Forces on Particles in Time-Varying Magnetic Fields[†]

Jaclyn D. Ray, James R. Nagel, Dave Cohrs and Raj K. Rajamani*

[†] Department of Metallurgical Engineering, University of Utah, USA

Abstract

Electrodynamic sorting is a process that sorts metals based on conductivity, density, and geometry. The process works by inducing electrical eddy currents within particles placed in a time-varying magnetic field. For the special case of a perfect, uniform sphere, an approximate equation can be used to predict the net force under a linear magnetic gradient. This paper explores the accuracy of that model by measuring the net force on spherical samples of copper, brass, and aluminum with varying sizes and excitation frequencies. Results consistently show strong agreement with the approximate models over all conditions. We also explore several non-spherical geometries, including cylinders, cubes, and disks. We found that they could be modeled as equivalent spheres, given an appropriate radius, and had reasonable accuracy over frequency.

Keywords: electrodynamics, eddy current, time-varying magnetic field, scrap metal waste

1. Introduction

Scrap metal recycling is a high-value industry that reduces pollution from primary production. As it can potentially be recycled an infinite number of times, scrap metal may even eliminate the mining of some metals altogether. This factor alone is significant since reducing the primary production of metals is becoming an essential means of decreasing man-made pollutants that aggravate the environment (Förstner et al., 2012). In 2016, for example, aluminum recovered from old scrap was equivalent to about 31 % of apparent consumption while copper scrap contributed similar values to the total US supply (USGS, 2017). Such low percentages of recycled material reveal the importance of investing in scrap metal recycling.

Since the value of scrap metal depends heavily on purity, a key aspect of the recycling process is sorting. Obtaining high-grade metal, however, can be economically difficult for many reasons. Alloys, for example, often have nearly indistinguishable properties and are thus difficult to tell apart. Furthermore, the material feedstock might vary from day-to-day. Automobiles, for instance, have changed dramatically in composition since the 1980s and continue to do so today (Wernick and Themelis, 1998). Consequently, there is strong economic incentive to find efficient new methods for purifying scrap metal.

To meet this challenge, several technologies exist to recover metals from mixed material streams. Magnetic separators, for example, remove ferrous material through the use of fixed permanent magnets (Oberteuffer, 1974). Dense media separation recovers certain nonferrous metals by separating light elements from the heavy (Weiss, 1985). Mechanical eddy current separators recover electrically conductive metals from non-conductive materials by rotating a drum of alternating north/south magnets (Rem et al., 1998). Metallic particles entering the magnetic field are excited with electrical eddy currents, which in turn accelerate the particle due to magnetic forces against the moving electrical charge. Even hand sorting by visual inspection can be effective in certain developing countries where the cost of human labor is very low (Wilson et al., 2006).

Despite the many available technologies, material recovery is still very limited. For example, magnetic separators can only recover ferrous materials from nonferrous. Density separation cannot distinguish between alloys with similar densities. Mechanical eddy current separators are generally only effective on particles greater than an inch in size and when the materials that are being separated have significantly different conductivity (Norrgran and Wernham, 1991). Hand sorters cannot easily distinguish between metals that look alike nor can they effectively process small particles in large quantity.

One emerging technology that addresses many of these problems is electrodynamic sorting (EDX) (Dholu et al., 2017; Smith et al., 2017). The process is similar to mechanical eddy current separation but, rather than spin an array of permanent magnets, it utilizes a fixed electromagnet.

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[†] 135 S 1460 E, ROOM 412, Salt Lake City, UT 84112-0102, USA

* Corresponding author: Raj K. Rajamani

E-mail: raj.rajamani@utah.edu

TEL: +01-801-587-8329

Unlike mechanical eddy current separators, however, EDX can reach far higher frequencies of excitation and thus operate on particles well below an inch in size. The tunability of frequency and intensity also allows for separation as a function of density, conductivity, and even geometry.

