A Review of the Modeling of High Pressure Grinding Rolls

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Abstract

High Pressure Grinding Rolls (HPGR) technology is accepted as an energy-efficient and cost-effective alternative for treating specific mineral ore types. HPGR technology has been advancing within manufacturing facilities and research centers since its first installation in 1985. Over the last three decades much of the literature on HPGR have focused on the industrial applications and trade-off studies in comparison with semi-autogenous and ball milling circuits. Literature on fundamental studies of HPGR technology has been very limited. This paper aims to provide a review of the modeling of high pressure grinding rolls.

Keywords: HPGR, modeling, interparticle breakage, particles compression, comminution

1. Introduction

More than thirty years have passed since the first industrial installation of high pressure grinding rolls (HPGR). Through this time, HPGR technology has advanced on many technical fronts. Nowadays, it is accepted as a standard technology for certain types of ore materials (Battersby et al., 1992; Patzelt, 1992).

The development of HPGR technology originated from the fundamental studies that the late Prof. Schönert, the inventor of HPGR, and his colleagues performed on interparticle breakage (Schönert, 1985, 1988). From his extensive research on single-particle breakage under compression and impact loading, Schönert (1988) concluded that the highest efficiency for particle breakage is only possible through slow compression loading of single particles. Schönert then extended his research to the breakage of beds of particles in a piston-die press. He observed that confined particle-bed breakage was less energy efficient than single-particle breakage yet was far more efficient than tumbling mill grinding (See Fig. 1). The dominant factors affecting the energy efficiency of the particle-bed stressing in a confined environment include the form of energy distribution to particles and the particle/particle interaction (Abouzeid and Fuerstenau, 2009).

Following this finding, the same results obtained in particle-bed loading could be repeated in a continuous mode with a small-scale HPGR. This led to a patent being issued to Schönert in 1977 (German Patent Number DE-2708053). In an oral history interview, Schönert stated that his patent was for a process and not for a machine¹. Schönert’s patent was for a process that compressed a bed of particles to a certain pressure that may require de-agglomeration of the product. He immediately began negotiations for licensing his patent to two German manufacturers, Polysius (currently known as thyssenkrupp Industrial Solutions) and KHD, to produce the required roller mills for industrial applications. Manufacturing an industrial-scale machine that would operate under the specific conditions of dry grinding circuits, found particularly in cement plants, proved to require extensive design and development. One of the challenges that led both companies to expend a substantial amount of effort was resolving how to handle wear rate on the rolls. Later, Köppern, another German manufacturer that had developed the technology of roller presses for the briquetting process, further contributed in promoting the HPGR technology. Briquetting machines are used for the compaction of granular materials that may share a few similarities with HPGR machines.

Fig. 1 The order of energy efficiency for different modes of breakage.

¹ Schönert’s interview is published in the appendix on pages 599 to 629 of Fuerstenau’s oral history (Fuerstenau, 2011).
At this point it might do well to comment on the terminology that has been used to designate high compression particle-bed comminution. When publishing in English, Schönert tended to call the technology either “high compression roller mills” or “high-pressure roller mills”, HPRM. Other designations were also used for the high pressure grinding technology with respect to a wide range of applications from coarse crushing to very fine grinding, such as ‘high pressure roller crusher’, HPRC. The generic term that has been widely settled upon is HPGR, high pressure grinding rolls, and is used in this paper as well. It should be noted that each manufacturer has its own registered trade name for their particular HPGR.

Quite rapidly, HPGRs were widely installed in cement plants throughout Germany and Europe. Schönert stated that he believed the rapid installations of energy-efficient HPGRs were due to the high cost of electrical power in Germany (Fuerstenau, 2011). The first major application of HPGRs by the mining industry was at the diamond mines of DeBeers in South Africa. The motivation in this case was the enhanced liberation of diamonds with HPGRs (Fuerstenau, 2011). The strong development and design efforts by the manufacturers led to an increasing number of installations in a wider range of industries.

The rapid utilization of HPGR technology in industrial applications led to publications that mainly presented general information about new installations, benefits achieved, as well as challenges. However, there were very few publications by manufacturers at the early stage of development that discussed the scale-up of HPGRs. That said, Schönert and his students at the Technical University of Clausthal were actively involved in the fundamental research that helped delineate interparticle breakage phenomena in a HPGR through systematic studies with a piston-die apparatus and a 200 mm diameter by 100 mm length lab-scale HPGR (Otte, 1988; Schönert, 1988, 1991; Schönert and Lubjuhn, 1990; Norgate and Weller, 1991; Lubjuhn and Schönert, 1993; Reichardt and Schönert, 2003). Schönert also developed fundamental relationships for defining the scale-up factors for the design of HPGR units. Additionally, by capturing high-speed videos, Schönert investigated the behavior of particle beds inside a HPGR (Schönert, 1991; Lubjuhn and Schönert, 1993).

By the early 1990s, Fuerstenau and his research team focused on this energy-efficient grinding technology by utilizing a single-particle roll mill as well as a 200 mm by 100 mm high pressure grinding rolls at the University of California, Berkeley, USA. They studied the effect of HPGR operating parameters and developed fundamental models to simulate high pressure breakage phenomena. (Fuerstenau et al., 1991; Fuerstenau et al., 1993; Fuerstenau et al., 1996; Fuerstenau and Abouzeid, 2007). As access to laboratory-scale HPGRs became prevalent, more research was dedicated to studying various aspects of high pressure grinding performance and attendant models (Austin et al., 1993; Lim et al., 1996b; Rashidi, 2014).

Although the majority of research about HPGR technology is limited to lab-scale units due to the large capacity of these machines, Klein and his research team at the University of British Columbia have been able to demonstrate interesting pilot-scale results through running tests in a 750 mm by 220 mm HPGR unit manufactured by Köppern (Drozdjak et al., 2011; Nadolski et al., 2011; Rosario et al., 2011). Daniel (2002) also obtained valuable information about HPGR scale-up through extensive tests using different sizes of HPGR machines.

As HPGR became accepted as a standard grinding technology, much of the research was generally directed toward establishing dimensionless numbers for characterizing grinding operations (Klymowsky and Liu, 1997; Daniel, 2003; Mcken and Williams, 2006; Morrell, 2006). Some research was also focused on applying discrete element modeling (DEM) to high pressure grinding techniques (Djordjevic and Morrison, 2006; Barrios et al., 2014). Even though DEM models are computationally demanding, they provide visual insight into the nature of particle breakage in the gap between the rolls.

Overall, HPGR is not yet considered mature in terms of modeling. In fact, modeling publications tend to be limited and scattered without proper verification. That being said however, there has not been a high demand within the industry for additional investigation and research on HPGR modeling. Rather, industrial publications show that the current focus is on developing new applications for HPGR technology as well as alternatives for improving the well-accepted layouts of this technology (Burchardt et al., 2011; Van der Meer, 2011).

