The Effects of Particle Properties on the Parameters of Impact Sound between Two Particles

Jusuke Hidaka, Atsuko Shimosaka and Shigeo Miwa
Department of Chemical Engineering
Doshisha University

Abstract

The parameters of impact sound between two particles are discussed to measure the particle size and velocity of colliding particles by the analysis of that impact sound. The radiation mechanism of the impact sound is considered theoretically, and the pressure waveform of the sound is estimated on the basis of the radiation mechanism. The calculation of pressure waveform takes account of the reflection of the impact sound from the surface of the colliding spherical particle. The estimated waveform agrees well with the measured one.

The parameters of impact sound related closely to the particle size and impact velocity between particles.

The results show that it is possible to measure the particle size and velocity of flowing particles by the analysis of flow noise.

1. Introduction

Many particles having comparatively large masses move vigorously in powder industrial process. The particles, therefore, cause a large collision and friction among particles and between particles and the container walls. This discharges energy with high sound out of the process line. That is, such radiated sound stems from the movement of particles inside the process system. The parameters of this radiated sound is determined by the properties and movement of these particles. This leads us to suppose that elucidating the relationship between the particle properties and parameters of radiated sound will make it possible to make particle size measurements using radiated noise waveforms.

One of the most widely-known pulverization measurement methods using radiated sound is the long-history measurement of the intra-mill residual quantity during cement pulverization. Another measurement, consisting of measuring rates of powder flow from hoppers, has been conducted experimentally. However, all these measurement methods use the empirical correlation between the target quantity of state and radiated sound parameters. No report has been released on the acoustic pulverization measurements.

Most of the radiated sound stemming from the flow or mixture of particles in a hopper or the vigorous movement of particles in an air transportation pipeline results from the noise released by particles colliding with one another. The first step in a basic study of such radiated sound should consist of elucidating the relationship between the parameters of two colliding particles, particle properties, and particle movement status. Several experiments on the impact sound released from two steel balls have been reported to basically examine ways to inhibit machine noise. Nishimura and Takahashi indicated that impact sound pressure is proportional to the gravity center acceleration and sectional area of steel balls as deformed by collision. Koss and Alfredson determined the impulsive velocity potential of a steel ball and synthesized a sound pressure waveform on the
assumption that two steel balls form a dipole. Tomita et al. estimated a sound pressure waveform by synthesizing sound released from each of the ultrasmall portions of the sphere surface, at the observation point.

Sound pressure waveforms estimated using the above methods agree comparatively well with observed waveforms in the direction of impact axis, where sound pressure peaks, but does not agree well in other directions. This, we believe, is because the above methods do not allow for effects of the reflection and diffraction conducted by colliding balls.

Our study consists of three parts:

1. estimating sound pressure waveforms of impact sound released from colliding particles, with due consideration given to the reflection effect of colliding particles; and
2. conducting collision tests on two steel balls and indicating that observed waveforms of impact sound agree well with calculated waveforms at all observation points;
3. examining the relationship between impact speed and resulting sound pressure, and between the diameters of colliding steel balls and resulting waveforms.

2. Impact sound generation mechanism and estimation of sound pressure waveforms

2.1 Acceleration during collision

Impact sound from spherical particles stems from the particle surfaces quickly accelerated by the colliding particles. An acceleration waveform acting on two particles colliding elastically can be obtained using Hertz's elastic contact theory described below. When spherical particles (1) and (2) having diameters of \( a_1 \) and \( a_2 \) respectively as shown in Fig. 1 collide with each other at relative speed \( v_0 \), the following equation holds for the elastic deformation \( \xi(t) \) and deformation acceleration \( \ddot{\xi}(t) \):

\[
\ddot{\xi}(t) = -q_1 q_2 \xi(t)^{\frac{3}{2}}
\]

where

\[
q_1 = \frac{m_1 + m_2}{m_1 m_2}, \quad q_2 = \frac{4}{3\pi} \left( \frac{1}{\delta_1 + \delta_2} \right) \sqrt{\frac{a_1 a_2}{a_1 + a_2}},
\]

\[
\delta_{1,2} = \frac{1 - \nu_{1,2}^2}{\pi E_{1,2}}
\]

and \( m_1 \) and \( m_2 \) are the masses of particles (1) and (2), \( \nu_1 \) and \( \nu_2 \) are their Poisson's ratios, and \( E_1 \) and \( E_2 \) are Young's moduli. Elastic deformation \( \xi(t) \) can be expressed approximately as:

\[
\xi(t) = \xi_m \sin \frac{1.068 v_0 t}{E m}
\]

where \( \xi_m \) is the maximum deformation of the colliding particles and can be expressed as:

\[
\xi_m = \frac{15\pi v_0^2 (\delta_1 + \delta_2) m_1 m_2}{16 (m_1 + m_2)} \frac{1}{a_1 a_2}
\]

\[
\times \left[ \frac{a_1 + a_2}{a_1 a_2} \right]^{\frac{1}{3}} v_0^{\frac{1}{3}}
\]