Although the basic principle of EDX has been demonstrated empirically, there does not yet exist a clear mathematical framework by which to model the forces on scrap metal particles. The most relevant contributions to date appear to stem from Peter Rony (1964) and George Lohöfer (1989), both of whom studied the heating effects of eddy current induction on uniform spheres. More recently, Bidinosti et al. (2007), developed a workable model for the eddy current distribution throughout a uniform metal sphere in a time-varying magnetic field. The recent work of Nagel (2017) has also helped to formalize the theory of eddy current induction in various canonical geometries. To date, however, there still has yet to be any direct application of this theory to the field of scrap metal recycling.

This paper seeks to validate the theoretical predictions of forces acting on metal particles in a time-varying magnetic field. We begin by reviewing the relevant theory on the subject and modeling the net force acting on a uniform metal sphere. We then test that model by measuring the forces on several spherical particles under excitation by an EDX magnet. Lastly, we expand on that analysis by fitting the same equations to non-spherical geometries in an effort to quantify the behavior of more realistic particles. Results show that even non-spherical particles can be reasonably represented by spheres, thus greatly enhancing our ability to predict the behavior of scrap metal particles as they travel through an EDX magnet.

2. Theory

We consider a sphere with known radius a and conductivity σ situated in a uniform magnetic field $\mathbf{B} = B_0 \cos(\omega t) \hat{z}$. Since the excitation field is in a sinusoidal steady state, we can express this using phasor notation as simply $\mathbf{B} = B_0 \hat{z}$. Building on the work done by Rony (1964), the magnetic dipole moment $\mathbf{m}$ of the sphere can be shown to satisfy

$$\mathbf{m} = \frac{2\pi B_0 a^3}{\mu_0} \left[1 - 3 \frac{ka \cosh(ka) - \sinh(ka)}{(ka)^2 \sinh(ka)} \right] \hat{z} \quad (1)$$

$$\mathbf{m} = m_z \hat{z}, \quad (2)$$

where $k = \sqrt{j\omega\mu_0\sigma}$ and μ_0 is the permeability of free space. Due to the presence of the applied magnetic field $\mathbf{B}$, there also exists a net force $\mathbf{F}$ acting on $\mathbf{m}$, given by Jackson (1999)

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B}). \quad (3)$$

It is important to note that Eqn. 2 is only a first-order approximation derived under the assumption of a linear magnetic field gradient. It also shows that a uniform magnetic field will produce no net force since the quantity $\mathbf{m} \cdot \mathbf{B}$ becomes constant. We therefore introduce a linear gradient α to the magnetic field such that $\mathbf{B} = (B_0 + \alpha x) \hat{z}$. The gradient α is assumed to introduce only a small perturbation, meaning that $\mathbf{m}$ does not change significantly in the applied gradient. Substituting back into Eqn. 2 therefore reveals

$$\mathbf{F} = \nabla[m_z (B_0 + \alpha x)] \quad (4)$$

$$\mathbf{F} = \frac{\partial}{\partial x} [m_z B_0 + m_z \alpha x] \hat{x} \quad (5)$$

$$\mathbf{F} = m_z \alpha \hat{x}. \quad (6)$$

Noting that $\mathbf{F}$ is still a phasor quantity in sinusoidal steady state, we next calculate the time-averaged force using

$$\mathbf{F}_{\text{avg}} = \frac{1}{2} \text{Re}\{\mathbf{F}\}, \quad (7)$$

where $\text{Re}\{x\}$ denotes the real part of x . After some lengthy derivation, the time-averaged force finally evaluates to

$$\mathbf{F}_{\text{avg}} = -\frac{3\pi B_0 \alpha a^3}{\mu_0} \left[\frac{1}{3} - \frac{1}{q} \frac{\sinh(q) - \sin(q)}{\cosh(q) - \cos(q)} \right] \hat{x}, \quad (8)$$

where $q = 2a/\delta$ and $\delta = \sqrt{2/(\omega\mu_0\sigma)}$ is the skin depth.

