Modified Ergun Equation for Airflow through Packed Bed of Loblolly Pine Grinds[†]

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Abstract

Biomass grinds are typically non-spherical and are composed of particles with wide range of sizes that may vary up to 10× between the smallest and largest particle. Since fluidized bed system is often used to convert biomass into fuels, chemicals and products, the viscous and kinetic energy losses' coefficients in the Ergun equation were determined to incorporate these unique characteristics of biomass grinds. The revised Ergun's equation, validated using loblolly pine wood grinds, and data from other published work resulted in estimated Ergun's K_1 and K_2 coefficients of 201 and 2.7 respectively. In addition, the relative mean deviation between experimental and predicted pressure drop was in general better with the modified Ergun's equation when compared to the original Ergun's equation.

Keywords: biomass, fluidization, physical properties, Ergun equation, pressure drop

1. Introduction

United States has the capacity to produce over one billion tonnes of biomass annually (DOE, 2016). These vast quantities of biomass feedstock can be processed into fuels, chemicals and products thereby reducing the dependency of the country on fossil fuels. One of the common reactors used for converting biomass to fuel and products is the fluidized bed system (FBS) because of relative uniformity in rate of heat transfer between biomass particles and fluidizing gases (Oliveira et al., 2013). Biomass materials properties such as moisture content (MC), density, particle shape and particle size distribution are needed to estimate the parameters (reactor pressure drop and minimum fluidization velocity— U_{mf}) needed to design, size, and operate FBS systems (Olatunde et al., 2017). There has been substantial progress made in quantifying the properties of biomass feedstock (Oginni et al, 2016; Olatunde et al., 2016) but the pressure drop and minimum fluidization velocity correlations developed for non-biomass materials are still currently used for biomass feedstocks. This has resulted in difficulties in sizing and designing equipment and reactors for fluidizing biomass

feedstocks (Allen et al., 2013; Koekemoer and Luckos, 2015; Kunii and Levenspiel, 1991). Therefore the main focus of this study is to develop reactor pressure drop and U_{mf} equations that utilize the properties of and are suitable for biomass feedstocks.

Reynolds et al. (1901) developed the first relationship between pressure loss and velocity of the fluid flowing through a packed bed using the Darcy concept.

$$\frac{\Delta P}{LU} = a + b\rho U \quad (1)$$

Ergun (1952) suggested that energy losses as fluid flows through a bed of particulate material is a function of fractional bed porosity (void) which are embedded in the coefficients 'a' and 'b' (Eqn. 1). Coefficients 'a' and 'b' were respectively regarded as the kinetic energy loss and viscous energy loss, and were related to the void space (ϵ) by Eqns. 2 and 3 (Ergun, 1952).

$$a = \alpha \frac{(1-\epsilon)^2}{\epsilon^3} \quad (2)$$

$$b = \alpha \frac{(1-\epsilon)}{\epsilon^3} \quad (3)$$

Combining Eqns. 1, 2 and 3 and further modification of Eqn. 1 to include average particle size resulted in the most widely used equation (Ergun's equation) for predicting packed bed pressure drop as a function of fluid velocity (Eqn. 4). The first term on the right-hand side of Eqn. 4 represents viscous energy loss at low fluid flow rate while

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the second term on the right hand side represent inertial energy loss due to high fluid flow rate. The values of the coefficient K_1 and K_2 in Eqn. 4 were estimated to be 150 and 1.75 for mono-sized spherical and nearly spherical particles (such as crushed coke, sand, glass) (Ergun, 1952).

$$\frac{\Delta P}{L} = K_1 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu U}{d^2} + K_2 \frac{1-\varepsilon}{\varepsilon^3} \frac{\rho U^2}{d} \quad (4)$$

As expected, Ergun equation (Eqn. 4) performs satisfactorily in predicting pressure drop in packed bed that contains uniform and spherical particles (Cloete et al., 2015; Mawatari et al., 2003; Nemec et al., 2001). Several authors have however documented that this equation does not satisfactorily predict the pressure drop of bed consisting of non-uniform and non-spherical particles. For example, Kunii and Levenspiel (1991) reported that Ergun equation resulted in under-prediction (greater than 25 % error) of pressure drop in packed bed that contains particles with sizes ranging between 4.75 mm and 37.5 mm. Similarly, Koekemoer and Luckos (2015) obtained 29 % pressure drop prediction error for a bed that contains coal, char and ash particles. Dolejs and Machac (1995) obtained 72.6 % and 24.9 % pressure drop prediction error for packed beds of polyhedral and cubes respectively while Gunarathne et al. (2014) obtained 35 % error from predicting pressure drop in biomass pellet (cylindrical) packed bed. Therefore there is a need to modify Ergun's equation in order to improve the accuracy of predicted pressure drops (subsequently U_{mf}) in beds that consist of non-uniform and non-spherical particles such as biomass grinds (Olatunde et al., 2016).

