

where H_{ij} is the so-called Hamaker constant between two particles i and j , z is the separation distance between two interacting surfaces, and d_{ij} is the radius of curvature at the point of contact which is equal to $(R_i \times R_j) / (R_i + R_j)$ for particle-particle interactions and R_i for particle-wall interactions.

In practice, the estimation of the van der Waals force acting on real particles is complicated by factors such as the surface geometry of contacting particles, local deformation of contact areas, hardness of particle materials, and gas adsorption onto particle surfaces. These factors can change the magnitude of the van der Waals force significantly and are difficult to quantify. In this work, $H_{ij} = \max(H_i, H_j)$, so that there is still an attraction force if a cohesive particle i (H_i) approaches a cohesionless particle j ($H_j = 0$). To avoid the singularity in applying equation (7), a minimum separation distance, $z_{min} = 1.0 \times 10^{-9}$ m [22], is used.

2.1.2. Interparticle torques

The tangential components of the forces due to the contact between particles i and j will generate a torque at the contact point, causing particle i to rotate. This interparticle torque can be calculated by

$$\mathbf{T}_{c,ij} = \mathbf{R}_i \times (\mathbf{f}_{ct,ij} + \mathbf{f}_{dt,ij}) \quad (8)$$

Moreover, the relative rotation among contacting particles will produce a rolling friction torque [24]. The effect of this rolling friction on the angle of repose of a sandpile has been demonstrated [25, 26] where a constant rolling friction coefficient was chosen. In this work, based on a simplified contact mechanics [19], the rolling friction torque is given by

$$|\mathbf{T}_{r,ij}| = \gamma_{r,ij} |\mathbf{f}_{cn,ij}| \quad (9)$$

where $\gamma_{r,ij}$ is the dynamic rolling friction coefficient, defined as $\sqrt{R_i^2 - (R_i - 0.5\delta_{ij})^2}$ for particle-particle contacts and $\sqrt{R_i^2 - (R_i - \delta_{ij})^2}$ for particle-wall contacts, which are valid as $\delta_{ij} < 2R_i$. This interparticle torque is dissipative in nature, i.e. $\mathbf{T}_{r,ij} = 0$ if $\boldsymbol{\omega}_i = 0$.

2.1.3. Fluid drag force

For gas fluidization, the particle-fluid interaction force is mainly attributed to the fluid drag, and the buoyancy force acting on a particle can be ignored as the density of particles is much larger than that of gas. The fluid drag force acting on individual particles depends not only on the relative velocity between particles and interstitial fluid, but also on the presence of other particles surrounding them. It is extremely difficult to determine this force analytically. On the other hand, empirical correlations have been established for the evaluation of this force in both fixed and fluidized beds over the full practical range of particle Reynolds numbers [27, 28]. According to Di Felice [28], the fluid drag force acting on a single particle in a fluid stream with the presence of other particles can be expressed as [10]

$$\mathbf{f}_{pf,i} = \mathbf{f}_{pf0,i} \varepsilon_i^{-(\chi_i+1)} \quad (10)$$

where ε_i is the porosity around particle i , taken as the porosity in a computational cell in which particle i is located. The fluid drag force acting on particle i in the absence of other particles, $\mathbf{f}_{pf0,i}$, and the equation coefficient, χ_i , are respectively given by

$$\mathbf{f}_{pf0,i} = 0.5c_{d0,i} \rho_f \pi R_i^2 \varepsilon_i^2 |\mathbf{u}_i - \mathbf{v}_i| (\mathbf{u}_i - \mathbf{v}_i) \quad (11)$$

and

$$\chi_i = 3.7 - 0.65 \exp \left[-\frac{(1.5 - \log_{10} Re_{p,i})^2}{2} \right] \quad (12)$$

where ρ_f is the fluid density. $c_{d0,i}$, the fluid drag coefficient for an isolated particle, and the particle Reynolds number, $Re_{p,i}$, are given by

$$c_{d0,i} = \left(0.63 + \frac{4.8}{Re_{p,i}^{0.5}} \right)^2 \quad (13)$$

and

$$Re_{p,i} = \frac{2\rho_f R_i \varepsilon_i |\mathbf{u}_i - \mathbf{v}_i|}{\mu_f} \quad (14)$$

where μ_f is the fluid viscosity.

