Comparing the Flow Properties of Bulk Solids by Tri-axial Shear, Unconfined Yield and Direct Shear Tests[†]

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Abstract

This paper discusses a comparison of the data of experiments obtained by using different shear tests methods in order to examine the confidence in the results of experiments on the flow properties of a bulk solid.

The angle of internal friction of dry granular materials, which has been measured by both a tri-axial shear tester and a fixed volume direct shear tester, increases with a decrease in the voidage of the specimen. Except for the extremely dense and loosely packed states, the experimental results of the angle of internal friction agree within about 3 degrees. There is however a difference of 3 to 5 degrees for a material in which the particle shape is deflective.

The flowability index of cohesive powders and wet materials is expressed by a ratio of the unconfined yield strength to the bulk density. The results of experiments on the flowability index are independent of the unconfined yield and direct shear tests.

1. Introduction

The flow properties of a bulk solid are important factors in designing various processes and operations relative to its use, as well as in evaluating its flowability. The flow properties are expressed in various ways, and the method and apparatus for their measurement are employed according to their relevance to the objective. From the viewpoint of designing

processes, therefore, the data on the flow properties must be handled in light of objectivity and confidence. That is to say, where a flow property is measured by differing devices, whether the results agree or differ is an important question.

In this paper, a comparison is made between the data obtained by three different research institutions with respect to the flow properties of common or similar samples subject to tri-axial shear tests, unconfined yield tests, and direct shear tests.

2. Subjects and organization of studies

The flow properties of a bulk solid are basically represented by a family of linear yield loci. However, it is not convenient to deal with the linear yield loci themselves as an object relevant to comparison of the data of the flow properties.

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† This report was originally printed in *J. Soc. Powder Technology, Japan*, **23**, 678-684 (1986) in Japanese, before being translated into English with the permission of the editorial committee of the Soc. Powder Technology, Japan.

Table 1 The system of the present study

Material	Flow property	Testing method	Research institute
Dry granule	Angle of internal friction	Tri-axial shear test	Shimizu Construction Co., Ltd.
		Direct shear test	Yokohama National University
Powder and wet granule	Flowability (a ratio of unconfined yield strength to bulk density)	Unconfined yield test	Mitsui Construction Co., Ltd. and Yokohama National University
		Direct shear test	Yokohama National University

Since the yield locus of a dry granular material generally assumes a straight line, it is convenient to deal with the angle of internal friction, which is the inclination formed by the line, as an object of comparison in considering the data of flow properties. However, with respect to a powder or wet granular material, the yield locus forms a curve. The flow properties cannot be represented singly by the angle of internal friction. The flow properties then characteristically include an unconfined yield strength into which the intergranular frictional force and cohesion are integrated. The flowability index, which is defined as a ratio of the unconfined yield strength to the bulk density, typically represents the flow properties of a cohesive bulk solid and must be dealt with by mutual comparison.

The test of the flow properties were undertaken by the three research institutions specified in Table 1. The data on the angle of internal friction of the dry granules was obtained by both tri-axial and direct shear tests, whereas the flowability indices of cohesive granules and powders were determined by unconfined yield and direct shear tests.

3. The angle of internal friction of dry granules

The reader is referred to the published related literature¹⁾ or reference book²⁾ for details about the tri-axial shear test, since this

Table 2 The conditions of the experiment of the tri-axial shear tests and the direct shear tests

Condition	Tri-axial	Direct
Specimen size [mm]	50 ^D × 100 ^H	100 ^D × 10 ^H
Stress range [kPa]	100 ~ 1900	0 ~ 100
Strain rate [%/min]	< 0.5	2.22

Table 3 The dry granules used in both the tri-axial and the direct shear tests

Material	Density [t/m ³]	Particle size [mm]
Soma sand	2.65	0.59 ~ 0.84
Yamakawa sand	2.64	0.35 ~ 0.84
Toyoura sand	2.64	0.105 ~ 0.297 (0.21 ~ 0.25)*
Silicon carbide	3.20	0.21 (average)
Glass beads	2.50	0.177 ~ 0.25 (0.21 ~ 0.25)*

*) These samples were used in direct shear tests.

test is widely used mainly in the field of soil engineering, and the testing method and devices applied are standardized. On the other hand, the direct shear test is carried out by various types of devices and has not yet been standardized yet. The test is limited to the fixed volume direct shear tester used in the present experiments. The testing method and devices used are described in the literature^{3,4)}.

Table 2 shows the main conditions of the experiment under which the tri-axial shear tests and the fixed volume direct shear tests were carried out. These conditions had no special features in common. Table 3 shows the five kinds of dry granules and their physical features that were subjected to the measurement of the angle of internal friction. Common samples were employed for the two testing methods with respect to Soma sand, Yamakawa sand, and silicon carbide, but the samples of Toyoura sand and glass beads used in the two tests were a little different in grain size.

