ON JACOBI FORMS OF WEIGHT 4

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0. Introduction

In [2] Bannai and Ozeki proved that many Jacobi forms (in the sense of Eichler and Zagier [6]) are constructed from certain multivariable polynomials which are a multivariable version of weight enumerators of certain codes, or, equivalently, from the polynomials which are invariant under a certain finite group action. This construction of Jacobi forms is a generalization of the well known Broué–Enguehard map from the space of weight enumerators of binary self-dual doubly even codes to a space of modular forms (see [3–5, 7]). We summarize one of the main results of [2] as follows.

THEOREM A. (Bannai and Ozeki [2, (3.3)]) *Let $G = \langle \sigma_1, \sigma_2 \rangle \subset GL(2l, \mathbb{C})$ with*

$$\sigma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & & & & \\ 1 & -1 & & & & \\ & & 1 & 1 & & \\ & & 1 & -1 & & \\ & & & & \ddots & \\ & & & & & 1 & 1 \\ & & & & & 1 & -1 \end{pmatrix},$$

and

$$\sigma_2 = \begin{pmatrix} \sqrt{-1} & & & & & \\ & 1 & & & & \\ & & \sqrt{-1} & & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & \sqrt{-1} \\ & & & & & & 1 \end{pmatrix}.$$

Let $f \in \mathbb{C}[x_1, y_1, x_2, y_2, \dots, x_l, y_l]_{s_1, s_2, \dots, s_l}^G$, i.e. f is a polynomial in $x_1, y_1, \dots, x_l, y_l$ invariant under the group $G = \langle \sigma_1, \sigma_2 \rangle$ and the degree of f in x_i and y_i is s_i

($1 \leq i \leq l$). (Here $n = s_1 + s_2 + \cdots + s_l \equiv 0 \pmod{8}$.) Then for any non-negative integers $m_1, m_2, \dots, m_l$, we have that

$$f(\varphi_3(\tau, m_1 z), \varphi_2(\tau, m_1 z), \varphi_3(\tau, m_2 z), \varphi_2(\tau, m_2 z), \dots, \varphi_3(\tau, m_l z), \varphi_2(\tau, m_l z))$$

is a Jacobi form of weight $k = n/2$ and index $m = \sum_{i=1}^l m_i^2 s_i$. Here note that $\varphi_2(\tau, z) = \theta_2(2\tau, 2z)$, $\varphi_3(\tau, z) = \theta_3(2\tau, 2z)$ and

$$\begin{cases} \theta_2(\tau, z) = \sum_{n \in \mathbb{Z}} e^{\pi i(n+(1/2))^2 \tau + (2n+1)\pi i z} \\ \theta_3(\tau, z) = \sum_{n \in \mathbb{Z}} e^{\pi i n^2 \tau + 2n\pi i z}, \end{cases}$$

are the Jacobi theta functions (cf. [6, 8, 11]).

In [2], we conjectured that for weight $k \equiv 0 \pmod{4}$ many (or most) of the Jacobi forms of weight $k = n/2$ and index m are constructed by Theorem A by taking appropriate non-negative integers $m_1, m_2, \dots, m_l$. The purpose of this paper is to provide numerical *evidence* for this conjecture for the first non-trivial case: $k = 4$. In this paper we exclusively treat the case $k = 4$; the cases for larger values of k (which are multiples of 4) will be discussed in a later paper. Now, we will formulate our conjecture more precisely, and state our main result (Theorem 1). In what follows, we assume that k and m are natural numbers with $k \equiv 0 \pmod{4}$ unless otherwise stated.

Let

$J_{k,m}$ = the space of Jacobi forms (on $SL(2, \mathbb{Z})$) of weight k and index m .

(The reader is referred to Eichler and Zagier [6, p. 1 or pp. 8–9] for the definition of Jacobi forms.) We also denote

$J_{k,m}^\natural$ = the subspace of Jacobi forms (on $SL(2, \mathbb{Z})$) of weight k and index m which are constructed by Theorem A (by choosing all appropriate $m_1, m_2, \dots, m_l$).