This paper aims to review the evolution of the modeling since the introduction of this technology. We emphasize only those models that have the potential for further development in the future.

2. Principle of high pressure grinding

The energy efficiency of comminution methods is determined by the mobility or confinement of the particles being comminuted (Fuerstenau and Kapur, 1995). Schönert (1996) clearly showed that the most energy-efficient method of breakage is the direct application of force to a single particle or at least a monolayer of particles. The breakage of particles in a confined bed is the second most energy-efficient method of comminution, at least at low reduction ratios (Gutsche, 1993). As exemplified by ball milling, loose-bed comminution has the lowest efficiency, which is due to the hit-and-miss mode of energy transfer during particle/ball impacts. Schönert quickly realized the opportunity of increasing the energy...
efficiency of comminution by employing particle-bed breakage.

The efficiency of this method comes from the direct transfer of energy to particles and the highly localized interparticle stresses that are generated at the contact points of each particle in the bed (De, 1995). In other words, particles are stressed by each other provided that the average size of the largest particle is smaller than the width and height of the bed and the applied force is higher than the particle strength (Schönert, 1988, 1991). During compression of the particle bed, smaller particles fill the void space and thereby participate in the transfer of energy and the breaking and re-breaking of larger particles to a certain fineness (Klymowsky, 2009). This mode of breakage requires fewer particles to be in contact with the grinding medium, that is, the roll surface (Klymowsky, 2009).

There is, however, a negative aspect of particle-bed grinding, namely a retardation effect. Grinding virtually comes to a halt in highly compressed beds at sufficiently high pressure. This was shown by Fuerstenau et al. (1996) in a detailed study of confined particle-bed breakage of brittle materials whilst using a piston-die apparatus. It seems that the presence of a sufficient amount of fines in the bed and the buildup of an isostatic-like pressure field can lead to a near cessation of particle breakage, as illustrated in Fig. 2. Schönert and Mueller (1990) developed an expression that gives the fraction of narrowly-sized feed particles that are broken as a function of compaction energy, including a maximum fraction that can be broken. For efficient HPGR comminution, this points to operating HPGRs at low reduction ratios and to the removal of fines in recycle systems if the HPGR is to produce a finished product.

A system consisting of rolls that operates at high pressure was conceived by Schönert to translate the breakage of a confined particle-bed into a continuous form. The three key criteria for this system include direct transfer of particles inside the machine, direct discharge of the product, and the capability of adjusting the level of the applied force and thereby the energy input (Schönert, 1991). The basic principle is similar to briquetting machines that operate with pressures low enough to compact the particles without necessarily increasing the portion of fines (Schewechten and Mulburn, 1990). When it comes to particle bed comminution however, a high grinding pressure is required in order to compress the bed of particles to a bulk specific gravity of 70 to 85 percent of the ore’s specific gravity, which will cause the particles to break and thereby generate fines in the product (Schönert, 1988).

An HPGR machine consists of a pair of counter-rotating rolls, which are mounted in a very sturdy frame. While the bearing block of one roll is fixed, the other one can float linearly to adjust the gap between the two rolls as the material properties, such as particle size distribution, moisture, and hardness, change. A hydro-pneumatic spring system applies the grinding force directly on the particle bed via the floating roll. Larger gap openings result in higher grinding pressures. The extent of the gap
opening is controlled by the preset grinding force and initial gap. Each roll unit is normally equipped with a single drive motor that powers the roll through a gearbox. Feed material is introduced from the top, passes through the gap where it experiences the grinding force, and then leaves the machine from the bottom in the form of either free-flowing particulates (as with quartz) or as a compacted cake (as with limestone, dolomite, coal, etc.).

In a confined mode of particle-bed breakage, energy absorption is a main factor for the size-reduction process (Schönert, 1996). The specific grinding force has a direct effect on the amount of energy absorbed to a certain level, beyond which the bed is saturated and little-to-no breakage occurs (Fuerstenau et al., 1996; Schönert, 1996).

Fig. 3 shows a schematic illustration of the multiple zones that particles pass through in between the two counter-rotating rolls. Feed material first enters the acceleration zone (also known as the de-aeration, the pre-crushing, or the pre-grinding zone) (Schwechten and Mulburn, 1990; Austin et al., 1993; Lubjuhn and Schönert, 1993) and transfers through the feed hopper into the next zone, where particle-bed breakage under high pressure occurs. This transfer is accompanied with compaction during which, re-arrangement of the particles takes place and smaller particles fill the void space between the larger ones. Air that is trapped in the void space, leaves the particle bed from the top. Primarily governed by gravity at the beginning, the downward movement of the feed can be resisted by the rate of the upward airflow. For this reason, efficient ventilation is critical to optimal operation of the HPGR, especially in bigger units. Particles get compacted as they move through and, hence, friction forces between particle/particle as well as particle/roll surface increases, which assists with the acceleration of the particle bed. Bulk density also increases as material goes down the HPGR, where it may reach up to 85 percent of the ore’s specific gravity.

The zone that starts with the nip angle is called the grinding zone (also known as the compression zone). True interparticle breakage takes place in the grinding zone, where the particles bed is exposed to the highest grinding pressure. The grinding zone is limited to the shortest distance between the two rolls namely the working gap and, on the other hand, the critical gap, which is defined by the nip angle. The nip angle is influenced by the material properties, the roll surface pattern, and the roll speed.

In both research and industrial operation of HPGRs, feed material may either be mono-sized or have a distribution of particles. In research, when comparing energy-efficiency with single-particle breakage, the feed must be narrowly sized. Extensive industrial application of HPGRs has been the comminution of granulated blast-furnace slag in the production of cement. In this case, the particle bed forms as the feed particles are broken in the grinding zone. On the other hand, when it comes to feeds that have a distributed particle size, voids in the particle bed are filled with finer particles as they enter the acceleration zone. This should lead to a somewhat more energy-efficient comminution. Among several factors that affect the performance of HPGR grinding, moisture in the feed has been found to have a significant effect on HPGR operation because it influences friction and adhesion between particles as they are being squeezed in the gap (Fuerstenau and Abouzeid, 1998). Depending on the feed characteristics, optimum moisture content would help with efficient grinding. Moreover, it also helps, up to a certain extent, with the build-up of an autogenous protection layer on the surface of the profiled or studded rolls when treating highly abrasive ores.

The third zone below the working gap is called the relaxation zone. Compacted particles face relatively no pressure as soon as they leave the grinding zone, and

![Fig. 3 Schematic of the gap between rolls showing Acceleration, Grinding and Relaxation zones.](image-url)
Therefore, the thickness of the flakes in the discharge tend to expand slightly right after exiting the grinding zone. If necessary, the thickness of the flakes provide a good estimate for the working gap. If, of course, the fine particles do not agglomerate however (as seen in the case of grinding quartz), then the particles exiting the grinding zone will be free flowing.