The following two equation holds for contact time \( T \) and maximum deformation acceleration \( \ddot{\xi}_m \) and impact velocity:

\[
T = 4.53 \left[ \frac{(\delta_1 + \delta_2) m_1 m_2}{m_1 + m_2} \right]^{\frac{2}{3}}
\]

\[
\times \left[ \frac{a_1 + a_2}{a_1 a_2} \right]^{\frac{1}{3}} v_0^{\frac{1}{3}}
\]

and

\[
\ddot{\xi}_m = 2\sigma_m = -q_1 q_2 \left[ \frac{15\pi (\delta_1 + \delta_2) m_1 m_2}{16 (m_1 + m_2)} \right]^{\frac{3}{5}}
\]

\[
\times \left[ \frac{a_1 + a_2}{a_1 a_2} \right]^{\frac{10}{3}} v_0^{\frac{2}{3}}
\]

Assuming that the particle deformation acceleration equals gravity center acceleration, the acceleration waveform \( [a(t) = \frac{1}{2} \ddot{\xi}(t)] \) of the particle surfaces due to collision can be obtained from Eqs. (1) and (2).

2.2 Estimation of sound pressure waveforms of impact sound

The first step is to determine the sound pressure waveform (impulse response) radiated by a particle when unit impulse acceleration acts on it. The velocity potential \( \phi'(r, \theta, t) \) of sound

Fig. 1 Collision between two elastic particles
occurring when a spherical particle oscillates reciprocatingly along the Z-axis at a speed of $U_0 \exp(jwt)$ satisfies the following wave equation:

$$\frac{\partial^2 \phi'}{\partial r^2} + \frac{2a}{r} \frac{\partial \phi'}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \phi'}{\partial \theta} \right) + k^2 \phi' = 0$$

(7)

where $k$ is the wave number. When one has solved Eq. (7) using the boundary condition which indicates that the speed of the particle surface equals that of the medium, the following velocity potential is obtained:

$$\phi' = \frac{a^3 U_0}{r^2} \frac{(1 + jkr) \cos \theta e^{jwt - k(r-a)}}{[2(1 + jka) - k^2 a^2]}$$

(8)

Based on Eq. (8), we can obtain sound pressure radiated by the particle subjected to unit impulse acceleration as follows using the method employed by Koss et al. 9:

$$P_{imp} = \frac{\rho_0 C \cos \theta}{r} \left[ \cos \left( \frac{C}{a} \tau' \right) - (1 - \frac{a}{r}) \sin \left( \frac{C}{a} \tau' \right) \right] e^{-\tau'/\alpha}$$

(9)

where $\tau' = t - (r - a)/C$, $\rho_0$ is medium density, while $C$ is sound speed.

The sound pressure waveform of sound radiated by a particle when subjected to a given acceleration $a(t)$ can generally be expressed as follows 6:

$$P(r, \theta, t) = \int_0^t P_{imp}(t - \tau) * a(\tau) d\tau$$

(10)

If we approximate the acceleration resulting from Eqs. (1) and (2) by $a(t) = \frac{1}{2} \xi_m \sin bt$ for easier calculation, we obtain the following equation 3 from Eqs. (9) and (10) where $b = \frac{\pi}{T}$:

$$P(r, \theta, t) = \frac{\rho_0 a_m a^3 \cos \theta}{8(b^4 + 4l^4)} \frac{1}{r^2}$$

$$\times \left[ \left( \frac{2r_t}{a_i} - 1 \right) \left\{ (4b^3 l_i - 8b^3 l_{i-'}) \cos \left( \frac{\pi}{2l_i} \right) T \right\} + [(4b^3 l_i - 8b^3 l_{i-'}) \cos \left( \frac{\pi}{2l_i} \right) T + \{ (4b^3 l_i - 8b^3 l_{i-'}) \cos \left( \frac{\pi}{2l_i} \right) T \} \exp(-l_{i-'}) \right]$$