Following Eichler and Zagier [6], we define

$$j(k, m) = \sum_{v=1}^m \dim M_{k+2v} - \sum_{v=1}^m \left\lceil \frac{v^2}{4m} \right\rceil,$$

where M_i is the space of modular forms of weight i . Hence

$$\dim M_i = \begin{cases} \left\lfloor \frac{i}{12} \right\rfloor & \text{if } i \equiv 2 \pmod{12} \\ \left\lfloor \frac{i}{12} \right\rfloor + 1 & \text{if } i \equiv 0, 4, 6, 8, 10 \pmod{12}. \end{cases}$$

Also, for a real number x ,

$$\lfloor x \rfloor = \text{the largest integer } \leq x \quad \text{and} \quad \lceil x \rceil = \text{the least integer } \geq x.$$

The following result is proved [6].

THEOREM B. (Eichler and Zagier [6, p. 105])

$$\dim J_{k,m} \geq j(k, m).$$

(Note that $\dim J_{k,m} = j(k, m)$ if $k \geq m$ [6, p. 121]. It seems that Eichler and Zagier conjecture that $\dim J_{k,m} = j(k, m)$ for all even integers $k \geq 3$ (see [6, p. 121]). It should be noted that Arakawa [1, p. 235, formula (5,4)] gives an explicit formula to calculate $\dim J_{k,m}$. At the time of writing this paper we do not know whether this value coincides with $j(k, m)$. A numerical calculation shows that this is the case at least for $k = 4$ and $m \leq 100$. We expect that this is true for all even $k \geq 4$ and all m . (See Remark 5 at the end of Section 2.)

We conjecture the following.

CONJECTURE 1.

$$\dim J_{k,m}^{\natural} = j(k, m) \quad \text{for all } k \equiv 0 \pmod{4} \text{ and } m.$$

(Also, we conjecture that $J_{k,m}^{\natural} = J_{k,m}$.)

The main result of this paper is the following.

THEOREM 1. For $k = 4$ and $m \leq 48$, Conjecture 1 is true. Namely, we have $\dim J_{4,m}^{\natural} = j(4, m)$ for $m \leq 48$. (Note that this also implies that $J_{4,m}^{\natural} = J_{4,m}$ for all $m \leq 48$.)

The explicit values of $j(4, m)$ ($m \leq 100$) are listed in Table 1 of the Appendix.

1. Outline proof of Theorem 1

Let $G = \langle \sigma_1, \sigma_2 \rangle \subset GL(2l, \mathbb{C})$, and let

$$\mathbb{C}[x_1, y_1, x_2, y_2, \dots, x_l, y_l]_{s_1, s_2, \dots, s_l}^G$$

be as defined in Theorem A. Here we consider the case $s_1 + s_2 + \dots + s_l = n = 8$.

Step 1

The space $J_{k,m}^{\natural}$ is spanned by the Jacobi forms constructed by Theorem A from those $f(x_1, y_1, \dots, x_l, y_l)$ with $(2k \Rightarrow) n = l$ and $s_1 = s_2 = \dots = s_l = 1$. (Note that the claim of this step is true for all $k \equiv 0 \pmod{4}$.)

Proof of step 1. We use the method of polarization in invariant theory (cf. Schur [12, Satz 1.6]). Namely, let

$$f \in \mathbb{C}[x_1, y_1, \dots, x_l, y_l]_{s_1, s_2, \dots, s_l}^G, \quad \text{with } s_i \geq 2.$$

Then

$$g = x_{l+1} \cdot \frac{\partial f}{\partial x_i} + y_{l+1} \cdot \frac{\partial f}{\partial y_i}$$

is in $\mathbb{C}[x_1, y_1, \dots, x_{l+1}, y_{l+1}]_{s_1, \dots, s_{i-1}, s_i-1, s_{i+1}, \dots, s_l, 1}^G$. Therefore, starting from the given f , by repeating this process we reach

$$h \in \mathbb{C}[x_1, y_1, \dots, x_n, y_n]_{\underbrace{1, 1, \dots, 1}_n}^G.$$

On the other hand, it is easy to see that the polynomial

$$f \in \mathbb{C}[x_1, y_1, \dots, x_l, y_l]_{s_1, s_2, \dots, s_l}^G$$

is obtained from the element

$$h \in \mathbb{C}[x_1, y_1, \dots, x_n, y_n]_{\underbrace{1, 1, \dots, 1}_n}^G$$

by finding a partition $\{S_1, S_2, \dots, S_l\}$ of the set $\{1, 2, \dots, n\}$ with $|S_i| = s_i$ and putting $x_j = x_k (= x_{S_i})$ if $j, k \in S_i$ ($1 \leq i \leq l$), and taking x_{S_i} as a new x_i .