Material moving between the two rolls is exposed to a gradient of pressure intensity along the gap. Schönert (1991) showed that the grinding pressure increases steeply in the compression zone, with the peak pressure being at the working gap or slightly above it. On the other hand, not all particles along the roll width experience the same pressure intensity. Because of this, multiple breakage modes take place depending on where the particles are passing through the gap and what is their relative size. Those particles in the bed that are larger than the critical gap, will break when they come in contact with the roll surface. This is because they are passing from the acceleration zone to the grinding zone. Depending on the construction of the housing of the rolls, there may also be an edge effect, where some of the feed particles experience partial compression in the grinding zone. In other words, particles passing through the center of the roll width are, due to the pressure gradient across the width, more compacted and therefore leave the rolls with a higher portion of fines than those passing through either end of the rolls.

3. Scale-up and sizing of HPGR

Soon after the introduction of HPGR, Schönert (1985) was the first to publish the key parameters, in the form of equations, for the scale-up of HPGR. Those equations have been widely quoted by others. Throughput, power, and the specific grinding force are the three main scale-up factors required for sizing a HPGR machine. To begin, a series of tests need to be carried out in order to obtain the necessary data for the quantification of the scale-up factors.

The energy efficiency of HPGR is achieved via true interparticle breakage. Measuring the nip angle allows one to suitably define the top size of the feed that would minimize single-particle contact breakage as much as possible and, instead, expose most of the feed to interparticle breakage mode. Any particle larger than the top size is normally broken in the pre-crushing zone repeatedly until being small enough to enter the compression zone. Nip angle is mostly a function of the material properties and both the roll dimensions and surface pattern. If material characteristics inside the grinding zone are known, then the nip angle can also allow one to measure the volumetric flowrate of particles. Klymowsky et al. (2002) used the geometry of the grinding zone to calculate the nip angle, $\alpha_c$:

$$\alpha_c = \cos^{-1}\left[1 - \left(\frac{\delta_1}{\delta_0} - 1\right) \times \left(\frac{x_g}{1000D}\right)\right]$$  \hspace{1cm} (1)

The nip angle theme is central to the models proposed in the literature. Austin et al. (1993, 1995) showed that the specific grinding pressure has an inverse effect on the specific throughput. Depending on the specific grinding pressure and the compressibility of the ore bed, the operating gap takes on a value of its own. Austin et al. (1993, 1995) also showed that a higher pressure reduces the working gap and thereby diminishes the volumetric flow through the rolls. The relationship that Austin et al. (1993, 1995) derived between the relative gap ($x_p/D$) and the specific throughput (see Eqn. 7 for definition) is as follows:

$$\dot{m}_v = \frac{x_p}{1000D} \left(1 - \theta_0\right) \cos\alpha_c + \left(1 - \theta_0\right) \cos\alpha_c (1 - \cos\alpha_c)$$  \hspace{1cm} (2)

Austin et al. (1993, 1995) relationship allows one to determine the nip angle and the critical gap (the distance between the two rolls at the nip angle) by fitting Eqn. 2. Several grinding tests with similar conditions under at least three substantially different levels of operating pressure are needed to find the linear relationship between the specific throughput and the relative gap. For details, refer to Dhawan et al. (2012). The nip angle, depending entirely on the feed characteristics and roll surface pattern, ranges from 4 degrees up to 10 or 11 degrees (Schewechten and Mulburn, 1990; Schönert, 1991; Lim and Weller, 1999).

The HPGR mass flowrate is proportional to the dimensions of the rolls because the geometry of the gap is directly related to the capacity of the machine. Schönert (1985) and Schönert and Lubjuhn (1990) quantified this relationship as follows:

$$M = \rho (1 - \theta_0) x_p \mu_m L 3.6 = \delta_f x_p \mu_m L 3.6$$  \hspace{1cm} (3)

Eqn. 3 is well quoted in the literature and is called the continuity equation. The mass flow between the rolls is illustrated in Fig. 4. As can be seen, Eqn. 3 can hold true as long as the cake (product) density remains constant along the roll width and the phenomenon of the edge effect is neglected (Lubjuhn and Schönert, 1993). Only small HPGR machines, with the roll diameter less than 300 mm, can provide uniform compaction through the roll width. As a result, a more practical approach to define mass flowrate is to conduct actual measurement during the operation. The volumetric flowrate can then be defined as follows (Schönert, 1988):

$$V = \frac{M}{\rho}$$  \hspace{1cm} (4)

With replacing Eq. 3, Schönert (1988) showed that the
The volumetric flowrate is a function of the rolls geometry and material characteristic:
\[ V = (1 - \theta_g) x_g u_m L 3.6 = k \left( \frac{u_m}{u} \right) V_p \quad (5) \]

With \( k = (1 - \theta_g)(x_g/1000) D \) being a material constant, knowing that the working gap is linearly related to the roll diameter of a given material, and assuming material velocity is equal to the roll velocity, volumetric flowrate can be measured based on the roll dimensions and the roll speed within a good approximation. Schönert (1988) called \( V_p \), the potential volumetric flow rate that replaces Eqn. 5, given as:
\[ V_p = uLD \quad (6) \]

In order to minimize the influence of the machine size on the measured throughput, the specific throughput parameter was introduced. The specific throughput parameter, also widely known as “m-dot” in the industry, provides the expected throughput for a HPGR, with 1 m diameter and 1 m length rolls running at 1m/s peripheral speed (Klymowsky et al., 2002):
\[ \dot{m}_c = \frac{x_g}{D} \cdot \delta_g \cdot 3.6 \quad (7) \]

The necessary data to calculate \( \dot{m}_c \) can be measured from the flake density, \( \delta_g \), in the HPGR discharge. At the same time however, this formula can only be used for lab-scale machines as long as the edge effect is negligible. For this reason, the following formula is more practical for real applications:
\[ \dot{m}_t = \frac{M}{uLD} \quad (8) \]

The value obtained from Eqn. 8 provides information about the response of the material to high pressure grinding. It should be noted that certain parameters, such as feed properties and specifics of the HPGR machine, influences the value. For more details refer to Rashidi (2014). Practice has shown that the specific throughput rate is influenced by the size of the machine, often caused by the size distribution of feed particles and the internal friction between particles (Burchardt et al., 2011). Evidently, larger feed sizes in the plant-scale HPGRs could behave differently under compression than finer feeds in pilot-scale HPGRs. This is the reason that most manufacturers prefer to run scale-up tests in large units because it minimizes the effect of machine size as much as possible.