3. Method

Fig. 1 shows a diagram of the experimental configuration. A signal generator sets a desired frequency and voltage, which is then fed to a high-current amplifier. In order to negate the high impedance from the magnetic induc-

Fig. 1 Schematic representation of the force measurement circuit.

Fig. 2 NiZn magnet and force gauge used for experiment.

tance, a capacitor bank is placed in series with the inductor to create a resonant RLC circuit. A $1.0\text{-}\Omega$ resistor is also placed in series with the capacitors to serve as a current-sense resistor for the entire circuit.

A custom-built 360-mm NiZn toroid magnet and a Mark-10 force gauge were used for our measurements. Depicted in **Fig. 2** the magnet is wound with 300 turns, using 18-gauge Teflon-insulated stranded-copper wire. At one end of the magnet, a specialized air gap was cut out so as to channel the magnetic field into a small volume of space. Once the system was activated, any test particles placed inside the gap were accelerated into the force gauge, thereby producing a force measurement. To negate gravity, the test particles were also glued to non-conductive string and suspended in the gap.

Because the probe on the force gauge is made out of metal, it could not be placed near the magnetic gap during excitation. To address this issue, a 3D-printed plastic extender was attached to the force gauge. The extender was then placed directly in front of the test particle, thereby catching it as soon as it was acted on by the magnetic field. The force then transferred directly down the rigid extender and into the gauge, resulting in a reliable force measurement.

Fig. 3 shows some metal samples used for our measurements. Though the theoretical model is technically only supposed to apply to spheres, industrial scrap metals rarely present as such perfect shapes. We were therefore particularly interested in testing the validity of Eqn. 8 on non-spherical geometries. The particle sizes used in this experiment ranged from 5 to 13 mm to represent typical sizes of fine scrap material. Each sample was also made from different metals and alloys, including copper, brass, and aluminum, which are commonly found in industrial scrap. For each metal shape, we likewise printed a unique plastic extender to fit the geometry. For example, an extender with a concave end would fit the spheres and cylinders, a flat end would hold the cubes, and a U-shaped end would steady the disks.

Fig. 4 shows the magnetic field profile that was measured at 10-A DC down the centerline of the gap. The

Fig. 3 Some test particles used in the force experiment.

Fig. 4 Magnetic field versus position at DC. The vertical lines indicate the volume of space where test particles were measured.

peak of the field at $x = 0$ indicates the rear of the gap at the inner radius to the toroid. Measurements at several frequencies between 0–10 kHz also showed a fairly consistent field profile over the bandwidth of interest. The vertical bars indicate the region where test particles were inserted and measured. This specific region was chosen due to its approximate linearity, high field intensity, and physical space into which the particles could fit.

Though the field profile was consistent across frequency, we did notice that the magnetic field decreased when particle samples were placed within the gap. This problem was caused by the eddy currents induced in the test sample itself, which tended to oppose the applied magnetic field of excitation. As a result, less magnetic flux would bridge the gap, thereby lowering the total inductance of the circuit. With the inductance lowered, the resonant frequency of the RLC circuit would shift slightly and thus lower the total drive current across the coils.

Since the excitation field is directly proportional to drive current, we compensated for it by simply increasing the drive voltage until the current reached its pretest amplitude.

Another problem we encountered during our measurements was thermal instability. When excited by a magnetic field, induced eddy currents tended to dissipate a great deal of heat throughout the particles. Even though measurements lasted for only a few seconds, the particle's temperature would often jump to over 100 °C. Since conductivity tends to somewhat vary with temperature, there was some worry about thermal consistency over the course of several measurements. To alleviate such concerns, all particles were immersed in a cup of ice water to ensure stable and reproducible temperatures before each test. Measurements were then repeated four times to ensure accuracy.