Step 2

$$\dim \mathbb{C}[x_1, y_1, \dots, x_8, y_8]_{\underbrace{1,1,\dots,1}_8}^G = 15.$$

Moreover, an explicit basis $\{f_i (1 \leq i \leq 15)\}$ of the space

$$\mathbb{C}[x_1, y_1, \dots, x_8, y_8]_{\underbrace{1,1,\dots,1}_8}^G$$

is given in Table 2 of the Appendix.

This basis was first obtained by hand calculation, then this was checked by computer calculation. The method of the calculation is as follows. From the invariance under the element σ_2 in G , f is a linear combination of those monomials with the sum of the degrees in $x_1, x_2, \dots, x_8$ being multiples of 4. There are $\binom{8}{0} + \binom{8}{4} + \binom{8}{8} = 72$ such monomials. Imposing the further condition that f is invariant under the element σ_1 , we obtain a system of linear equations of the 72 coefficients, and then find the dimension of the solution space as well as an explicit basis of the solution space.

Remark 1. The symmetric group S_8 (which acts as permutations on indices $1, 2, \dots, 8$) acts on the space $\mathbb{C}[x_1, y_1, \dots, x_8, y_8]_{1,1,\dots,1}^G$. This representation of S_8 is decomposed into the sum of the two irreducible representations of S_8 of degree 1 and degree 14 corresponding to the partition (Young diagram) (8) and (4, 4) respectively. It is also possible to prove $\dim \mathbb{C}[x_1, y_1, \dots, x_8, y_8]_{1,1,\dots,1}^G = 15$ directly by using group theory. Namely, this dimension corresponds to the inner product $(1, \rho^8)_G$, where ρ is the natural two-dimensional representation of $G (\subset GL(2, \mathbb{C}))$.

Remark 2. Since it may be interesting to know the dimensions of the spaces $\mathbb{C}[x_1, y_1, \dots, x_l, y_l]_{s_1, s_2, \dots, s_l}^G$ explicitly, we list the results in Table 3 of the Appendix.

Step 3

Calculations of $f_i(\varphi_3(\tau, m_1 z), \varphi_2(\tau, m_1 z), \dots, \varphi_3(\tau, m_8 z), \varphi_2(\tau, m_8 z)) \in J_{4,m}^{\natural}$, and of $\dim J_{k,m}^{\natural}$ (for $m \leq 48$).

Fix $m (\leq 48)$. For each $f_i \in \mathbb{C}[x_1, y_1, \dots, x_8, y_8]_{1,1,\dots,1}^G$ ($1 \leq i \leq 15$), and for each solution $(m_1, m_2, \dots, m_8)$ of $m = \sum_{i=1}^8 m_i^2$ (here we may assume that $m_1 \leq m_2 \leq \dots \leq m_8$), we calculate

$$f_i(\varphi_3(\tau, m_1 z), \varphi_2(\tau, m_1 z), \dots, \varphi_3(\tau, m_8 z), \varphi_2(\tau, m_8 z)) \in J_{4,m}^{\natural}$$

in the form of a Fourier expansion. (This calculation was carried out using the computer packages Gap, Magma and Mathematica.) Next we determine how many of the elements in $J_{4,m}^{\natural}$ thus constructed are linearly independent, then we get

$$\dim J_{4,m}^{\natural} = j(4, m),$$

the result stated in Theorem 1. Here we give an explicit basis of $J_{4,m}^{\natural}$ ($= J_{4,m}$) for each $m \leq 20$ in Table 4 of the Appendix.

2. Concluding remarks and open problems

Here we collect some concluding remarks and some questions.