In the interparticle mode of breakage, grinding occurs when the compressive force is higher than the particle strength. A higher specific grinding force increases the rate of fines generated in the product at a diminishing rate (Schönert, 1996), as discussed in Section 2. The optimum HPGR specific grinding force can be defined via HPGR grinding tests. It is broadly discussed across the HPGR publications that the compressive milling force is the main controlling parameter for the fineness of the HPGR product. The specific grinding pressure is considered to be one of the scale-up factors as the effect from the roll dimensions is taken off through dividing the milling force by the roll width and length:
\[ F_{sp} = \frac{F}{DL \times 10^6} \quad (9) \]

The specific grinding force is the average force applied on the projected area of the rolls. At the same time however, there is a pressure gradient along the width. Further, the pressure across the gap rises to a very high value at the center of the working gap and then drops very quickly. De and Fuertestenau (2012) presented a model for calculating the pressure profile across the gap at different angular positions in the vicinity of the nip angle. This study, verified with a piston-die apparatus, showed that the maximum pressure inside the bed can reach as high as 200 to 250 MPa (De, 1995). Schönert and Lubjuhn (1990) estimated it to be 40 to 60 times the applied specific grinding force:
\[ F_{max} = \left( \frac{1}{a_{c,F}} \right) F_{sp} \quad (10) \]

Extensive results from lab-scale to industrial-size machines show that the required power for driving rolls increases with the specific grinding force. HPGRs are designed in such a way that the grinding force is applied on the floating roll, perpendicular to the projected area of the roll. The applied force is split into two force components: tangential and radial. The tangential component creates a torque on the roll, which determines the required power to drive the roll. One of the rolls is normally exposed to less force compared to the other one, however Schönert (1988) simplifies it by assuming that equal force is applied to both rolls. Therefore, the total torque needed to drive both rolls is given by:
\[ T = 2\left( \frac{D}{2} \right) F \sin \beta = DFB \beta \quad (11) \]
The point at which the grinding force is applied on the roll is determined by the force-acting angle $\beta$. The force-acting angle should be less than the nip angle in order to efficiently apply the grinding force on the particle bed. The power required for driving the roll is determined by the force and the force-acting angle. With $\sin \beta = \beta$ for small angles, the power is calculated as follows:

$$P = \omega T = \left(\frac{2\pi N_u}{60}\right) \left(DF\beta\right) = 2\beta V \mu F_p = 2\mu F$$  (12)

There are several necessary measurements to define the scale-up factors, which include the specific throughput, the optimum specific grinding force in relation to the required fitness, and the specific power consumption, that can be determined through a series of tests. Fig. 5 summarizes the interrelationship between the scale-up parameters. HPGR design and, to a great extent, material properties majorly affect these parameters. Overall, the scale-up of HPGRs has not faced any significant changes since the introduction of the technology.

4. Particle breakage modeling

Developing a realistic mathematical model for the size reduction in HPGRs has really not been possible due to a variety of reasons. The fact that HPGR was put into commercial use so quickly and given the restricted access to scale-up as well as the industrial datasets are just a few of the reasons why the progress in this area has been slow. In this section, those models that have shown potential for further development are discussed.

As was pointed out, grinding in interparticle bed breakage is directly controlled by the specific energy input. With respect to HPGR grinding, Fuerstenau et al. (1993) showed that the reduction ratio for a single pass of material varies linearly with the specific energy input, which holds true for a limited range:

$$\frac{X_f}{X_{s0}} = f E_{sc} + e$$  (13)

Unlike in ball milling, the residence time for individual particles in the bed is the same in HPGR. All particles in the bed experience a fixed-time of residence (Fuerstenau et al., 1991). Even then, Gutsche and Fuerstenau (2004) showed that similar to ball mills, the self-similar behavior holds true for the product size distribution of the HPGR. In other words, different product cumulative size distributions follow one master curve (Fuerstenau and Kapur, 1995) when expressed as a function of relative size (size of particle divided by the product median size) which is given by:

$$F(x, E) = Z\left(\frac{x}{X_{50}}\right)$$  (14)

Utilizing the self-similarity concept, some researchers proposed that by fitting an empirical master curve to the self-similar distributions, the product size distribution can be easily predicted at any specific energy input as long as the median size is determined (Lim et al., 1996a; Daniel, 2002). Based on these observations, Fuerstenau et al. (1991) modified the ball mill grinding kinetics equation in terms of the cumulative energy input instead of the grinding time to predict the product size distributions in HPGR:

$$\frac{dM_j(E)}{dE} = -S_j^E M_j(E) + \sum_{j=1}^{i} S_j^E M_j(E); j = 1, \ldots, i-1$$  (15)

Assuming the breakage function can be normalized, the specific selection function can then be based on the actual energy that goes into grinding by incorporating an energy dissipation exponent, $\gamma$ (Fuerstenau et al., 1991):

$$S_j^E = \frac{P}{E_{sc}}; \quad 0 \leq y < 1$$  (16)

For this full batch-grinding model, both selection function and breakage function had to be back-calculated from experimental data, resulting in a total of eight exponents and constants to be simultaneously fitted (Fuerstenau et al., 1991; Fuerstenau et al., 1993). An important finding was that the specific selection function is nearly constant, independent of particle size, in contrast to ball milling. This should be expected since the same amount of energy is transmitted to all particle sizes in confined particle-bed comminution. This model was successfully verified using the experimental data obtained in a lab-scale HPGR that was developed in collaboration between Prof. Schönert at the Technical University of Clausthal and Prof. Fuerstenau at the University of California, Berkeley (Gutsche, 1993). This lab unit with 200 mm diameter by 100 mm width rolls can process sample as small as 10 kg. This machine was later handed over to the University of Utah under the supervision of Prof. Rajamani.

The assumption of using the energy-normalized selection function for grinding in HPGRs could only hold true for low specific energy input where there is a linear function between the energy input and the fineness of the
product. Unlike ball milling, where energy is a direct function of time, higher specific energy in particle-bed breakage does not necessarily relate to a higher reduction ratio (Austin et al., 1993). Moreover, the application of this model needs to be further investigated for larger machines where, for a given specific energy input, the product size distribution varies along the rolls width.

In a separate study, Austin and Trubelja (1994) modified the roll crusher model to predict the action of grinding in HPGR. The roll crusher model was previously developed by Austin et al. (1980). It should be noted that grinding in roll crushers is completely different from HPGR grinding. In roll crushers, the gap is fixed and no compressive pressure is applied to the rolls. Hence, breakage occurs through contact with the rolls. Individual particles larger than the gap repeatedly break into smaller sizes until they pass through the rolls.