2.2. Continuum model

The continuum fluid field is calculated from the local-averaged continuity and the Navier-Stokes equations based on the mean variables over a computational cell. Under isothermal and incompressible conditions, these equations are given by

$$\frac{\partial e}{\partial t} + \nabla \cdot (\varepsilon \mathbf{u}) = 0 \quad (15)$$

and

$$\frac{\partial (\rho_f \varepsilon \mathbf{u})}{\partial t} + \nabla \cdot (\rho_f \varepsilon \mathbf{u} \mathbf{u}) = -\nabla p - \mathbf{F} + \nabla \cdot (\mathbb{T}) + \rho_f \varepsilon \mathbf{g} \quad (16)$$

where $\mathbf{u}$, p and $\mathbf{F}$ are, respectively, the fluid velocity, pressure and volumetric particle-fluid interaction force; $\mathbb{T}$ and ε are the fluid viscous stress tensor and porosity in a computational cell which are given by

$$\mathbb{T} = \left[\left(-\frac{2}{3} \mu_f \right) \nabla \cdot \mathbf{u} \right] \delta_K + \mu_f \left[(\nabla \mathbf{u}) + (\nabla \mathbf{u})^{-1} \right] \quad (17)$$

and

$$\varepsilon = 1 - \frac{\sum_{i=1}^{k_c} V_i}{DV} \quad (18)$$

where δ_K is the Kronecker delta. ΔV and V_i are, respectively, the volume of a computational cell and the volume of particle i inside this cell. k_c is the number of particles in the cell. In the present work,

$\Delta V = 2\Delta x \Delta y R_i$ and Δx and Δy are, respectively, the lengths of a computational cell in x and y directions.

2.3. Coupling between continuum and discrete models

Mathematically, the coupling between continuum and discrete models is reflected by the calculation of the volumetric particle-fluid interaction force. This is realised by Newton's third law of motion so that the fluid drag force acting on individual particles will react on the fluid phase from the particles. As the fluid drag force is known for each particle, the volumetric particle-fluid interaction force in a computational cell can be determined by

$$\mathbf{F} = \frac{\sum_{i=1}^{k_c} \mathbf{f}_{pf,i}}{DV} \quad (19)$$

2.4. Solution schemes

The explicit time integration method is used to solve the translational and rotational motions of a system of particles in the discrete model. The interparticle force models are also applicable to interactions between a particle and a wall, with the corresponding wall properties used. However, the wall is assumed to be so rigid that no displacement and movement result from this interaction. The SIMPLE method [29] is used to solve the equations for the fluid phase in the continuum model. The second-order central difference scheme is used for the pressure gradient and divergence terms. A third-order upwind and bounded scheme [30] is used for the convection term, and a second-order Crank-Nicolson scheme is used for the time derivative. The no-slip boundary condition applies to the bed walls, and a uniform gas velocity is specified at the bottom of the bed. The zero normal gradient condition applies along the boundaries for the other parameters and at the top exit for the gas velocity.

Table 1 lists the parameters used in the present

Table 1 Parameters used for the present simulation*

Solid phase			Gas phase		
Particle shape	spherical		Type of gas	air	
Number of particles	45,000		Viscosity, μ_f	1.8×10^{-5}	$kgm^{-1}s^{-1}$
Particle diameter	1.0×10^{-4}	m	Density, ρ_f	1.205	kgm^{-3}
Particle density, ρ_p	1,440	kgm^{-3}	Bed width	2.5×10^{-2}	m
Spring constant, κ	50	Nm^{-1}	Bed height	6.0×10^{-2}	m
Sliding friction coefficient, γ_s	0.3		Bed thickness	1.0×10^{-4}	m
Contact damping coefficient, η	1.65×10^{-5}	$kg s^{-1}$	Cell width, Δx	1.0×10^{-3}	m
Hamaker constant, H	$6.00 \times 10^{-19}, 0$	Nm	Cell height, Δy	1.0×10^{-3}	m

*. The wall properties such as κ , γ and η are the same as those for particles.

simulations. The method suggested by Xu and Yu [9] is used to determine the computational time step and viscous contact damping coefficient, which are determined to be 1.25×10^{-6} s and 1.65×10^{-5} kgs $^{-1}$, respectively. To generate an initial particle configuration in the calculation domain, the bed is divided into a set of square cells with its length equal to the diameter of particles. Along the height of the bed, each adjacent cell is offset by a distance of one particle radius. Then 45,000 particles are randomly positioned in these cells and allowed to settle to form a packing under gravity. The packed bed thus generated is then used as a base condition for the later simulations of fluidization where van der Waals forces are introduced in particle-particle and particle-wall interactions.