Remark 3. Calculation of $\dim J_{8,m}^{\natural}$ for small m seems possible, and we are in the process of calculating them. Note that we have

$$\dim \mathbb{C}[x_1, y_1, \dots, x_{16}, y_{16}]_{\underbrace{1,1,\dots,1}_{16}}^G = 2796,$$

and this number is the sum of the degrees of 4 irreducible characters of S_{16} corresponding to the partitions (Young diagrams) (16), (14, 2), (12, 4) and (8, 8). That is, $2796 = 1 + 104 + 1260 + 1430$. (We remark that we have now succeeded in calculating the decomposition into irreducible characters of the action of S_l on $\mathbb{C}[x_1, y_1, \dots, x_l, y_l]_{\underbrace{1,1,\dots,1}_l}^G$ for arbitrary l .)

Question 1. For $k \equiv 0 \pmod{4}$, starting from $\mathbb{C}[x_1, y_1]_{2k}^G = \mathbb{C}[e_8, g_{24}]_{2k}$, by taking all possible polarizations $2k - 1$ times, can we obtain all the elements of

$$\mathbb{C}[x_1, y_1, \dots, x_k, y_k]_{\underbrace{1,1,\dots,1}_k}^G?$$

Also, can one obtain a basis of this space explicitly by this method? (The answer to the first question turned out to be negative. We have now succeeded in obtaining the solution to the second question. This result is to be published in a joint paper by Eiichi Bannai, Etsuko Bannai, Michio Ozeki and Yasuo Teranishi, which is in preparation.)

Question 2. The polarization process in the polynomials should correspond to certain operations of constructing Jacobi forms from modular forms. Can one describe these operations explicitly?

Question 3. What is known about $\dim(J_{k,m}(\Gamma(N)))$, where $\Gamma(N)$ is a principal congruence modular group (or more generally about $\dim(J_{k,m}(\Gamma_1))$ for a finite index subgroup Γ_1 of $\Gamma = SL(2, \mathbb{Z})$)?

Question 4. Obviously $J_{k,m} \subset J_{k,m}(\Gamma(N))$ (or more generally $J_{k,m} \subset J_{k,m}(\Gamma_1)$). Is there any natural way to find $J_{k,m}$ if we know $J_{k,m}(\Gamma(N))$ or $J_{k,m}(\Gamma_1)$ explicitly?

Question 5. For $k \equiv 2 \pmod{4}$, our construction like Theorem 1 gives elements of $J_{k,m}$ from

$$\mathbb{C}[x_1, y_1, \dots, x_{2k}, y_{2k}]_{\underbrace{1,1,\dots,1}_{2k}}^H,$$

where H is the subgroup (of index 2) of G mentioned in [2]. Can one calculate $\dim J_{k,m}^{\natural}$ for $k \equiv 2 \pmod{4}$?

Remark 4. In view of [2] and the present paper, it would be interesting to study the following problem in general. Let G be a finite u.g.r. with $G \subset GL(n, \mathbb{C})$. Let

$$\tilde{G} = \left\{ \left(\begin{pmatrix} g & & & \\ & g & & \\ & & g & \\ & & & \ddots \\ & & & & g \end{pmatrix} \in GL(nl, \mathbb{C}) \mid g \in G \right\}$$

(of course, $\tilde{G}$ is isomorphic to G as an abstract group) and study the structure of the invariant ring $\mathbb{C}[x_{11}, x_{12}, \dots, x_{1l}, x_{21}, x_{22}, \dots, x_{2l}, \dots, x_{n1}, x_{n2}, \dots, x_{nl}]^{\tilde{G}}$, the polynomial ring of nl variables under the natural action of $\tilde{G}$. Oura and Tanabe [9] have recently studied the Molien series of this invariant ring for each of the irreducible finite u.g.r. G in the list of Shephard and Todd [13] for $l = 2$.

Remark 5. It was recently proved by Ozeki (unpublished) that $\dim J_{k,m}$ calculated by Arakawa [1] coincides with $j(k, m)$ for all even integers $k \geq 3$. So our Conjecture 1 is in fact equivalent to the conjecture that $J_{k,m} = J_{k,m}^{\natural}$.