Figure 1 shows the results obtained from the tri-axial shear tests on Toyoura sand. The specimens had a voidage at the initial packing e_0 of 0.463. The minor principal stress (lateral pressure) σ_3 had four values – 0.098, 0.196, 0.294,

Fig. 1 The result of the tri-axial shear tests on Toyoura sand

and 0.392 MPa. These values were common to each of the tests. The difference in the principal stress, change in the volume, and change in the voidage were each plotted in relationship to the compressive strain. The voidage during failure was calculated by the equation.

$$\epsilon = 1 - (1 - \epsilon_0)/(1 - v_0)(1 - v) \quad (1)$$

In this equation, ϵ_0 represents the voidage at the initial packing, v_0 is the change in the volume after consolidation (when only σ_3 is applied), and v is the change in the volume at the time of failure.

Upon determination of the maximum difference in principal stress as marked by ▼ in Fig. 1, Mohr's stress circles can be drawn as shown in Fig. 2. The straight line passing through the origin of the coordinates and tangent to the stress circles in the figure represents a yield locus. The angle of internal friction is represented as 35° in Fig. 2. With respect to the voidage corresponding to the maximum difference in the principal stress, the values differ slightly from one another with the change in σ_3 , as shown in Fig. 1. Thus, the voidages are shown as 0.452, 0.449, 0.448, and 0.447 in the figure. It is a defect of the tri-axial shear testing method that the voidage cannot be made exactly the same for all tests. There-

Fig. 2 The yield locus of Toyoura sand obtained from the tri-axial shear tests

Table 4 The results of the tri-axial shear tests

Material	Voidage		Angle of internal friction δ [deg.]
	at initial packing ϵ_0 [-]	at failure (average) ϵ [-]	
Soma sand	0.463	0.449	32.0
	0.433	0.417	36.0
	0.376	0.354	39.5
	0.354	0.329	40.25
Yamakawa sand	0.434	0.420	33.75
	0.417	0.400	34.5
	0.365	0.348	38.5
	0.357	0.335	40.0
Toyouura sand	0.482	0.470	34.0
	0.463	0.449	35.0
	0.405	0.387	39.75
	0.395	0.375	40.25
Silicon carbide	0.519	0.509	31.5
	0.496	0.483	32.0
	0.427	0.408	35.5
	0.415	0.393	36.25
Glass beads	0.408	0.408	23.0*

*) Quick test

fore, the mean value of the voidages is used as the voidage during failure in the processing data. Table 4 shows both the voidages and angles of internal friction obtained from the tri-axial shear tests.

Figure 3 shows a yield locus of Toyoura sand obtained by a fixed volume direct shear test. The yield locus can be determined by a single shear test where the vertical and horizontal components of the force generated at the step of shearing a specimen are simultaneously

Fig. 3 The yield locus of Toyoura sand measured by the fixed volume direct shear tester

detected and output on an X-Y recorder. A yield locus can be regarded as forming a straight line passing through the origin. In Fig. 3, the angle of internal friction is shown as 36° . The voidage of the specimen at the initial packing ϵ_0 is shown as 0.440. In a fixed volume shear test, the voidage does not change during failure but remains the same as the value at the initial packing.

A comparison of the test results in Figs. 2 and 3 shows that even though there is a great difference in the ranges of the stresses applied for the measurement, the angles of internal friction show closeness in values due to the virtual uniformity of the voidage.

Figure 4 shows a comparison of the results obtained by tri-axial shear tests with those obtained by fixed volume direct shear tests on the relationship between the voidage and the angle of internal friction. In past tri-axial shear tests, the angle of internal friction was considered in relationship to the voidage at the initial packing ϵ_0 , but since the voidage during failure changes with compressive strain, as shown in Fig. 1, the voidage corresponding to the maximum difference in the principal stress must be considered.

The angle of internal friction increases with a decrease in the voidage independent of the testing methods. The results of two types of tests on Soma sand, Toyoura sand, and glass beads agree in the range from medium to loose in the packed state, but the values obtained by the direct shear tests are larger than those obtained by the other tests in the densely packed

Fig. 4 A comparison of the angle of internal friction obtained from the tri-axial shear tests and the fixed volume direct shear tests

range. With respect to Yamakawa sand, the results agree on the whole, but there is a difference of up to 3° in the loosely packed range.

With respect to silicon carbide, however, the values obtained clearly show differences of $3^\circ \sim 5^\circ$ between the tri-axial shear tests and the fixed volume direct shear tests. The specimen of silicon carbide is a pulverized product, and the particles include those shaped almost like needles and flat particles. The distinct differences in the values obtained are considered to be due to a difference in how the shearing direction bears upon the disposition of the packed particles of the specimen between the two testing methods.