When $0 < \tau' < T$,

$$P(r_i, \theta_i, t_i) = \frac{\rho_0 a_m a^3 \cos \theta_i}{8(b^4 + 4l^4)} \frac{1}{r_i^2}$$

$$\times \left[ \left( \frac{2r_t}{a_i} - 1 \right) \left\{ (4b^3 l_i - 8b^3 l_{i-'}) \cos \left( \frac{\pi}{2l_i} \right) T \right\} + [(4b^3 l_i - 8b^3 l_{i-'}) \cos \left( \frac{\pi}{2l_i} \right) T + \{ (4b^3 l_i - 8b^3 l_{i-'}) \cos \left( \frac{\pi}{2l_i} \right) T \} \exp(-l_{i-'}) \right]$$

When $T < \tau'$,

$$P(r_i, \theta_i, t_i) = \frac{\rho_0 a_m a^3 \cos \theta_i}{8(b^4 + 4l^4)} \frac{1}{r_i^2}$$

$$\times \left[ \left( \frac{2r_t}{a_i} - 1 \right) \left\{ (4b^3 l_i - 8b^3 l_{i-'}) \cos \left( \frac{\pi}{2l_i} \right) T + [(4b^3 l_i - 8b^3 l_{i-'}) \cos \left( \frac{\pi}{2l_i} \right) T \} \exp(-l_{i-'}) \right]$$
\[ \times \exp[-l_i(t'_i - T)] \\
- [(8b_i^2 - 4b^3l_i)\cos l_i(t'_i - \frac{\pi}{2l_i}) \\
+ (8b_i^2 + 4b^3l_i)\sin l_i(t'_i - \frac{\pi}{2l_i})] \exp(-l_i t'_i) \] \\
(12)

where the subscript \(i\) is an index of spherical particle, \(l_i = \frac{C}{a_i}\). Sound pressure \(P(r, \theta, t)\) from two colliding particles at the observation point \(M(r, \theta)\) is shown in Fig. 3 must allow for the sum of sound pressures radiated from particles (1) and (2) and waves reflected between the particles.

2. 3 Sound pressure waveforms of reflected waves

Calculations related to the scattering of one particle’s sound wave due to the other are very complex. This paper, therefore, considers reflected waves alone.

Let’s now consider the case in which impact sound radiated from particle (1) is reflected particle (2). The velocity potential \(\phi(r, \theta, t)\) of sound radiated by one particle subjected to Hertz’s acceleration can be obtained in the same manner as in Eqs. (10) and (11) and expressed as follows:

When \(0 < t' < T\),
\[ \phi_i(r_i, \theta_i, t_i) = -\frac{\alpha_m a_i^3 \cos \theta}{4(b^4 + 4l_i^4)2r_i^2} \left[ \left( \frac{2r_i}{a_i} - 1 \right) \right. \\
\times (4l_i b \sin bt'_i - 8l_i^3 \sin bt'_i + 8b_i^2 \cos bt'_i) \\
- (4b_i^2 \cos bt'_i - 4l_i b \cos bt'_i - 8l_i^3 \sin bt'_i) \\
- \left( \frac{2r_i}{a_i} - 1 \right) (8b_i^2 \cos l_i t'_i + 4b^3 \sin l_i t'_i) \\
\times \exp(-l_i t'_i) + \left[8b_i^2 \cos l_i(t'_i - \frac{\pi}{2l_i}) \\
+ 4b^3 \sin l_i(t'_i - \frac{\pi}{2l_i}) \right] \exp(-l_i t_i) \] \\
+ \frac{\alpha_m a_i^3 \cos \theta}{2r_i^2 b} (1 - \cos bt'_i) \] \\
(13)

When \(T < t'\),
\[ \phi_i(r_i, \theta_i, t_i) = -\frac{\alpha_m a_i^3 \cos \theta}{4(b^4 + 4l_i^4)2r_i^2} \left[ \left( 1 - \frac{2r_i}{a_i} \right) \right. \\
\times [8b_i^2 \cos l_i(t'_i - T) + 4b^3 \sin l_i(t'_i - T)] \]

The velocity component \(u_n\), in the direction of its surface normal, of the sound wave from particle (1) at a given point \(Q\) of the surface of particle (2) can be expressed as per Fig. 4 as:

\[ u_n = -\frac{\partial \phi_1 (s, \beta, t)}{\partial s} a_1 - (a_1 + a_2) \cos \gamma \] \\
(15)

By expanding the above \(u_n\) using the Legendre function, we obtain:

\[ u_n = \sum_{n=0}^{\infty} G_n P_n (\cos \gamma) \] \\
(16)

where
\[ G_n = (n + \frac{1}{2}) \int_{-1}^{1} -\frac{\partial \phi_1 (s, \beta, t)}{\partial s} \frac{a_1 - (a_1 + a_2) h}{s} \] \\
\times P_n (h) \, dh \] \\
(17)

\(P_n (h)\) refers to the Legendre function.