4. Results and Discussion

The first set of experiments focused solely on the spherical particles, with parameters displayed in **Table 1**. The radius r of each sphere was measured using a caliper. The magnetic field intensity B was measured from the center of the sphere using a Gauss probe and fixed to a value of 49.5 mT peak (or equivalently, 35 mT RMS) over all experiments. The field gradient α was then obtained by calculating the central difference around the center of each sphere. Note that there are small differences in α even though each measurement had the same magnetic field intensity. These differences are due to variations in the particle's placement in the gap, which was primarily determined by its size. Lastly, electrical conductivity σ of each particle was measured with a Fisher Sigmascopes conductivity probe. Results from the spherical force measurements are summarized in **Fig. 5** and strongly agree with the predictions of Eqn. 8.

Table 1 Test parameters used in the force calculations of sphere samples.

Material	r [mm]	B [mT]	α [T/m]	σ [MS/m]
Al 2017	2.8	49.5	−1.7	16.1
Al 6061	6.1	49.5	−1.8	25.5
Al 2024	6.3	49.5	−1.7	17.3
Al 1100	6.3	49.5	−1.7	24.3
Brass	3.2	49.5	−1.7	13.8
Brass	9.5	49.5	−1.8	12.7
Copper	2.8	49.5	−1.8	46.6
Copper	7.5	49.5	−1.8	49.6

Fig. 5 Spherical force measurements compared to analytic calculations.

While encouraging, it is important to remember that scrap metal waste is rarely spherical. At the same time however, due to the strong predictability and convenience of Eqn. 8, we would still like to try and approximate non-spherical geometries as spheres. To that end, we note that for any arbitrary geometry at a given excitation frequency, there exists a hypothetical sphere which will experience the same force under identical conditions. If the radius of such a sphere were to remain consistent over a large bandwidth, then we can use that sphere as an approximate model for the non-spherical particle. To test this hypothesis, we measured the force on several non-spherical geometries and plotted their frequency response against an ideal sphere with an appropriate radius.

To derive an equivalent spherical radius, we began by calculating the equivalent volume of some non-spherical particle. For example, a cylinder could satisfy

$$\pi r_{\text{cyl}}^2 h = \frac{4}{3} \pi r_s^3, \quad (9)$$

where r_{cyl} is the measured radius of the cylinder, h is the cylinder height, and r_s is the radius of some equal-volume sphere. Solving for r_s , we have

$$r_s = \sqrt[3]{\frac{4}{3} r_{\text{cyl}}^2 h} \quad (10)$$

We then multiply r_s by a correction coefficient ε to find

$$r_0 = \varepsilon r_s, \quad (11)$$

where r_0 is the equivalent radius of a sphere that would produce the same force as the measured particle under identical conditions. If $\varepsilon < 1$, then it can be understood that $r_0 < r_s$, thus implying a lower force density than would have been achieved if the particle were compacted into a sphere of equal volume. Likewise, for $\varepsilon > 1$, the particle has a higher force density than its equal-volume sphere. To find ε for a given particle, we simply searched over increments of 0.01 until the theoretical curve generated by Eqn. 8 reasonably fit the measured data.

Results of this experiment are summarized in **Fig. 6–8** with the parameters for each experiment summarized in **Tables 2–4**. For the cylinders, we found that ε was generally between 0.95–1.05. The only exception was a small brass cylinder, which may have been an experimental outlier. Cubes, however, were consistently between 0.92–0.99. The disks consistently resulted in ε values well above 1.2, which is most likely the result of their high cross-sectional area relative to their volume. If the disks were instead oriented along their minimum cross-section, then ε would naturally decrease accordingly. The key finding here, however, is that once a suitable r_0 was derived, Eqn. 8 seemed to produce a very good fit to the measured data. This seems to strongly indicate that even non-spherical particles can, in principle, be modeled in terms of some equivalent sphere over a large bandwidth.

Fig. 6 Cylinder force measurements compared to analytical calculations.