There have been attempts to revise Ergun's equation for non-spherical and non-uniform particles size bed (Dolejs and Machac, 1995; Gunarathne et al., 2014; Innocentini et al., 1999). Some of the authors refitted Ergun's equation for a specific material and obtain new values for constant (K_1 and K_2) but retained the overall structure (the porosity correlation—Eqns. 2 and 3). For instance, Cloete et al. (2015) proposed $K_1 = 250$ and $K_2 = 2.5$ for cylindrical particle of $\gamma\text{-Al}_2\text{O}_3$. Quinn (2014) proposed a value of 267 for K_1 while K_2 was found to be 2.14, 2.51, 4.02 for leadshot, glass beads and white sand, respectively. Also, Ozahi et al. (2008) proposed a constant of $K_1 = 160$ and $K_2 = 1.61$ for zeolite and chickpeas. It is important to note that these samples have porosity less than 0.5. The porosity for biomass grinds is significantly higher (typically 0.8 and above) (Olatunde et al., 2016; Fasina, 2006). We hypothesize that the porosity correlations (Eqns. 2 and 3) may not be suitable for materials such as biomass grinds with high void space, and that improvements in the porosity correlations may improve the predictions of pressure drop in packed beds containing non-uniform and non-spherical particles.

2. Methodology

Clean loblolly pine wood chips were obtained from trees harvested in Alabama, U.S. Using standard E871-82 (ASTM, 2006), the moisture content of the chips was 8.5 %. The chips were ground through a hammer mill (Model No. 10HBLPK, Sheldon Manufacturing, Tiffin, OH) fitted with one of the following screens sizes: size 22.23 mm, 19.05 mm, 15.88 mm, 12.7 mm, 9.53 mm, 6.35 mm, or 3.18 mm. This resulted in seven samples with different bulk densities and porosities. The physical properties of each sample were determined as described below.

2.1 Particle size distribution and shape

Particle size distribution of each sample was determined with a digital image-based particle size analyzer (Camsizer[®], Retsch Technology, Haan, Germany). An example of the image obtained from the particle size analyzer is shown in **Fig. 1**. For this analysis, about 100 g of a sample was poured into the hopper of the analyzer from where the sample was conveyed through a vibratory feeder to the measurement chamber of the system. The chamber is equipped with two cameras that capture pictures of the particles of the samples falling through the measurement field of the chamber. The software provided by the manufacturer of the analyzer was used to read, store and process the captured images. The size parameters that were retrieved from the software for these studies were: sphericity (ϕ), x_{84} , x_{16} , x_{50} and coefficient of variation (γ), and are defined below.

$$\phi = \frac{4\pi A}{P^2} \quad (5)$$

$$\gamma = 50 \frac{x_{84} - x_{16}}{x_{50}} \quad (6)$$

where P is measured perimeter or circumference of a projected particle (mm), A is measured surface area covered by the projected particle (mm^2), and x_{84} , x_{16} and x_{50} are diameters (mm) at which 84 %, 16 % and 50 % of particles in a sample is comprised of smaller particles respectively.

2.2 Particle size distribution and shape

Particle density of each sample was measured with a gas pycnometer (Accupyc 1330, Micromeritics Instrument Corp., Norcross, Ga.) that uses helium to estimate the pressure difference between a reference cell and a cell containing the sample. The pressure difference was used by the pycnometer to estimate the volume of a known mass of sample. A digital weighing scale (Model AR3130, Ohaus Corp, Pinebrook, NJ) was used to measure the sample mass. Particle density was estimated as the ratio

Fig. 1 Image showing the variation in particle size and shape of biomass grinds.