Assuming that velocity potential \(\phi_r\) of a wave reflected by particle (2) is an ordinary is a
spherical wave, the following equation holds\(^7\):

\[
\phi_r(r, \theta, t_r) = \sum_{n=0}^{\infty} C_n [J_n(kr) - jY_n(kr)]
\]

\[
x \times P_n(\cos \theta) e^{j \omega r t}
\]  

(18)

where \( t_r = (r_2 - a_2)/C \), \( J_n(kr) \) is the spherical Bessel function, \( Y_n(kr) \) is the spherical Neumann function, and \( C_n \) is an undetermined coefficient. Velocity component \( u_r \) of a scattered wave in the direction of its normal on the spherical surface can be expressed in the same manner as in Eq. (15) as:

\[
u_r = -\frac{\partial \phi_r}{\partial r_{rz}} = -\sum_{n=0}^{\infty} C_n [J_n'(kr) - jY_n'(kr)]
\]

\[
x \times kP_n(\cos \theta)
\]  

(19)

At point \( Q \) on the particle surface, \( u_n + u_r = 0 \) holds. This determines coefficient \( C_n \) as:

\[
C_n = \frac{aG_n}{[nh_n^{(2)}(ka) - kah_{n+1}^{(2)}(ka)]}
\]  

(20)

where \( h_n^{(2)} \) is the Bessel function of the second kind of the Hankel type. As a result, velocity potential \( \phi(r, \theta_2, t) \) of a wave scattered by particle \( (2) \) at observation point \( M \) can be expressed as:

\[
\phi_r = \sum_{n=0}^{\infty} \frac{aG_n h_n^{(2)}(kr)P_n(\cos \theta_2) e^{j \omega r t_2}}{[nh_n^{(2)}(ka) - kah_{n+1}^{(2)}(ka)]}
\]  

(21)

The wave \( P_{r21} \) from particle \( (2) \) reflected by particle \( (1) \) can be calculated in exactly the same way.

The sound pressure waveform \( P(r, \theta, t) \) of sound from two colliding particles can be expressed as:

\[
P = p_1 + p_2 + p_{r12} + p_{r21}
\]  

(22)

3. Experiment Method

Figure 5 outlines the experimental apparatus that we employed to measure impact sound from two colliding steel balls. We hung two balls, lifted one of them to a relative height \( h \)
with regard to the other, and then made them collide. We calculated impact velocity \( v_0 \) using equation \( v_0 = \sqrt{2gh} \) and confirmed the result using a laser-type measuring instrument illustrated in the figure. Time \( T \) of the contact (or collision duration time) between two particles was observed using an electric circuit illustrated in the figure. The waveform of acceleration acting on the steel balls was observed using an acceleration pickup (manufactured by Lion; Type PV-90A; weight: 1 g) installed on the opposite side of the collision point. Impact sound was detected using a condenser microphone (manufactured by Bruel & Kjaer, Type 4135), passed through an amplifier (manufactured by B & K, Type 2607), stored in a waveform analyzer (manufactured by Iwatsu Electric, SM-2100), and then subjected to peak sound pressure measurements and frequency analyses.

The steel balls used measured 0.0508, 0.0381, 0.0284, 0.0183, 0.0161, and 0.0111 m. They had a density \( \rho_p \) of 7.8 kg/m\(^3\), a Young’s modulus \( E \) of \( 2.09 \times 10^{11} \) N/m\(^2\), and a Poisson’s ratio \( v \) of 0.287.