Appendix

TABLE 1. List of $J(4, k)$ ($0 \leq m \leq 100$).

m	0	1	2	3	4	5	6	7	8	9	10
$j(4, m)$	1	1	1	1	2	1	1	2	2	2	2
m		11	12	13	14	15	16	17	18	19	20
$j(4, m)$		2	2	3	2	2	4	3	3	4	3
m		21	22	23	24	25	26	27	28	29	30
$j(4, m)$		3	4	4	3	6	4	5	5	5	3
m		31	32	33	34	35	36	37	38	39	40
$j(4, m)$		6	6	5	6	5	6	8	6	6	6
m		41	42	43	44	45	46	47	48	49	50
$j(4, m)$		7	5	9	7	7	8	8	7	11	9
m		51	52	53	54	55	56	57	58	59	60
$j(4, m)$		8	9	10	8	9	8	10	11	10	7
m		61	62	63	64	65	66	67	68	69	70
$j(4, m)$		12	10	10	13	11	9	14	11	11	10
m		71	72	73	74	75	76	77	78	79	80
$j(4, m)$		12	11	15	13	13	13	13	11	15	12
m		81	82	83	84	85	86	87	88	89	90
$j(4, m)$		16	15	15	11	16	15	14	15	16	12
m		91	92	93	94	95	96	97	98	99	100
$j(4, m)$		17	16	17	16	15	14	20	18	17	19

TABLE 2. A basis $\{f_i \ (1 \leq i \leq 15)\}$ of $\mathbb{C}[x_1, y_1, \dots, x_8, y_8]_{1,1,\dots,1}^G$.

$$\begin{aligned}
f_1 = & x_1x_2x_3x_4x_5x_6x_7x_8 + x_5x_6x_7x_8y_1y_2y_3y_4 + x_1x_4x_7x_8y_2y_3y_5y_6 \\
& + x_2x_3x_7x_8y_1y_4y_5y_6 + x_1x_4x_6x_8y_2y_3y_5y_7 + x_2x_3x_6x_8y_1y_4y_5y_7 \\
& + x_3x_4x_5x_8y_1y_2y_6y_7 + x_2x_4x_5x_8y_1y_3y_6y_7 - x_1x_4x_5x_8y_2y_3y_6y_7 \\
& - x_2x_3x_5x_8y_1y_4y_6y_7 + x_1x_3x_5x_8y_2y_4y_6y_7 + x_1x_2x_5x_8y_3y_4y_6y_7 \\
& + x_3x_4x_6x_7y_1y_2y_5y_8 + x_2x_4x_6x_7y_1y_3y_5y_8 - x_1x_4x_6x_7y_2y_3y_5y_8 \\
& - x_2x_3x_6x_7y_1y_4y_5y_8 + x_1x_3x_6x_7y_2y_4y_5y_8 + x_1x_2x_6x_7y_3y_4y_5y_8 \\
& + x_1x_4x_5x_7y_2y_3y_6y_8 + x_2x_3x_5x_7y_1y_4y_6y_8 + x_1x_4x_5x_6y_2y_3y_7y_8 \\