In Fig. 5, the results of shearing tests⁵⁾ carried out extensively by the Soil Engineering Society on sand (obtained at Onahama beach in Fukushima Prefecture, Japan) are shown as a relationship of the angle of internal friction to the initial void ratio. The sand had particle sizes of 0.1 ~ 0.5 mm, and with the 50% diameter of 0.22 mm, if somewhat resembled Toyoura sand. Figure 5 also shows the test results of Toyoura sand in the present study for comparison. Although the angles of internal friction differ slightly because of differences in the samples used, a close similarity is observed

Fig. 5 A comparison of the angle of internal friction for Onahama sand and Toyoura sand measured by various types of shear testers

between the results of the Society's tests and those obtained in the present study for both the tri-axial and direct shear tests (the results of the Society's tests being based on upper-movable cells). Therefore, it is considered that there were no technical problems relating to the measurement in the present experiments.

It is said⁵⁾ that in the Society's extensive tests the results obtained by the use of lower-movable cell type devices for the direct shear tests showed larger deviations than all of the other comparable results, as well as a large degree of scatter, but the results were found

lacking in confidence. The device used for the fixed volume direct shear tests in the present experiments was also of the lower-movable cell type, but the obtained results were similar to those obtained by the tri-axial shear tests and the upper-movable cell-based direct shear tests.

Whether the shearing cell is of the upper-movable type or of the lower-movable type does not seem to significantly affect the results of the direct shear tests. The problem is considered to lie in the structure of the testing device, or in the dimensions of the shearing cells, etc.

4. The flowability of cohesive powder and granules

The unconfined yield strength of a cohesive powder or granular material is an important property to consider^{6,7)} when a storage vessel for it is designed. This is because this property is closely related to blocking which occurs at or near the outlet of the vessel. The most direct method for measuring the unconfined yield strength is the unconfined yield test, which is standardized mainly in the field of soil engineering. It is also possible to obtain the unconfined yield strength from the data of a yield locus measured by the direct shear test. Next, a comparison is made of the results obtained from the two testing methods.

Table 5 shows the samples and dimensions of the specimens, etc. which were used in the experiments. Common samples were employed in the two testing methods with respect to calcium carbonate, limestone, and coal (A), whereas the samples of coal (B) used for the two respective testing methods were similar but differed in particle size and free moisture.

Figure 6 shows a record of the compressive

Table 5 The conditions of the experiment and materials used in the unconfined yield tests and the direct shear tests

Material Condition	Calcium carbonate		Lime stone		Coal (A)		Coal (B)	
	U.Y.T.	D.S.T.	U.Y.T.	D.S.T.	U.Y.T.	D.S.T.	U.Y.T.	D.S.T.
Testing method	U.Y.T.	D.S.T.	U.Y.T.	D.S.T.	U.Y.T.	D.S.T.	U.Y.T.	D.S.T.
Particle size	2.6 μ m	2.6 μ m	< 3 mm	< 3mm	< 3mm	< 3mm	< 10 mm	< 5.6 mm
Free moisture (%)	0.20	0.18	4.1	4.07	12.6	12.6	8.3	7.8
Specimen size [mm]	18.8D $\times$ 50H	100D $\times$ 10H	50D $\times$ 100H	100D $\times$ 10H	50D $\times$ 100H	100D $\times$ 10H	100D $\times$ 200H	100D $\times$ 10H

(Note: U.Y.T. shows the unconfined yield test and D.S.T. the direct shear test)

Fig. 6 Record of unconfined yield tests for coal (B)

stress applied to the coal (B) in an unconfined yield test. The peak value of the compressive stress was regarded as the unconfined yield strength f . The test was carried out five times, more or less, for the same bulk density, and then it was repeated likewise for a different bulk density. The specimens were composed of three to five vertical layers. The specimens had different sizes depending on the particle size, as shown in Table 5. The relationship between the consolidation stress and the bulk density was examined by a separate measurement. The details of the testing method are described in the literature⁷⁾.

Figure 7 shows a yield locus of the coal (B) obtained by a direct shear test. From the end point of the preliminary shearing step of a specimen, a continuous line representing the relationship between the normal stress and the shear stress was recorded to show the yield locus. The unconfined yield stress f can be obtained as the diameter of the Mohr circle that

Fig. 8 A comparison of the unconfined yield strength of coal (A) and limestone obtained from the unconfined yield tests and the direct shear tests

passes through the origin of the coordinates and meets the yield locus at one point. The major consolidation stress σ_1 is shown by the Mohr circle which meets the yield locus at the end point. The bulk density is calculated from the packed volume and the mass of the specimen at the time when the preliminary shearing has ended. The details of the testing method etc. are described in the literature³⁾.