Austin et al. (1993) defined two sets of selection functions based on the geometry of the gap in a HPGR. The first selection function was defined for large particles broken by contact, which normally happens above the critical gap in the acceleration zone. This selection function was normalizable by the working gap as a function of specific grinding force and therefore, the energy input. The second selection function was defined for particles with equal or smaller size compared to the critical gap, which are nipped into the compression zone and broken by interparticle stresses. It is important to note that the largest particle present in the feed is defined by the working gap. Austin and Trubelja (1994) at the time did not suggest any particular compression selection function and, instead, used an equation based on the calculated particle size distribution to determine this selection function. Later, Schneider et al. (2009) incorporated the specific grinding pressure into Austin’s model and defined the compression selection function.

Austin and Trubelja (1994) proposed a sequential discrete-size model to simulate the product size distribution. It is based on two sets of particle-size distributions: particles that are left unbroken and are denoted as \( P_i \), and particles that enter from larger size fractions after breakage and are denoted as \( P_i^* \). With \( 1 - a_i \) and \( 1 - a_i^* \) to be the mass of the fraction of particles left unbroken after nipping (pre-crushing) and compression, respectively, the product size distribution is then calculated as follows:

\[
P_i = \begin{cases} 
(1-a_i)(1-a_i^*); & i = 1 \\
(1-a_i)(1-a_i^*)P_i^*; & 1 < i \leq n
\end{cases}
\]

(17)

where,

\[
P_i^* = \begin{cases} 
0; & i = 1 \\
-a_i^*b_{ij} + \sum_{j=1}^{i-1} b_{ij}a_j^*P_j^*; & 1 < i \leq n
\end{cases}
\]

(18)

with

\[
a_i^* = a_i + a_i'(1-a_i)
\]

(19)

Similarly, two sets of normalizable breakage functions were considered based on the two modes of breakage, namely contact (nipping) and compression. To simplify the model however, they assumed that the values for both breakage functions are the same. The experimental data used for fitting the model were obtained in a 100 mm by 30 mm lab-scale HPGR manufactured by Polysius (Austin et al., 1993). To verify this model, Schneider et al. (2009) used six sets of experimental data that were obtained in a 250 mm in diameter by 100 mm in length lab-scale HPGR equipped with studded rolls, also manufactured by Polysius.

The multi-component model developed by Austin and Trubelja (1994) is primarily empirical and employs a large number of model parameters. It is difficult to regenerate the results by back-calculations unless some constraints are imposed on some of the model parameters. Also, the edge effect that is a characteristic of larger machines is not addressed in this model.

Since the dominant form of breakage in a HPGR unit is particle-bed comminution, feed properties, especially feed size distribution profoundly influences the final product size distribution. Various arrangements of particles in the bed lead to distinctive stressing patterns under the force loading, which accordingly result in different product size distributions. This led Liu and Schönert (1996) to pursue the application of the well-developed population balance models for predicting the product size distribution in a confined particle bed as a function of energy input. They introduced an energy split function in order to determine the energy absorption for each size fraction via an iterative algorithm:

\[
k_j = \frac{E_j}{E} = \sum_{j=1}^{n} k_j M_j(E) = 1
\]

(20)

Assuming that breakage and selection functions are characteristic features of material, these two parameters were quantified via controlled tests with mono-sized particles in a piston-die apparatus. Liu and Schönert (1996) tests were focused on brittle material such as quartz. They showed that the influence of the energy input increases as the distribution of particle sizes widens. However, due to the complex iterative nature of the method, Liu and Schönert (1996) did not recommend it for a wide feed size distribution since the errors between the measured and the fitted values would inherently increase.

Klymowsky and Liu (1997) further developed Liu and Schönert’s model to simulate the entire spectrum of product size distribution for an industrial HPGR unit. As explained earlier, the final HPGR product is a combined
result of three grinding actions that take place as material passes through the two rolls. Those include pre-crushing, compression, and edge effect. Correspondingly, Klymowsky and Liu (1997) multi-component HPGR model consisted of three steps for quantifying the product size distribution: (1) predicting the resulting particle size distribution in the pre-crushing zone through sequential breakage similar to Austin and Trubelaia (1994) method; (2) predicting the resulting particle size distribution in the center using the population balance model similar to Liu and Schöntart (1996) model; (3) defining the theoretical particle size distribution for the edge product and solving the energy consumption balance between all three breakage zones based on the fact that a pressure gradient exists between the two rolls.

Klymowsky and Liu (1997) created a large experimental dataset based on a 900 mm diameter by 250 mm pilot-scale HPGR. They set the selection and breakage functions and the energy split function according to the results that Liu and Schöntart (1996) had obtained in their ideal particle-bed tests on quartz. Nonetheless, around 10 remaining model parameters still needed fitting via back-calculation. Therefore, to avoid large variations, Klymowsky and Liu (1997) proposed splitting the model parameters into secondary and primary sets. They suggested that secondary parameters should be determined through a few fundamental tests such as running controlled experiments with narrow-sized feed. Then the value for each secondary parameter would be fixed by averaging it over the range obtained in each experimental dataset. Following this, primary parameters could be further refined through a new set of data and an averaging technique would be used to fix a value for each primary model parameter. They showed that simulation results based on this technique were promising. This model was further evaluated by Van der Meer (2010).

Similarly, Morrell et al. (1997b) adopted a modular structure to predict throughput, power, and product size distribution. The product size distributions for three defined breakage zones within a HPGR are independently calculated and combined at the end to produce the final results. To do so, Morrell et al. (1997b) individually applied a modified version of the Whiten-Awachi-Anderson crusher model (Anderson, 1988) for each grinding zone. The Whiten-Awachi-Anderson crusher model features a classification function that defines the probability that particles would break in a classifier based on the preset gap openings given as:

\[ C(x) = 1 - \left( \frac{K_3 - x}{K_2} \right)^{K_3} \]

Crushers operate in a stepwise action and, therefore, particles bigger than the gap openings \( K_2 \) and \( K_3 \) in Eqn. 21 would break repeatedly until become small enough to pass through the gap without further size reduction. This is similar to the breakage action in the pre-crushing zone, where large particles breaks sequentially in order to be smaller than the critical gap to enter the grinding zone (Morrell et al., 1997b; Daniel and Morrell, 2004). Morrell et al. (1997a) defined the critical gap based on the geometry of the grinding zones as follows:

\[ x_c = 0.5 \left( D + x_g \right) \left( \left( D + x_g \right)^2 - \frac{45Dx_g}{\delta v} \right)^{-0.5} \]

Inside the compression zone, Morrell et al. (1997b) considered two sub breakage mechanism namely edge-effect and compression. While in reality all particles inside the compression zone go under compression to some extent, with the highest force exerted on particles in the center of the gap, in their model, Morrell et al. (1997b) assumed that resulting particles from edge-effect go under single-particle breakage similar to what was assumed for the pre-crushing zone. In order to determine the fraction of particles that undergo the edge effect, Morrell et al. (1997a) defined a split function as follows:

\[ f = \gamma \frac{x_g}{L} \]

The classification function used in Whiten-Awachi-Anderson crusher model employs three parameters and with the addition of the fourth parameter, \( t_{10} \), a total of 12 parameters need to be fitted to experimental data to produce the final product size distribution. As shown by Fuerstenau and Kapur (1995), the HPGR product size distribution follows a master curve, which makes it logical to use \( t_{10} \) as a breakage characteristic parameter. The value of \( t_{10} \) varies with the energy input. However, as mentioned earlier, the energy input and grinding force as the dominant factor for controlling the product fineness, forms a linear relationship for only a limited range.