Fig. 2 Schematic diagram of experimental setup: (1)—Manometer, (2)—Lower pressure tap, (3) Bed support, (4)—Fluidization test chamber having 101.6 mm diameter pipe, (5)—Bed material on distributor plate, (6)—Nikon digital camera (7)—Flow rate probe, (8)—RS232 cable, (9)—Computer system, (10)—50.4 mm (D) PVC pipe, (11)—Air blower, (12) Air blower controller, (13) Power source.

of the mass of the sample in the cell and the volume estimated by the pycnometer.

Sample bulk density was determined using an apparatus that consists of a funnel through which the sample freely falls onto 1137 mm³ cup. The ratio of the mass of the sample in the container to the volume of the container was used to compute the estimated bulk density. The inter-granular porosity (ε) of the ground wood sample was calculated from the measured values of bulk density and particle density as follows:

$$\varepsilon = 1 - \frac{\rho_b}{\rho_p} \quad (7)$$

2.3 Pressure drop test

The experimental setup (**Fig. 2**) utilized for the pressure drop measurements consists an acrylic cylindrical pipe with 0.1 m diameter and 1.0 m bed height. A perforated-plate having 100 μ m holes (Purolator, Model UNS 530403, Sacramento, CA 95828) that serves as the distributor is located at the base end bed. The amount of ambient air supplied by a blower (Black and Decker, Model LH5000, Antioch, CA 94509) to the experimental unit was varied by means of a fan speed controller (Lutron electronic, MFG part S2-LFSQH-WH Monroe, NJ) and was measured by a vane anemometer (model 407113, Ex-tech Instruments, Nashua, NH 03063). The pressure drop

across the bed was measured by connecting a U-tube manometer into the upper (800 mm above the distributor) and lower pressure taps (200 mm below the distributor).

Fluidization experiments were conducted by adding 2 kg sample into the fluidization chamber. The blower was turned on and the fan speed controller was used to gradually increase airflow rate through the bed. At least, 10 airflow data (pressure drop and air velocity) were recorded for each sample before the onset of entrainment of the bed material. At each airflow velocity, 60 seconds was allowed for stabilization before the pressure drop across the chamber and the corresponding air velocity through the bed were recorded. The pressure drop across the chamber was determined for each superficial gas velocity starting from fixed bed condition until complete bed mixing was achieved.

2.4 Data analysis

All experiments were conducted in triplicates and results are presented in the relevant sections as mean values and standard deviation. Statistical significance of the following variables—screen size, porosity, particle and bulk density, particle size, sphericity and coefficient of variation was tested using the analysis of variance (ANOVA) procedures (SAS, 2011). Tukey multiple range test was used to compare means. Differences were considered to be statistically significant when $p < 0.05$. Mean Relative Deviation (MRD) (Eqn. 8) was used as statistical indicator to compare the predictive ability of the original Ergun and the modified Ergun (developed in this study) equations

$$MRD(\%) = \frac{1}{N} 100 \sum_{i=1}^N \frac{|\Delta P_{i,calc} - \Delta P_{i,exp}|}{\Delta P_{i,exp}} \quad (8)$$

3. Results and discussion

3.1 Physical properties

The size, density, porosity and shape of the seven loblolly pine grind samples are summarized in **Table 1**. Coefficients of variation for all the samples were higher than 60 % thus further confirming that size of particles in each sample varied widely. The results in **Table 1** also affirm the high porosity and the non-spherical nature of biomass grinds (> 0.80) as earlier discussed in the introductory section. The particle density did not significantly vary with increase in particle size but bulk densities significantly decreased ($p < 0.05$) as particle size increased thereby indicating that the amount of inter-particle space (void or pore space) increased with increase in particle size. However, the increase in porosity was not significantly influenced by screen (or particle) size.

3.2 Pressure drop and airflow rate

The plots of ratio of pressure gradient to velocity against air mass flow rate (Eqn. 1) for 5 samples of loblolly pine wood grinds are presented in **Fig. 3**. It can be deduced from the plots that reduction in particle size resulted in higher pressure losses because there were less pore spaces for the air to flow through thereby confirming the bulk density, particle density and porosity results presented in **Table 1**.

The slopes, intercepts and the corresponding R^2 value of each of the plot in **Fig. 3** are summarized in **Table 2**. When the nominal screen size reduced by 89.8 %, the slope and intercept increased by 67.7 and 38.2 % respectively. Increase in the value of intercept or slope indicate

Table 1 Physical properties of loblolly pine wood grinds ground through different screen sizes.