& + x_2x_3x_5x_6y_1y_4y_7y_8 + x_1x_2x_3x_4y_5y_6y_7y_8 + y_1y_2y_3y_4y_5y_6y_7y_8 \\
\\
f_2 = & x_1x_2x_3x_4x_5x_6x_7x_8 + x_4x_6x_7x_8y_1y_2y_3y_5 + x_1x_5x_7x_8y_2y_3y_4y_6 \\
& + x_2x_3x_7x_8y_1y_4y_5y_6 + x_1x_5x_6x_8y_2y_3y_4y_7 + x_2x_3x_6x_8y_1y_4y_5y_7 \\
& + x_3x_4x_5x_8y_1y_2y_6y_7 + x_2x_4x_5x_8y_1y_3y_6y_7 - x_1x_4x_5x_8y_2y_3y_6y_7 \\
& - x_2x_3x_4x_8y_1y_5y_6y_7 + x_1x_3x_4x_8y_2y_5y_6y_7 + x_1x_2x_4x_8y_3y_5y_6y_7 \\
& + x_3x_5x_6x_7y_1y_2y_4y_8 + x_2x_5x_6x_7y_1y_3y_4y_8 - x_1x_5x_6x_7y_2y_3y_4y_8 \\
& - x_2x_3x_6x_7y_1y_4y_5y_8 + x_1x_3x_6x_7y_2y_4y_5y_8 + x_1x_2x_6x_7y_3y_4y_5y_8 \\
& + x_1x_4x_5x_7y_2y_3y_6y_8 + x_2x_3x_4x_7y_1y_5y_6y_8 + x_1x_4x_5x_6y_2y_3y_7y_8 \\
& + x_2x_3x_4x_6y_1y_5y_7y_8 + x_1x_2x_3x_5y_4y_6y_7y_8 + y_1y_2y_3y_4y_5y_6y_7y_8 \\
\\
f_3 = & x_1x_2x_3x_4x_5x_6x_7x_8 + x_1x_6x_7x_8y_2y_3y_4y_5 + x_4x_5x_7x_8y_1y_2y_3y_6 \\
& + x_2x_3x_7x_8y_1y_4y_5y_6 + x_1x_5x_6x_8y_2y_3y_4y_7 + x_1x_4x_6x_8y_2y_3y_5y_7 \\
& + 2x_2x_3x_6x_8y_1y_4y_5y_7 - x_1x_3x_6x_8y_2y_4y_5y_7 - x_1x_2x_6x_8y_3y_4y_5y_7 \\
& + x_3x_4x_5x_8y_1y_2y_6y_7 + x_2x_4x_5x_8y_1y_3y_6y_7 - 2x_1x_4x_5x_8y_2y_3y_6y_7 \\
& - x_2x_3x_5x_8y_1y_4y_6y_7 + x_1x_3x_5x_8y_2y_4y_6y_7 + x_1x_2x_5x_8y_3y_4y_6y_7 \\
& - x_2x_3x_4x_8y_1y_5y_6y_7 + x_1x_3x_4x_8y_2y_5y_6y_7 + x_1x_2x_4x_8y_3y_5y_6y_7 \\
& + x_3x_5x_6x_7y_1y_2y_4y_8 + x_2x_5x_6x_7y_1y_3y_4y_8 - x_1x_5x_6x_7y_2y_3y_4y_8 \\
& + x_3x_4x_6x_7y_1y_2y_5y_8 + x_2x_4x_6x_7y_1y_3y_5y_8 - x_1x_4x_6x_7y_2y_3y_5y_8
\end{aligned}$$