A large number of the parameters are given fixed values to simplify the approach. However, the remaining critical parameters are determined via conducting breakage characteristics tests such as drop-weigh test in addition to HPGR tests. The application of breakage characteristics tests could be referred as an important feature for the Morrell-Tondo-Shi model. Among the parameters, \( K_2 \) takes the value of critical gap for the pre-crushing zone and the value of working gap for the portion of material going through the edge-effect zone. In other words, \( K_2 \) is the controlling factor for the top size of the product. Meanwhile, the value of \( K_3 \), which is an ore characteristic, is only determined for the compression zone and along with \( t_{10} \) has to be fitted to experimental data obtained from HPGR tests. The iterative back-calculation continues until the difference between the simulated and measured energy input reaches a minimum.

Morrell-Tondo-Shi model has been further reviewed by...
Danish (2002) and Benzer et al. (2001) by vast data from various sizes of HPGR machines, putting it in a unique position compared to the rest of the work done so far. Nevertheless, it does not fully include the fundamental breakage mechanism that takes place in the compression zone with respect to all parameters contributing to the final product size distribution.

Torres and Casali (2009) developed a modular structure on the basis of Morrell-Tondo-Shi model to simulate throughput, power, and product size distribution from pilot-scale HPGR tests. Their model for predicting product size distribution shares a similar structure with Morrell-Tondo-Shi model, but with a different approach as it features a discretized system for the particle bed. With this approach, the gap between the two rolls is divided into \( N \) blocks. These blocks start from the critical gap and end at the working gap.

To predict the product size distribution, Torres and Casali (2009) benefited from the population balance modeling technique. They assumed that all particles larger than the critical gap would instantaneously break and enter the compression zone along with the rest of the particles that are already small enough to form the feed for the compression zone. The final product size distribution is the combined particle size distributions from each block. The resultant particle size distribution is not only a function of the feed particle size, but also depends upon the position of the particles inside each block in addition to the position of the corresponding block along the gap. Therefore, the population balance model was modified accordingly (Torres and Casali, 2009):

\[
\frac{dM_{ik}(z)}{dz} = -S_{ik}M_{ik}(z) + \sum_{j=1}^{i-1} h_{ij}S_{jk}M_{jk}(z) \tag{24}
\]

Eqn. 24 results in \( N \times n \) matrix of ordinary differential equations with \( N \) and \( n \) being the total number of blocks and particle size classes, respectively. Torres and Casali (2009) suggested an analytical solution for Eqn. 24 as follows:

\[
p_{ik} = \sum_{j=1}^{i} A_{ij} \exp \left( \frac{S_{jk}}{u_k} - z \right) \tag{25}
\]

where

\[
A_{ij} = \begin{cases} 
0 & i < j \\
\sum_{j=1}^{i} b_{ij}S_{ik} - S_{ik}A_{ik} & i > j \\
f_k^{\text{BP}} - \sum_{i=1}^{i-1} A_{ik} & i = j 
\end{cases} \tag{26}
\]

It was assumed that the breakage function is the same for each particular ore independent of the position inside the compression zone. The breakage function needed to be calibrated with the experimental data. Using the functional format of selection function, three additional parameters including the specific breakage rate, \( S_{ik} \), required fitting to experimental data.

Unlike Morrell-Tondo-Shi model, Torres and Casali (2009) considered that compression takes place along the roll width even though the intensity follows a parabola patter resulting in coarser product towards the ends. However, their model does not offer a way to quantify the fraction of edge product that does not undergo true inter-particle breakage as opposed to those particles passing through the center of the compression zone.

In a simplified approach to apply the population balance model to HPGR grinding, Dundar et al. (2013) assumed that the dominant grinding mechanism in HPGR is true compression bed. Dundar et al. (2013) determined the breakage function through running tests in a piston-die setup using a narrow-size feed. The breakage function parameters were estimated using the Narayanan (1986) single-particle impact breakage model. With the breakage function being considered as the characteristics of materials and constant for each ore type, the selection function was estimated for the total HPGR discharge product via back-calculation. Therefore, the selection function was fitted to experimental HPGR data using Austin et al. (1976) power equation. In contrast to previous models, Dundar et al. (2013) model simplifies the grinding action in HPGRs to only true particle-bed breakage. Although this model does not consider individual grinding actions in industrial HPGRs, the published results are quite promising for further evaluation.

In another effort to simplify the population balance model for compression grinding, Esnault et al. (2015) defined the breakage mechanism of the particle-bed compression as a function of porosity. Porosity in the particle-bed changes with the applied energy. Esnault et al. (2015) assumed that breakage via compression can be defined by the generation of new surface area referring to Rittinger’s law:

\[
SS(F_{a+1}(x)) - SS(F_a(x)) = \frac{dSS(F(x))}{dE} \left( \theta_{a+1} - \theta_a \right) \tag{27}
\]

Esnault et al. (2015) emphasized that the tests for determining the breakage function values need to be performed under low pressure to minimise the occurrence of re-breakage. Esnault et al. (2015) results were based on the fine brittle material, which fits with the applicable range of particle sizes that is suggested for the Rittinger’s law. Esnault et al. (2015) noted the increased error for those tests under very high pressure and referred that to the effect of particle cushioning. Even though, grinding in HPGR units is more complicated than piston-die apparatus, Esnault et al. (2015) model provided a great under-
standing for measuring the breakage in particles bed related with porosity.

Looking at overall particle size simulation modeling works (Summarized in Table 1); the focus has been mainly on adapting the population balance model that has been well-developed for ball mills to the breakage behavior in HPGR. This approach has had limitations with the fact that particle-bed breakage rate does not necessarily increases with the energy input beyond the saturation point. Results published so far indicate that the ideal controlled way to determine the selection and breakage function for particle-bed breakage is through the use of piston-die apparatus. The major difficulty in such modeling is to include edge effect.

5. HPGR characteristics tests

As discussed earlier, the scale-up studies are mostly based on HPGR testing in pilot-scale machines with at least a minimum of 500 to 700 mm roll diameter. The particle-bed breakage is very sensitive to the feed properties and a real test in a HPGR machine would provide the most realistic results about the response of high compression on any specific material. However, HPGRs are known for their high capacity making it difficult to conduct feasibility studies, requiring large mass of ore material. Even the smallest lab-scale HPGR requires at least 20 to 25 kg material to maintain a long enough feed stream to log data under steady-state operation.