3. Results and Discussion

As demonstrated in the previous work [19, 20], the behaviour of fine particles in a gas-fluidized bed depends strongly on the ratio of the magnitude of interparticle cohesive forces to the particle weight. When this ratio is less than 30, smooth fluidization is achieved with bubbles of relatively small size rising through the bed. When the ratio is in the range of 40 to 100, although the bed is still fluidizable, the fluidization quality deteriorates significantly and severe channelling occurs, giving the so-called quasi-fluidization. When the ratio is greater than 250, fluidization is impossible by normal means to such an extent that the gas simply flows through gaps between solid blocks made from primary cohesive particles, and the bed is de-fluidized. The predicted trends agree with the experimental observations where the bed changes from smooth, quasi-, to de-fluidization state when the size of particles is decreased [21].

Figure 1 shows the gas-solid flow patterns with

an initial two layers of 5,000 cohesionless particles placed alternatively in the bed of 40,000 cohesive particles. When the gas is introduced uniformly from the bottom of the bed, the particle assembly is lifted and detached from the bottom. During its upwards movement, the solid segments across the bed start to bend, which causes fractures at the interfaces along the layers of cohesionless particles. These fractures promote preferential gas flow and lead to the final breakage of solid segments into solid blocks. After the release of a large gas vortex, these solid blocks fall back to the bottom of the bed under gravity. A macroscopically stable de-fluidized bed is quickly established where gas simply flows through the gaps between the solid blocks. However, as highlighted in **Figure 2**, localised vigorous solids motion can still exist in such a bed: cohesionless particles trapped in the gaps between solid blocks can form a well-defined solid vortex. This localised solids motion is caused by the free flow of cohesionless particles under gravity and the strong underneath gas flow through the gap between the solid block and the side wall. This phenomenon is similar to the so-called raceway phenomenon in blast furnace iron making [16]. It is also noticed that there are layers of cohesive particles stuck onto the side wall, giving a very rough surface to the gas and solids flows.

From the above results, it is apparent that to facilitate the fluidization of a bed of cohesive particles, more cohesionless particles are needed. **Figure 3** shows the gas-solid flow patterns with an initial five layers of 12,500 cohesionless particles placed alternatively in the bed of 32,500 cohesive particles. Similar to the above two-layer case (see **Figure 1**), the preferential gas flow breaks the solid segments across the bed layer by layer along with the upward movement of the particle assembly. However, a segregated bed is established after initial disruption to the bed.

Fig. 1 The gas-solid flow patterns with an initial two layers of 5,000 cohesionless (white) particles placed alternatively in the bed of 40,000 cohesive (black) particles at $u/u_{mf}=4.8$. The ratio of the maximum cohesive force at z_{min} to particle weight is 338.

Fig. 2 The localised solids motion in a macroscopically de-fluidized bed at $t=4s$ and $u/u_{mf}=4.8$, showing the circulation of cohesionless (white) particles in the gap between blocks of cohesive (black) particles and side wall. The ratio of the maximum cohesive force at z_{min} to particle weight is 338.

The cohesionless particles are mainly found at the upper bed where they are fluidized, while the solid blocks are at the lower bed where they are de-fluidized. In the middle of the bed, there are some cohesionless particles trapped in the gaps among solid

blocks, they may be transported further upwards by the gas flow or downwards under gravity through these gaps. The simulated solid segregation patterns are comparable to the segregation experiments when particles of different sizes are fluidized [31]. However, after such an initial solid segregation, solid blocks at the upper bed are further migrated towards the side walls, leaving a V-shape zone at the centre for cohesionless particles where they are fluidized freely.