Figure 8 shows the unconfined yield strength of the coal (A) and limestone obtained by both the unconfined yield and direct shear tests. In this figure, the results of the unconfined yield tests are shown as the relationship of the unconfined yield strength f to the consolidation stress σ_n , and the results of the direct shear

Fig. 7 The yield locus of coal (B) measured by the direct shear tester, and determination of the unconfined yield strength, f , and the major consolidation stress, σ_1

Fig. 9 A comparison of the bulk density of coal (A) and limestone obtained from the consolidation tests and the direct shear tests

tests are shown as the relationship of the unconfined yield strength f to the major consolidation stress σ_1 . The results of the two testing methods were relatively close, with the values obtained from the direct shear tests being slightly larger on the whole than those obtained from the other tests. This difference is considered to be due to shearing at the unconfined state of one method and a difference in the molding method for the specimens between the two testing methods. The latter reason lies in the fact that whereas the specimens used in the unconfined yield tests were molded by simple compression, those for the direct shear tests were molded through preliminary shearing.

Figure 9 shows the results of the bulk density γ of coal (A) and limestone, obtained by the two testing methods. It is shown that the bulk density was larger, and the specimens were packed more densely for the direct shear test than for the unconfined yield test. As a result, the unconfined yield strength is considered to have shown a larger value during the direct shear test.

It was already reported³⁾ that the flowability of a cohesive powder or granular material can be evaluated by a flowability index expressed as $f/\gamma g$. This flowability index means the critical height up to which a molded body of powder maintains its shape and an increase in the value, which occurs with an increase in

Fig. 10 A comparison of the flowability of calcium carbonate obtained from the unconfined yield tests and the direct shear tests

Fig. 11 A comparison of the flowability of wet coals and limestone obtained from both the unconfined yield tests and the direct shear tests

the cohesive force and frictional force between particles, signifies a degradation in the flowing property. The respective flowability indices obtained from both the unconfined yield and direct shear tests are shown in Figs. 10 and 11 for comparison. Although they show some scatter, the results of calcium carbonate, are almost in agreement for both testing methods. The unconfined yield tests of calcium carbonate were carried out by Yokohama National University, and those of the other three kinds of specimens were done by Mitsui Construction Co.

The flowability indices obtained by the unconfined yield tests in Fig. 11 represent the averages of the results of tests carried out five times, as shown in Fig. 6. In Fig. 11, a fairly good agreement can be observed between the results obtained by the two testing methods with respect to the flowability of wet coals and limestone.

The relationship between the flowability index and the major compressive stress in the results of direct shear tests can be represented by Eq. (2), as already reported^{3,8)}

$$f/\gamma g = a\sigma_1^b \quad (2)$$

where a and b are coefficients proper to a powder, and g is the acceleration of gravity.

As is clear from Fig. 11, Eq. (2) is also applicable to the results of unconfined yield tests if the compressive stress σ_n is substituted for the major compressive stress σ_1 . From the results obtained by the two testing methods, it is seen that Eq. (2) is valid for σ_1 or σ_n in a range of approximately 20 ~ 500 kPa. If the results previously discussed^{3,8)} are also taken into consideration, Eq. (2) is valid for a range of 1 ~ 500 kPa.

5. Summary

With a view to examining the results of differing testing methods in terms of their consistency, tri-axial shear tests were compared with fixed volume direct shear tests in the measurement of the angle of internal friction of dry granules. Unconfined yield tests were compared with direct shear tests in the measurement of the flowability indices of cohesive powder and granules. As a result, it was found that:

1) whereas the angle of internal friction of dry granules is dependent on the voidage, the values obtained by the two testing methods

agree to within a difference of about 3°, except for the extremes of the voidage range where, even though the specimen grains have exceptional shapes, a difference of more than 3° may arise depending on the testing method.

2) with respect to the flowability index of cohesive powder or granules, the values obtainable by the two testing methods agree relatively well. The flowability index can be expressed by Eq. (2) for the compressive stress or major compressive stress in the range of up to 500 kPa.

Nomenclature

a	: coefficient in Eq. (2)	[—]
b	: coefficient in Eq. (2)	[—]
f	: unconfined yield strength	[kPa]
g	: acceleration due to gravity	[m/s ²]
v	: volume change during failure	[%]
v_0	: initial volume change	[%]
γ	: bulk density	[t/m ³]
δ	: angle of internal friction	[deg]
ϵ	: voidage during failure	[—]
ϵ_0	: voidage at initial packing	[—]
σ_1	: major stress	[kPa]
σ_3	: minor stress	[kPa]
σ_n	: compressive stress	[kPa]

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