4. Results and discussion
4.1 Acceleration waveform and contact time

Figure 6 shows a typical acceleration waveform observed. The broken lines in the figure have been calculated using Eqs. (1) and (2). High-frequency oscillation included in the observed waveform corresponds to the characteristic frequency of the particles. \( T \) indicated in the figure is contact time. Figure 7 indicates the relationship between contact time \( T \) and impact velocity \( v_0 \) measured using the above electric circuit. The values calculated using Eq. (5) agree very well with the observed values. These results prove that, as reported by Nishimura et al. \(^2\), Hertz’s elastic contact theory is sufficiently applicable to this collision. These results also confirm that the following equation, obtained by deforming Eq. (5), holds:

\[
T \propto D_p^2
\]

When the two steel balls have the same diameter,

\[
T \propto D_p \quad (23)
\]

When the diameter ratio is \( \lambda \),

\[
T \propto \lambda \left[ \frac{(1 + \lambda)}{(1 + \lambda^3)^2} \right]^{0.2} \propto \lambda^{0.96} \quad (24)
\]

where \( \lambda = a_1/a_2 \), \( a_1 \leq a_2 \).

4.2 Sound pressure waveforms of impact sound

Figure 8 and 9 gives typical examples of the sound pressure waveforms of impact sound and

![Fig. 8 Pressure waveforms and frequency spectra of the sound radiated from the collision of two steel balls](image-url)
their frequency spectra. As the figure indicates, the particle impact sound consists of the pulsatory sound during the initial stage of collision (transitory sound) and the subsequent characteristic oscillation of the balls. This paper focuses on the transitory sound. The relationship between the peak sound pressures $P_1 \sim P_3$, along with $P_1' \sim P_3'$, and the impact velocity $v_0$ satisfies the following relational expression derived from Eqs. (5) and (9), in all cases as shown in Fig. 10.

$$P_m \propto v_0^{\frac{6}{5}}$$

(25)

Between particle diameter $\lambda$ and peak sound pressure $P_m$, the following relational expression holds:

$$P_m \propto \lambda^{0.85}$$

(26)

Figures 8 and 9 indicate the sound pressure waveform of impact sound in the direction along the collision axis ($\theta = 0^\circ$) and in the di-
rection perpendicular (θ = 90°) to it. The waveform varies depending on the observation point. As is evident from the given typical frequency spectra, however, the peak frequency remains almost constant on a specific spectrum regardless of changes in sound pressure waveform or impact velocity according to the observation point. Although with slight changes depending on the impact velocity and observation point as will be mentioned later, the following relational expressions hold between the diameter of the colliding balls and their frequency as shown in Fig. 11.

When the two balls have the same diameter,
\[ f \propto D_p^{-1} \quad (27) \]

When the diameter ratio is \( \lambda \),
\[ f \propto \lambda \left( \frac{1 + \lambda}{1 + \lambda^3} \right)^{-0.2} \propto \lambda^{-0.96} \quad (28) \]

The above indicates that measuring particle impact sound makes it possible to measure its sound pressure, particle motion speed, frequency, and particle diameter \( D_p \).

4.3 Sound pressure waveform calculations

Figure 12 indicates a typical example of sound pressure waveform calculations under the conditions shown in Fig. 8. The dotted line in the figure represents the waveform calculated using the aforementioned method allowing for scattering. The continuous line gives the same observed waveform as that in Fig. 8. The alternate long and short dash line is the waveform calculated using Koss's method. Since the waveforms correspond in the direction in which the maximum acceleration acts, the waveform at the observation point (θ = 0°) indicating the maximum sound pressure agrees comparatively well in every case of calculation. Koss's waveforms, however, slip on the time axis. With the value θ being large, or being equal to 90° in particular, calculated values differ greatly from observed values. Our calculated values allowing for reflection approach the observed values much more than they do Koss's values. Okamoto took photos of the
compression status of the air around balls at the moment they collided, using the Schlieren method. His photos show how complexly the air density changes between two balls. This, we believe, is partly due to the disturbance of the air near the colliding balls and partly due to considerably strong reflection between the balls. The effect of this reflection increases as θ rises (Fig. 13). Calculated values, therefore, become smaller than observed values. This leads us to suppose that more strict calculations, allowing for frequent reflection between the balls, will achieve results nearer to the observed waveforms. Taking into account calculated values applicable to impact sound from a number of colliding particles, this paper considers one-time reflection alone.

4.4 Relationship between impact sound frequency and particle diameters

Figure 14 indicates the structure of the impact sound pressure waveforms shown in Fig. 12. These sound pressure waveforms are composed by overlapping

- radiated sound wave $p_1$ from steel ball (1);
- sound wave $p_2$, resulting from steel balls (2) — which has its center away from ball (1) by the distance equal to the latter’s diameter — accelerated in the opposite direction of steel ball (1); and
- sound wave $p_{r_{12}}$ reflected by ball (2). The above calculations excludes sound wave $p_{r_{12}}$ reflected by steel ball (1), because the ball’s reflection face is geometrically hidden from the viewpoint of the observation point. When $θ = 90°$, the above calculations include both $p_{r_{12}}$ and $p_{r_{21}}$.