Figure 4 shows the gas fluidization of a reversed particle configuration as shown in **Figure 1**. Two layers of 5,000 cohesive particles were placed alternatively in the bed of 40,000 cohesionless particles. A stable fluidized bed is established after these two layers of cohesive particles are broken into straw-like solid blocks by the strong gas flow. These straw-like solid blocks further reduce their size by shearing action from fluidizing cohesionless particles. This results in a wide size and shape distribution of solid blocks made from primary spherical cohesive particles. **Figure 5** shows the details of such a size reduction mechanism where the ‘tail’ of a solid block is about to be torn apart by the shearing action from fluidizing cohesionless particles. From **Figure 4**, it is noticed that the solid blocks with a high aspect ratio tend to settle to the bottom of the bed, while the solid blocks with a low aspect ratio are fluidized smoothly with the cohesionless particles. It is also found that solid blocks with a high aspect ratio can only be dragged into the upper bed when they are tilted on the bottom of the bed. However, they do not rise in the wake of the bubbles as for primary spherical particles, but in a series of jerks as successive bubbles pass through the bed. These results are in good agreement with experimental observations by Bilbao et al. [32] where straw and sand particles were fluidized.

Fig. 3 The gas-solid flow patterns with an initial five layers of 12,500 cohesionless (white) particles placed alternatively in the bed of 32,500 cohesive (black) particles at $u/u_{mf}=4.8$. The ratio of the maximum cohesive force at z_{min} to particle weight is 338.

Fig. 4 The gas-solid flow patterns with an initial two layers of 5,000 cohesive (black) particles placed alternatively in the bed of 40,000 cohesionless (white) particles at $u/u_{mf}=4.8$. The ratio of the maximum cohesive force at z_{min} to particle weight is 338.

Fig. 5 The details of a size reduction mechanism where the 'tail' of a solid block made from primary cohesive (black) particles is about to be torn apart by the fluidizing cohesionless (white) particles at $t=4s$ and $u/u_{mf}=4.8$. The ratio of the maximum cohesive force at z_{min} to particle weight is 338.

4. Conclusions

The work on cohesive particle flows has been extended to study the possible ways of improving the flowability of cohesive particles in a gas-fluidized bed.

It is shown that by adding a sufficient amount of cohesionless particles, sustainable fluidization can be achieved for cohesive particles which are difficult to fluidize by normal means. Further study is needed to find an optimised procedure for improving the flowability of cohesive particles.

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Nomenclature

c_{d0}	coefficient of fluid drag for an isolated particle	<i>dimensionless</i>
d	radius of curvature at point of contact	<i>m</i>
D	diameter of a particle	<i>m</i>
f	force acting on a particle	<i>N</i>
F	volumetric particle-fluid interaction force	Nm^{-3}
f_{pf0}	fluid drag force acting on an isolated particle	<i>N</i>
g	gravitational acceleration	ms^{-2}
H	Hamaker constant	<i>Nm</i>
I	moment of inertia of a particle, defined as $I=2/5mR^2$	kgm^2
k_c	number of particles in a computational cell	<i>dimensionless</i>
k_i	number of particles interacting with particle i	<i>dimensionless</i>
m	mass of a particle	<i>kg</i>
p	pressure	<i>Pa</i>
r	position vector of a particle	<i>m</i>
R	radius of a particle	<i>m</i>
Re	Reynolds number	<i>dimensionless</i>
t	time	<i>s</i>
T	torque acting on a particle	<i>Nm</i>

u	fluid velocity	ms^{-1}
v	translational velocity of a particle	ms^{-1}
V	volume of a particle	m^3
z	separation distance between two interacting surfaces	m

Greek letters

ρ	density	kgm^{-3}
δ	displacement between two contacting particles	m
χ	equation coefficient defined in Eq. (12)	<i>dimensionless</i>
$\mathbb{T}$	fluid viscous stress tensor	$kgm^{-1}s^{-2}$
ε	porosity	<i>dimensionless</i>
ω	rotational velocity of a particle	s^{-1}
κ	spring constant of a particle	Nm^{-1}
μ	viscosity	$kgm^{-1}s^{-1}$
η	viscous contact damping coefficient of a particle	kgs^{-1}
Δt_c	actual contact time between two colliding particles	s
δ_K	Kronecker delta	<i>dimensionless</i>
ΔV	volume of a computational cell	m^3
Δx	computational cell length in x direction	m
Δy	computational cell length in y direction	m
$\Delta \delta$	increment of displacement between two contacting particles	m
γ_r	rolling friction coefficient	m
γ_s	sliding friction coefficient	<i>dimensionless</i>

Subscripts

c	contact
d	damping
f	fluid phase
i	particle i
ij	between particles i and j
j	particle j
mf	minimum fluidization
n	normal component
p	particle phase
pf	particle-fluid
pp	particle-particle
r	rolling
s	sliding
t	tangential component
v	van der Waals

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Author's short biography

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