Screen size (mm)	Particle size (mm)	Sphericity	Bulk density (kg/m ³)	Particle density (kg/m ³)	Porosity	γ^*
6.35	1.64 ^b (0.10)	0.37 ^c (0.10)	250.9 ^b (2.40)	1453.3 ^{ab} (44.10)	0.83 ^b (0.01)	72.6 ^{bc} (0.01)
9.53	1.95 ^a (0.10)	0.37 ^a (0.10)	242.8 ^b (2.80)	1420.4 ^b (12.10)	0.84 ^{ab} (0.01)	72.8 ^c (0.1)
15.88	2.09 ^a (0.30)	0.39 ^b (0.10)	229.9 ^d (1.60)	1448.2 ^{ab} (3.03)	0.84 ^{ab} (0.01)	72.5 ^{abc} (0.1)
19.05	2.02 ^a (0.02)	0.42 ^a (0.10)	228.8 ^d (2.90)	1424.9 ^b (12.15)	0.84 ^{ab} (0.01)	67.9 ^c (0.06)
22.23	2.10 ^a (0.10)	0.41 ^a (0.01)	222.7 ^e (3.30)	1438.8 ^{ab} (11.70)	0.85 ^a (0.01)	64.0 ^c (0.02)

Values are means of triplicates experimental runs and are based on d_{50} from the particle size distribution data

Numbers in parentheses are standard deviation

Means with the different superscript (alphabet) in a column are significantly different ($p < 0.05$)

*coefficient of variation (calculated using Eqn. 6)

Fig. 3 Linear form of pressure–drop equation (Eqn. 1) using loblolly pine wood grind through different screen sizes.

Table 2 Estimation of slope and intercept parameters for loblolly pine wood at different particle sizes.

Screen size (mm)	Bulk density (kg/m ³)	⁺ Intercept (a)	⁺ Slope (b)	⁺ R ²	Void correlations based on	
					(Eqn. 2)	(Eqn. 3)
6.35	250.89	883.2	3346.2	0.90	0.053	0.31
9.53	242.80	811.1	3248.3	0.95	0.046	0.28
15.88	229.93	687.7	3053.4	0.93	0.042	0.27
19.05	228.27	670.5	3071.1	0.98	0.041	0.26
22.23	222.72	633.1	2999.8	0.97	0.039	0.25

⁺Values of (a), (b) and R² were obtained from Fig. 4 and based on Eqn. 1

that resistance of the bed material to stress-shear deformation at viscous level and bed entropy (particle-particle collision per unit area due to kinetic energy) increased with reduction in size (Ergun, 1952). The last two columns in **Table 2** were calculated from the fraction correlation (kinetic energy loss (Eqn. 2) and viscous energy loss (Eqn. 3)) as proposed by Ergun (1952). The values were determined by substituting the experimentally determined porosity (values presented in **Table 1**) for each of the screen size into the corresponding Eqns. 2 and 3. The result showed that void fraction for kinetic energy loss were higher than viscous energy losses. This implies that energy losses due to particle-particle movement dominated the system.

Fig. 4 shows the plot of slope versus kinetic fractional correlation (Eqn. 2) on the primary axis and the plot of intercept values versus viscous void fractional correlation (Eqn. 3) on the secondary axis. The result shows a good fit of R² values of 0.95 for viscous energy loss but a weak R² values of 0.61 kinetic energy loss—an indication that

Fig. 4 Influence of kinetic and viscous energy losses on fractional void volume.

the kinetic energy expression in the Ergun equation does not appear to be adequate for pressure drop-airflow relationship of irregular shape typical of biomass grinds.

To improve the void fraction correlation for kinetic energy losses, the void fraction was modified into the expression below.

$$kinetic = \frac{(1-\varepsilon)^n}{\varepsilon^k} \quad (9)$$

The values of parameters ‘ n ’ and ‘ k ’ were obtained by using the Microsoft Excel® nonlinear solver based on generalized reduced gradient algorithm such that the error sum of square between kinetic void fraction correlation data and the corresponding slope data (**Table 2**) was minimized. The plot of slope versus modified kinetic fractional correlation (Eqn. 9) using the new values of ‘ n ’ (0.3) and ‘ k ’ (3.4) resulted in improved R^2 value of 0.94 (a 35 % improvement). These values of ‘ n ’ and ‘ k ’ indicate that other factors (in addition to turbulent flow) such as particle-to-particle cohesive force, particle-wall interactions, variations in the minimum fluidization velocity may be contributing to pressure drop during fluidization of bed that is composed of non-spherical and non-uniform particles. (Srivastava and Sundaresan, 2002; Olatunde et al., 2016)