For frequency $f_m$, which peaks in the frequency spectrum of impact sound, $f_m \propto D_p^{-1}$ holds as mentioned before when the colliding particles have the same diameter. Furthermore, experiments show that the frequency $f_m$ equals the inverse number of time $τ$ (Fig. 8) from the rise of the sound pressure waveform to its second intersection with the time axis[8]. The time $τ$ in Figs. 8 and 14 is proportional to $D_p$.

This is because the sound pressure waveforms from particles (1) and (2), as mentioned before, result from the convolution of the impulse sound pressure $P_{imp}$ and the acceleration wave-
Form \( a(t) \) indicated in Fig. 15. The basic waveform of \( P_{imp} \) is determined by the particle diameter as shown in Fig. 15, while the degree of its sound pressure is determined by the location of the observation point. The time during which acceleration acts equals the contact time, which is proportional to \( D_p \) when the particles have the same diameter. The duration of sound radiated by one particle in the form of sound pressure waveform on the time axis is therefore proportional to \( D_p \). The distance between the particles is very small. With the sound attenuation due to distance neglected, the result is that two waveforms with different phases are given. We find that the frequency of resulting synthetic waveform is proportional to \( D_p \). When the diameter ratio is \( \lambda \), the comparable frequency is determined by the relationship between contact time and \( \lambda \), so that it relates to the diameter ratio \( \lambda \) as indicated in Eq. (24).

Contact time relates to impact velocity as follows:

\[
T \propto v^{-0.2}
\]

Furthermore, sound wave arrival time varies slightly depending on the observation point. Impact sound frequency is therefore expected to vary when impact velocity varies greatly or impact sound is observed from a nearby point.

5. Conclusion

As a basic study of pulverization measurements using particle impact sound, we examined the relationship between the parameters of impact sound stemming from two colliding particles, particle properties and particle motion velocity.

Particle impact sound (transitory sound) results from the quick acceleration of the particle surfaces due to collision. To check the radiation mechanism, we calculated impact sound pressure waveforms, allowing for reflection between the particles. Waveforms thus calculated agreed well with observed values.

A quantitative relationship holds between particle impact sound pressure and particle impact velocity, and between frequency and particle diameter. We can therefore measure the motion velocity and diameters of colliding particles by analyzing impact sound.

**Nomenclature**

- \( a \) : radius of spherical particle [m]
- \( b = b/r/T \) : [rad/s]
- \( C \) : sound speed [m/s]
- \( D_p \) : particle diameter [m]
- \( E \) : modulus of elasticity [N/m²]
- \( f \) : frequency of impact sound [Hz]
- \( h_n(kr) \) : spherical Bessel function of the third kind
- \( i \) : index of spherical particle [-]
- \( j \) : imaginary unit [-]
- \( J_n(kr) \) : spherical Bessel function of the first kind
- \( k \) : wave number \( (= \omega/C) \) [kg]
- \( m \) : mass of spherical particle [kg]
- \( P_{imp} \) : impulse sound pressure [Pa]
- \( P_n(h) \) : Legendre function
- \( r \) : distance between particle and observation point [m]
- \( t \) : time [s]
- \( T \) : contact time between two spherical particles [s]
- \( u \) : particle velocity of fluid [m/s]
- \( U_0 \) : amplitude of velocity of oscillating particle [m/s]
- \( v_0 \) : impact velocity [m/s]
- \( \alpha(t) \) : acceleration of colliding particles [m/s²]
- \( \beta \) : angle [degree]
- \( \gamma \) : angle [degree]
- \( \omega \) : angle [degree]
- \( \lambda \) : ratio of particle radius \( (= a_1/a_2) \) [-]
- \( \nu \) : Poisson's ratio [-]
- \( \xi(t) \) : elastic deformation of colliding particles [m]
- \( \xi_m \) : maximum deformation of colliding particle [m]
$\rho_0$: density of fluid [kg/m$^3$]
$\rho_p$: particle density [kg/m$^3$]
$t$: characteristic time in Fig. 8 [s]
$\phi$: velocity potential of impact sound [m$^2$/s]
$\omega$: angular frequency [rad/s]

References

8) Okamoto, H.: *J. of the Faculty of Eng., The Univ. of Tokyo*, 36, [1], 37 (1981).