A considerable amount of research has been put into investigating the possibility of running small-scale batch tests as an indication for the amenability of HPGR. Piston-die apparatus represents the batch mode of interparticle breakage. Besides, it requires a small mass of sample, allows controlled compression, and applies compression at a desired rate. These advantages made the piston-die a standard tool for measuring the breakage rates and the breakage distribution functions in many of the modeling works.

De and Fuerstenau (2012) showed that the rotation angle is the key parameter to define the grinding zone, where the compression happens. They derived an equation to determine this angle based on the load-displacement profile in a piston-die test provided that the nip angle is known (De, 1995):

$$\alpha = \cos^{-1}\left(1 - \frac{h - h_i}{h_0 - h_i}(1 - \cos \alpha_i)\right)$$  \hspace{1cm} (28)

Knowing the angle of rotation, De and Fuerstenau (2012) related the pressure profile in an HPGR to the piston-die test data, and subsequently predicted the scale-up factors of HPGRs including force, torque, working gap, throughput, and the specific energy input. They also demonstrated that the normalized particle size distributions obtained from a lab-scale HPGR unit and piston-die setup are matching if the specific energy input is kept constant.

Daniel (2003) also presented interesting findings to relate the piston-die operating parameters to lab-scale HPGR operation including the critical gap, working gap, and the grinding force. He showed that the product size distribution in both machines for the same feed size distribution and the same material were matching under the specific energy input range of 2.5–3.5 kWh/t. Similar results have been reported by Kalala et al. (2011). In a novel approach, Rashidi (2014) defined a new series of controlled tests to estimate the selection and breakage functions via running tests in a lab-scale HPGR.

The most practical meaning for a characteristic test would be the formulation of a work index. A value that can show the performance of the HPGR unit to a specific ore regardless of any parameter involved such as feed size distribution and so on. Bond work index is the world-wide known factor that with all the limitations serves as a well-established technique to evaluate the response of tumbling mills to grinding a specific ore. However, this index itself is not directly applicable to HPGR grinding technology. Morrell (2004) formulated a general work index formula that could be applied to numerous grinding equipment including HPGR:

$$W_h = KM_a 4\left(x_i^{(c_2)} - x_i^{(c_1)}\right)$$  \hspace{1cm} (29)

Where $M_a$ is a breakage related index (HPGR work index) that is obtained through a specialized test known as SMC Test®. However, it is not clear how this parameter is defined and how it is related to the strength of the particles. $K$ is a constant that takes two different values depending on the mode of operation ($K = 1$ for a HPGR in closed circuit with a screen and $K = 1.19$ for an open circuit HPGR). Morrell (2006) showed that the exponent $f(x)$ in Eqn. 29 takes different values depending on the particle size as follows:

$$f(x) = -\left(0.295 + \frac{x_i}{1000000}\right)$$  \hspace{1cm} (30)

Data that Morrell (2006) used for the verification of this model were obtained through a lab-scale HPGR under the limited range of 2.5–3.5 kWh/t specific energy input. Later, Morrell (2010) introduced a new parameter, $S_m$, the material strength parameter, in Eqn. 29, in case feed particle size was greater than 25 mm relating to the fact that the particle strength changes with the size:

$$W_h = S_m KM_a 4\left(x_i^{(c_2)} - x_i^{(c_1)}\right)$$  \hspace{1cm} (31)

with
### Table 1  Summary of HPGR particle size simulation modeling works.

<table>
<thead>
<tr>
<th>Model structure</th>
<th>Salient features</th>
<th>Areas for improvement</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Modifying ball mill grinding kinetics equation in terms of energy input.</em>&lt;br&gt;<em>Introducing an energy dissipation component to normalize the specific selection function.</em></td>
<td><em>Important observation that selection function is constant for all particle sizes.</em>&lt;br&gt;<em>Fully representing the similarities between ball mill and HPGR products.</em></td>
<td><em>Depends on the feed material properties.</em>&lt;br&gt;<em>Does not address the variation of particle size in different HPGR sizes.</em>&lt;br&gt;<em>Applicable to low energy input levels.</em></td>
<td>Fuerstenau et al., 1991; Fuerstenau et al., 1993</td>
</tr>
<tr>
<td><em>Modifying roll crusher model individually for two dominant breakage mechanisms in HPGRs: Contact and particle-bed.</em>&lt;br&gt;<em>The final product is a combination of broken and unbroken particles under two separate breakage mechanisms.</em></td>
<td><em>Model accounts for the fact that not all particles break as a result of the bed compression.</em>&lt;br&gt;<em>It adopts roll crusher model that represents the crushing mode in the acceleration zone.</em></td>
<td><em>Empirical only.</em>&lt;br&gt;<em>Depends on the feed material properties.</em>&lt;br&gt;<em>Does not account for the edge-effect (partial compression).</em></td>
<td>Austin et al., 1993; Austin and Trubelja, 1994</td>
</tr>
<tr>
<td><em>Providing two mass and energy balance equation to account for the effect of energy adsorption in different size classes.</em>&lt;br&gt;<em>Selection and function are defined by controlled tests in a piston-die apparatus using mono-sized particles.</em></td>
<td><em>First application of the PBM model for HPGR grinding.</em>&lt;br&gt;<em>Using controlled confined particle-bed breakage tests to quantify breakage characteristics.</em></td>
<td><em>Not suitable for wide feed size distributions.</em>&lt;br&gt;<em>Does not consider the pre-crusher and the edge-effect zones.</em></td>
<td>Liu and Schönert, 1996</td>
</tr>
<tr>
<td><em>A modular model that introduces an energy distribution function.</em>&lt;br&gt;<em>HPGR tests are combined with piston-die tests to simulate the product size distributions.</em></td>
<td><em>Accounting for three grinding actions that take place in an industrial HPGR.</em>&lt;br&gt;<em>Defining the grinding parameters through basic grinding tests.</em></td>
<td><em>Incorporating large number of model parameters.</em></td>
<td>Klymowsky and Liu, 1997</td>
</tr>
<tr>
<td><em>A modular model considering three grinding zones in an HPGR.</em>&lt;br&gt;<em>Adopting cone crusher model individually for each grinding zone.</em>&lt;br&gt;<em>Applying drop-weight test along with HPGR tests to fit the model.</em></td>
<td><em>Reconciling results from drop-weight test (breakage characteristic test) with the HPGR model.</em>&lt;br&gt;<em>Widely evaluated in industrial applications based on published material.</em></td>
<td><em>Large number of model parameters.</em>&lt;br&gt;<em>Does not consider the partial compression in the edge effect zone.</em>&lt;br&gt;<em>Limited to low energy input levels.</em></td>
<td>Morrell et al., 1997a; Daniel and Morrell, 2004</td>
</tr>
<tr>
<td><em>A modular discretized PBM model based on Morrell-Tondo-Shi model.</em></td>
<td><em>Embodied in the model the pressure gradient that exists in the compression zone.</em></td>
<td><em>Depends on the feed material properties.</em></td>
<td>Torres and Casali, 2009</td>
</tr>
<tr>
<td><em>Application of population balance model.</em>&lt;br&gt;<em>Measuring the breakage function via the piston-die test.</em>&lt;br&gt;<em>Smoothing out selection function based on material and operating conditions.</em></td>
<td><em>Quantifying selection function in accordance with the operating conditions and material properties.</em></td>
<td><em>Does not fully cover the multiple breakage actions in industrial and semi-industrial HPGRs.</em></td>
<td>Dundar et al., 2013</td>
</tr>
<tr>
<td><em>Modifying population balance model for the compression-bed breakage based on the creation of new surface area.</em></td>
<td><em>Application of Rittinger’s law to define the particle-bed breakage under the compressive load.</em></td>
<td><em>Applicable to low energy input due to the particle cushioning under higher pressure.</em></td>
<td>Ensault et al., 2015</td>
</tr>
</tbody>
</table>
Morrell work index is limited to the higher end of specific energy input range, under which there is a linear relationship between energy input and size reduction. This model being empirical keeps evolving.