3.3 Model development

As mentioned in the introductory section, the most common form of predicting packed bed pressure drop is the Ergun equation (Eqn. 4). The Ergun equation was modified by incorporating sphericity (ϕ) and coefficient of variation (γ) expression (Eqn. 10) below (Anderson and Warburton, 1949) and the kinetic energy loss expression developed in the previous section thereby extending the use of the Ergun equation to packed beds containing particles that are non-spherical and have wide particle size distribution (Eqn. 11).

$$d \cong \bar{d}\phi(1+\gamma^2) \quad (10)$$

$$\frac{\Delta P}{L} = K_1 \frac{(1-\varepsilon)^2}{\varepsilon^3 [\bar{d}\phi(1+\gamma^2)]^2} \mu U + K_2 \frac{(1-\varepsilon)^{0.3}}{\varepsilon^{3.4} \bar{d}\phi(1+\gamma^2)} \rho U^2 \quad (11)$$

$$\frac{\Delta P}{LU} \frac{\varepsilon^3 d^2}{\mu(1-\varepsilon)^2} = K_1 + K_2 \frac{N_{Re}}{\varepsilon^{0.4}(1-\varepsilon)^{1.7}} \quad (12)$$

where

$$N_{Re} = \frac{\rho U d}{\mu}$$

If

$$f_v = \frac{\Delta P}{LU} \frac{\varepsilon^3 d^2}{\mu(1-\varepsilon)^2} \quad (13)$$

then

$$f_v = K_1 + K_2 \frac{N_{Re}}{\varepsilon^{0.4}(1-\varepsilon)^{1.7}} \quad (14)$$

Based on the linear plot (**Fig. 5**) of

Fig. 5 A general plot for a single system having different void fraction using Ergun's concept using chord diameter scheme.

$$f_v \text{ versus, } \frac{N_{Re}}{\varepsilon^{0.4}(1-\varepsilon)^{1.7}}$$

coefficients K_1 and K_2 were estimated to be 201.6 and 2.7 for loblolly pine grinds, and are similar to values that have been reported in literature for non-spherical materials (K_1 varying from 160 to 267, and K_2 varying from 1.6 to 4.0; Quinn, 2004; Nemec and Levec, 2005; Ozahi et al., 2008; Cloete et al., 2015; Koekemoer and Luckos, 2015). Therefore, the modified Ergun equation for loblolly pine grinds is:

$$\frac{\Delta P}{L} = 201.6 \times \frac{(1-\varepsilon)^2}{\varepsilon^3 (d\phi(1+\gamma^2))^2} \mu U + 2.7 \times \frac{(1-\varepsilon)^{0.3}}{\varepsilon^{3.4} d\phi(1+\gamma^2)} \rho U^2 \quad (15)$$

3.4 Model validation

Validation of the modified equation (Eqn. 15) was achieved by comparing the pressure drop prediction from this equation to the values obtained from the Ergun equation of Eqn. 16 (a version of Ergun equation that has sphericity factor).

$$\frac{\Delta P}{L} = K_1 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu U}{\phi^2 \bar{d}^2} + K_2 \frac{1-\varepsilon}{\varepsilon^3} \frac{\rho U^2}{\phi \bar{d}} \quad (16)$$

A new set of loblolly pine wood chips were prepared with hammer screen sizes that were different from those used to prepare the samples utilized in the above model development. The physical attributes of the new samples are presented in **Table 3**. Similar effects of particle size on particle density, bulk density, and porosity were obtained as described earlier on. The MRD values from the predicted pressure drop values using the modified Ergun equation (Eqn. 16) and the original Ergun equation (Eqn. 4) are summarized in **Table 4**. The lower MRD values for the modified Ergun equation for all the hammer mill screen sizes provide proof that the modified Ergun equation is more suitable for predicting pressure drop in

Table 3 Physical properties of loblolly pine wood grinds used for validation.