$$\begin{aligned}
& -2x_2x_3x_6x_7y_1y_4y_5y_8 + x_1x_3x_6x_7y_2y_4y_5y_8 + x_1x_2x_6x_7y_3y_4y_5y_8 \\
& -x_3x_4x_5x_7y_1y_2y_6y_8 - x_2x_4x_5x_7y_1y_3y_6y_8 + 2x_1x_4x_5x_7y_2y_3y_6y_8 \\
& +x_2x_3x_5x_7y_1y_4y_6y_8 + x_2x_3x_4x_7y_1y_5y_6y_8 + x_1x_4x_5x_6y_2y_3y_7y_8 \\
& +x_1x_2x_3x_6y_4y_5y_7y_8 + x_2x_3x_4x_5y_1y_6y_7y_8 + y_1y_2y_3y_4y_5y_6y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_4 = & x_1x_2x_3x_4x_5x_6x_7x_8 + x_1x_6x_7x_8y_2y_3y_4y_5 + x_1x_5x_7x_8y_2y_3y_4y_6 \\
& +x_1x_4x_7x_8y_2y_3y_5y_6 + 2x_2x_3x_7x_8y_1y_4y_5y_6 - x_1x_3x_7x_8y_2y_4y_5y_6 \\
& -x_1x_2x_7x_8y_3y_4y_5y_6 + x_4x_5x_6x_8y_1y_2y_3y_7 + x_2x_3x_6x_8y_1y_4y_5y_7 \\
& +x_3x_4x_5x_8y_1y_2y_6y_7 + x_2x_4x_5x_8y_1y_3y_6y_7 - 2x_1x_4x_5x_8y_2y_3y_6y_7 \\
& -x_2x_3x_5x_8y_1y_4y_6y_7 + x_1x_3x_5x_8y_2y_4y_6y_7 + x_1x_2x_5x_8y_3y_4y_6y_7 \\
& -x_2x_3x_4x_8y_1y_5y_6y_7 + x_1x_3x_4x_8y_2y_5y_6y_7 + x_1x_2x_4x_8y_3y_5y_6y_7 \\
& +x_3x_5x_6x_7y_1y_2y_4y_8 + x_2x_5x_6x_7y_1y_3y_4y_8 - x_1x_5x_6x_7y_2y_3y_4y_8 \\
& +x_3x_4x_6x_7y_1y_2y_5y_8 + x_2x_4x_6x_7y_1y_3y_5y_8 - x_1x_4x_6x_7y_2y_3y_5y_8 \\
& -2x_2x_3x_6x_7y_1y_4y_5y_8 + x_1x_3x_6x_7y_2y_4y_5y_8 + x_1x_2x_6x_7y_3y_4y_5y_8 \\
& +x_1x_4x_5x_7y_2y_3y_6y_8 + x_1x_2x_3x_7y_4y_5y_6y_8 - x_3x_4x_5x_6y_1y_2y_7y_8 \\
& -x_2x_4x_5x_6y_1y_3y_7y_8 + 2x_1x_4x_5x_6y_2y_3y_7y_8 + x_2x_3x_5x_6y_1y_4y_7y_8 \\
& +x_2x_3x_4x_6y_1y_5y_7y_8 + x_2x_3x_4x_5y_1y_6y_7y_8 + y_1y_2y_3y_4y_5y_6y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_5 = & x_1x_2x_3x_4x_5x_6x_7x_8 + x_1x_6x_7x_8y_2y_3y_4y_5 + x_1x_5x_7x_8y_2y_3y_4y_6 \\
& +x_1x_4x_7x_8y_2y_3y_5y_6 + 2x_2x_3x_7x_8y_1y_4y_5y_6 - x_1x_3x_7x_8y_2y_4y_5y_6 \\
& -x_1x_2x_7x_8y_3y_4y_5y_6 + x_1x_5x_6x_8y_2y_3y_4y_7 + x_1x_4x_6x_8y_2y_3y_5y_7 \\
& +2x_2x_3x_6x_8y_1y_4y_5y_7 - x_1x_3x_6x_8y_2y_4y_5y_7 - x_1x_2x_6x_8y_3y_4y_5y_7 \\
& +2x_3x_4x_5x_8y_1y_2y_6y_7 + 2x_2x_4x_5x_8y_1y_3y_6y_7 - 3x_1x_4x_5x_8y_2y_3y_6y_7 \\
& -2x_2x_3x_5x_8y_1y_4y_6y_7 + x_1x_3x_5x_8y_2y_4y_6y_7 + x_1x_2x_5x_8y_3y_4y_6y_7 \\
& -2x_2x_3x_4x_8y_1y_5y_6y_7 + x_1x_3x_4x_8y_2y_5y_6y_7 + x_1x_2x_4x_8y_3y_5y_6y_7 \\
& +x_1x_2x_3x_8y_4y_5y_6y_7 + x_4x_5x_6x_7y_1y_2y_3y_8 + x_3x_5x_6x_7y_1y_2y_4y_8 \\
& +x_2x_5x_6x_7y_1y_3y_4y_8 - 2x_1x_5x_6x_7y_2y_3y_4y_8 + x_3x_4x_6x_7y_1y_2y_5y_8 \\
& +x_2x_4x_6x_7y_1y_3y_5y_8 - 2x_1x_4x_6x_7y_2y_3y_5y_8 - 3x_2x_3x_6x_7y_1y_4y_5y_8 \\
& +2x_1x_3x_6x_7y_2y_4y_5y_8 + 2x_1x_2x_6x_7y_3y_4y_5y_8 - x_3x_4x_5x_7y_1y_2y_6y_8
\end{aligned}$$