6. Summary

The modeling of HPGR via the principles of selection and breakage function modeling concept has been partly successful. Many published works show internal consistency with the data generated in their works. However, the difficulty arises when scaling up the lab-model parameters to large machines. The principal difficulty has been the postulation of two different grinding zones in the radial direction and two different zones in the axial direction of the roll. Moreover, an experimental procedure for determining breakage functions independently is lacking. Hence, estimation of numerous parameters befuddles the modeling effort. Next, the models based on a work-index like structure, will have to depend on a single size to represent the entire size distribution. Therefore, more empirical coefficients are introduced in such models. Nevertheless, all the models have explained the internal dynamics of the high compression bed loading and the effect of machine parameters on loading. As of this date, the modeling effort continues to advance.

Nomenclature

\(a_i\) Nipping selection function (Austin model)
\(a'_i\) Compression selection function (Austin model)
\(b_{ij}\) Breakage function
\(c_F\) Material constant
\(C(x)\) Classification function
\(D\) Roll diameter [m]
\(E\) Energy input [kWh]
\(E_j\) Energy absorption in size class \(j\) [kWh/t]
\(E_{sc}\) Specific energy input [kWh/t]
\(f\) Split function (Morrell-Tondo-Shi model)
\(f^{IP}\) Feed fraction factor going under single-particle breakage (Torres-Casali model)
\(F\) Grinding force [N]
\(F(x)\) Particle size distribution
\(F(x, E)\) Particle size distribution function as a function of specific energy input
\(F_{wp}\) Specific grinding pressure [N/mm²]

\[ F_{\text{max}} \quad \text{Maximum specific grinding pressure [N/mm}^2\text{]} \]
\[ F(x_i) \quad \text{Exponent for Morrell work index formula} \]
\(h\) Particle-bed height at an applied force [mm]
\(h_0\) Initial particle-bed height [mm]
\(h_f\) Final particle-bed height [mm]
\(j\) Particle-bed grindability [t/kWh]
\(k\) Material constant
\(k_f\) Energy split function (Schöner model)
\(K_1\) Whiten-Awachi-Anderson crusher model parameter
\(K_2\) Whiten-Awachi-Anderson crusher model parameter
\(K_3\) Whiten-Awachi-Anderson crusher model parameter
\(L\) Roll length [m]
\(M\) Mass flowrate [t/h]
\(\dot{m}_c\) Specific throughput based on cake characteristic [ts/m³h]
\(\dot{m}_f\) Specific throughput based on flowrate [ts/m³h]
\(M_{hi}\) Morrell HPGR work index
\(M_i(E)\) Mass fraction of particles in size class \(i\) per unit of Energy input
\(M_{i,k}(z)\) Mass fraction of particles in size class \(i\) based on its vertical position \((z)\) in block \(k\)
\(P\) Power [kW]
\(P_i\) Mass fraction of particles left unbroken at size \(i\)
\(P_i^*\) Mass fraction of particles that enter size \(i\)
\(p_{i,k}\) Mass fraction of particles entering size \(i\) in block \(k\)
\(SS(x)\) Specific surface function
\(S_i^E\) The energy-normalized selection function at size \(i\)
\(S_i\) Selection function at size \(i\)
\(T\) Torque [Nm]
\(u\) Roll peripheral speed [m/s]
\(u_m\) Material peripheral velocity in the gap [m/s]
\(u_r\) Roll rotational speed [rpm]
\(u_\alpha\) Material peripheral velocity in the \(\alpha\) direction [m/s]
\(V\) Volumetric flowrate [m³/h]
\(V_p\) Potential volumetric flowrate [m³/h]
\(W_h\) Morrell work index
\(x\) Particle size
\(x_c\) Critical gap at the nip angle [mm]
\(x_g\) Working gap [mm]
\(X_f\) Median size in the feed [μm]
\(X_{50}\) Median size in the product [μm]
\(x_2\) 80 percent passing size for product [μm]
80 percent passing size for feed [μm]
γ Energy dissipation exponent
z Particle position inside gap (Torres-Casali model)
Ω Self-similar distribution function
α Angle of rotation
αᵣ Nip angle
β Force-acting angle
γ Split factor (Morrell-Tondo-Shi model)
δₖ Gap (cake) density [t/m³]
δᵢ Feed bulk density [t/m³]
θ Porosity
θₖ Gap (cake) porosity
ρ Material density [t/m³]
ω Roll radial velocity [1/s]

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Author’s short biography

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Samira Rashidi received her Ph.D. from the University of Utah in 2014. The principal emphasis of Samira’s research has been about improving efficiency of the grinding equipment and circuits via mathematical modeling and simulation. Her Ph.D. research was focused primarily on energy-size relationship and wear-rate measurement in HPGRs under the supervision of Prof. Raj Rajamani. Samira currently is a Process Engineer at thyssenkrupp Industrial Solutions (USA). She is involved in all processing aspects of new and existing grinding systems in the mining, minerals, and cement applications.

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After receiving his Sc.D. degree at MIT, Dr. Fuerstenau spent a six-year period teaching at MIT and working in industry, after which time he joined the faculty of the University of California. At Berkeley he established an extensive program of teaching and research in minerals processing, applied surface chemistry and particle technology. With his graduate students and postdoctoral researchers, he has published a wide range of seminal papers in this field. He currently is P. Malozemoff Professor Emeritus of Mineral Engineering in the University of California.