Screen size (mm)	Particle size (mm)	Sphericity	Bulk density (kg/m ³)	Particle density (kg/m ³)	Porosity	γ^*
22.40	2.90 ^a (0.20)	0.45 ^a (0.01)	222.7 ^e (2.2)	1438.5 ^a (3.1)	0.85 ^a (0.003)	66.1 ^a (2.6)
19.00	2.20 ^b (0.10)	0.45 ^a (0.01)	232.1 ^d (2.1)	1438.1 ^a (2.3)	0.84 ^a (0.003)	67.0 ^a (3.0)
15.80	2.09 ^b (0.30)	0.45 ^a (0.01)	275.4 ^c (2.62)	1437.7 ^a (4.9)	0.81 ^a (0.001)	60.0 ^a (4.4)
12.70	2.02 ^b (0.20)	0.46 ^a (0.01)	282.4 ^c (4.25)	1436.4 ^a (9.0)	0.80 ^a (0.001)	60.5 ^a (2.7)
6.35	0.82 ^d (0.03)	0.38 ^{cb} (0.02)	311.1 ^{ab} (1.57)	1422.3 ^{ba} (2.4)	0.78 ^c (0.001)	65.5 ^a (11.9)

Values are means of triplicates experimental runs and are based on x_{50} from the particle size distribution data

Numbers in parentheses are standard deviation

Means with the different superscript (alphabet) in a column are significantly different ($p < 0.05$)

*coefficient of variation (calculated using Eqn. 6)

Table 4 Comparison of the overall MRD (%) between predicted and the experimental data.

Screen size (mm)	Modified Ergun (Eqn. 16)	Original Ergun (Eqn. 4)
6.35	18.4	56.7
12.70	25.5	44.1
15.80	49.1	81.8
19.00	31.2	76.6
22.40	32.8	76.9

packed beds containing loblolly pine grinds in comparison to the original Ergun equation. The MRD values of the modified Ergun equation were about 2× lower than that of the original Ergun equation, and are similar to the values reported by others that have attempted to customize the Ergun equation for a particular material. For instance, Nemec and Levec (2005) used neural network approach to modify Ergun using material shaped into various geometry sphere (size ranged between 1.66 mm and 3.50 mm, porosity 0.40 to 0.44), cylinder (size ranged between 2.62 and 3.50 mm, porosity 0.32 to 0.68) and quadralobes (size 2.13 porosity 0.47–0.50) and obtained MRD of 42.2, 20.6 and 60.0 %. Harrison et al. (2013) re-fitted Ergun's expression and obtained 119.8 for K_1 and K_2 to be 4.63 and author validated the equation over a wide range of Reynolds number and tube-diameter to particle diameter ratio. The author reported an absolute mean relative deviation of up to 50.7 %. Despite the significant improvement in the prediction of pressure drop during fluidization of biomass grinds using the modified Ergun equation developed in this study, there is the need to con-

tinue to conduct studies that will lead to further improvement and development of equations for predicting the fluidization behavior of non-spherical and non-uniform particles.

4. Conclusions

Ergun's equation has attracted the attention of several researchers since it was first developed. Some of the authors showed that the equation is best suited for uniformly sized particles while others concluded that the void fraction correlation, the coefficients (K_1 and K_2) and a term of introducing the effect of size distribution need to be carefully determined before Ergun's equation can be used for a bed consisting of non-uniform particles having non-spherical shape. In this study, we introduced a new concept of determining the void fraction correlation suitable for non-uniform particle size distribution. We also incorporated coefficient of variation to capture the effect of particle distribution. Accordingly, we estimated a new coefficient K_1 and K_2 to be 201.6 and 2.7 respectively. We also proposed new frictional loss equation using ground loblolly pine wood. The result showed that the new equation resulted in lower overall mean relative deviation of pressure drop data compared with original Ergun equation.

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Nomenclature

FBS	fluidized bed system
MRD	mean relative deviation (%)
a, b	constants
A	surface area (m^2)
d	particle diameter (m)
$\bar{d}$	average particle diameter (m)
P	perimeter (m)
ΔP	pressure drop (Pa)
g	acceleration due to gravity (m^2/s)
L	height of packed bed (m)
N_{Re}	Reynolds number
U_{mf}	minimum fluidization velocity (m/s)
k, n	constants
K_1, K_2	constants
x_{84}	particle diameter 84 % percentile
x_{50}	particle diameter 50 % percentile
x_{16}	particle diameter 16 % percentile
ρ	fluid density (kg/m^3)
ρ_b	bulk density (kg/m^3)
ρ_p	particle density (kg/m^3)
μ	fluid viscosity (Pa s)
ϕ	sphericity factor
ε	porosity
f_v	friction factor
γ	coefficient of variation

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