In summary, we found that Eqn. 8 very accurately predicts the expected force on conductive spheres in the presence of a time-varying magnetic field. We also found that, given the appropriate equivalent radius, non-spherical geometries could likewise follow curves similar to Eqn. 8. Such knowledge has tremendous practical application in the field of magnetic separation of nonferrous metals.

To illustrate, consider the limit as $F \rightarrow \infty$. Since the net force found in Eqn. 8 is a function of the product between conductivity and frequency, all particles of similar geometry must eventually converge to the same value. Density, however, is a fixed property of the material itself and does not vary. For example, copper is roughly three times denser than aluminum, meaning an aluminum particle

Fig. 7 Cube force measurements compared to analytical calculations.

Fig. 8 Disk force measurements compared to analytical calculations.

Table 2 Test parameters used in the force measurements of cylinder samples with radius r and height h .

Material	r [mm]	h [mm]	B [mT]	α [T/m]	σ [MS/m]
Al 2017	3.0	6.1	49.5	−1.7	25.1
Al 6061	6.4	13.1	49.5	−1.8	12.8
Brass	3.3	6.2	49.5	−1.8	12.8
Brass	6.3	13.1	49.5	−1.7	12.8
Cu alloy	3.4	6.2	49.5	−1.7	54.1
Cu alloy	6.3	12.4	49.5	−1.7	50.1

Table 3 Test parameters used in the force measurements of cube samples with side length ℓ .

Material	ℓ [mm]	B [mT]	α [T/m]	σ [MS/m]
Al 6061	5.5	49.5	−2.3	25.1
Al 6061	11.3	49.5	−2.0	25.0
Brass	5.6	49.5	−2.3	12.8
Brass	11.7	49.5	−2.3	12.8
Copper	5.5	49.5	−2.3	54.0
Copper	11.5	49.5	−2.0	54.1

Table 4 Test parameters used in the force measurements of disk samples with radius r and height h .

Material	r [mm]	h [mm]	B [mT]	α [T/m]	σ [MS/m]
Al 7075	4.5	1.7	49.5	−1.5	18.2
Copper	5.0	1.7	49.5	−1.3	54.1
Brass	4.7	1.9	49.5	−1.9	12.8

will experience three times the acceleration under the same magnetic field. As was demonstrated by Dholu (2017), this allows us to separate aluminum from copper by simply throwing the aluminum particles a much further distance. Further research will be necessary to explore the practical limits of this concept on real-world industrial applications.

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Author's short biography

Jaclyn Ray

Jaclyn Ray completed her undergraduate degree in Physics from the University of Utah in Salt Lake City, in 2015. In 2016, she graduated with MS degree in Metallurgical Engineering also from the University of Utah. She currently works for Lockheed Martin Aerospace Skunk Works.

James Nagel

Dr James Nagel completed his undergraduate degree in 2004 and MS degree in 2006, both in Electrical Engineering, from Brigham Young University in Provo, Utah. He then completed his PhD in 2011, also in Electrical Engineering, from the University of Utah in Salt Lake City, Utah, and was awarded the Stockham Medal of Excellence for conspicuously effective teaching. He now works as a research associate for the University of Utah, where his research focuses on applied electromagnetics with a specialty in numerical methods.

Dave Cohrs

Dave studied Environmental Science with emphasis on Analytical Chemistry at University of Maryland, Baltimore County in Maryland. He managed water quality laboratories at public aquariums for 16 years and has consulted for aquariums, zoos, and academia on water chemistry and life support topics. He now works for the University of Utah as a research associate.

Raj Rajamani

Raj Rajamani has been on the faculty of the Metallurgical Engineering department of the University of Utah, Salt Lake City, Utah, USA since 1980. Currently he holds the position of professor. His research interest include population balance modeling of tumbling mills, computational fluid dynamics of hydrocyclones, discrete element modeling of semi-autogenous grinding mills, eddy current sorting of metallic particles and modeling of high pressure grinding rolls. He received the Antoine M. Gaudin Award, presented by the Society of Mining, Metallurgy and Exploration Engineers Inc. in the year 2009.