$$\begin{aligned}
& -x_2x_4x_5x_7y_1y_3y_6y_8 + 2x_1x_4x_5x_7y_2y_3y_6y_8 + x_2x_3x_5x_7y_1y_4y_6y_8 \\
& + x_2x_3x_4x_7y_1y_5y_6y_8 - x_3x_4x_5x_6y_1y_2y_7y_8 - x_2x_4x_5x_6y_1y_3y_7y_8 \\
& + 2x_1x_4x_5x_6y_2y_3y_7y_8 + x_2x_3x_5x_6y_1y_4y_7y_8 + x_2x_3x_4x_6y_1y_5y_7y_8 \\
& + x_2x_3x_4x_5y_1y_6y_7y_8 + y_1y_2y_3y_4y_5y_6y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_6 = & x_3x_6x_7x_8y_1y_2y_4y_5 - x_1x_6x_7x_8y_2y_3y_4y_5 - x_2x_3x_7x_8y_1y_4y_5y_6 \\
& + x_1x_2x_7x_8y_3y_4y_5y_6 - x_2x_3x_6x_8y_1y_4y_5y_7 + x_1x_2x_6x_8y_3y_4y_5y_7 \\
& - x_3x_4x_5x_8y_1y_2y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7 + x_2x_3x_5x_8y_1y_4y_6y_7 \\
& - x_1x_2x_5x_8y_3y_4y_6y_7 + x_2x_3x_4x_8y_1y_5y_6y_7 - x_1x_2x_4x_8y_3y_5y_6y_7 \\
& - x_3x_5x_6x_7y_1y_2y_4y_8 + x_1x_5x_6x_7y_2y_3y_4y_8 - x_3x_4x_6x_7y_1y_2y_5y_8 \\
& + x_1x_4x_6x_7y_2y_3y_5y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_2x_6x_7y_3y_4y_5y_8 \\
& + x_3x_4x_5x_7y_1y_2y_6y_8 - x_1x_4x_5x_7y_2y_3y_6y_8 + x_3x_4x_5x_6y_1y_2y_7y_8 \\
& - x_1x_4x_5x_6y_2y_3y_7y_8 - x_2x_3x_4x_5y_1y_6y_7y_8 + x_1x_2x_4x_5y_3y_6y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_7 = & x_3x_5x_7x_8y_1y_2y_4y_6 - x_1x_5x_7x_8y_2y_3y_4y_6 - x_2x_3x_7x_8y_1y_4y_5y_6 \\
& + x_1x_2x_7x_8y_3y_4y_5y_6 - x_3x_4x_5x_8y_1y_2y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7 \\
& + x_2x_3x_4x_8y_1y_5y_6y_7 - x_1x_2x_4x_8y_3y_5y_6y_7 - x_3x_5x_6x_7y_1y_2y_4y_8 \\
& + x_1x_5x_6x_7y_2y_3y_4y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_2x_6x_7y_3y_4y_5y_8 \\
& + x_3x_4x_5x_6y_1y_2y_7y_8 - x_1x_4x_5x_6y_2y_3y_7y_8 - x_2x_3x_4x_6y_1y_5y_7y_8 \\
& + x_1x_2x_4x_6y_3y_5y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_8 = & x_3x_5x_6x_8y_1y_2y_4y_7 - x_1x_5x_6x_8y_2y_3y_4y_7 - x_2x_3x_6x_8y_1y_4y_5y_7 \\
& + x_1x_2x_6x_8y_3y_4y_5y_7 - x_3x_4x_5x_8y_1y_2y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7 \\
& + x_2x_3x_4x_8y_1y_5y_6y_7 - x_1x_2x_4x_8y_3y_5y_6y_7 - x_3x_5x_6x_7y_1y_2y_4y_8 \\
& + x_1x_5x_6x_7y_2y_3y_4y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_2x_6x_7y_3y_4y_5y_8 \\
& + x_3x_4x_5x_7y_1y_2y_6y_8 - x_1x_4x_5x_7y_2y_3y_6y_8 - x_2x_3x_4x_7y_1y_5y_6y_8 \\
& + x_1x_2x_4x_7y_3y_5y_6y_8
\end{aligned}$$

$$\begin{aligned}
f_9 = & x_3x_4x_7x_8y_1y_2y_5y_6 - x_1x_4x_7x_8y_2y_3y_5y_6 - x_2x_3x_7x_8y_1y_4y_5y_6 \\
& + x_1x_2x_7x_8y_3y_4y_5y_6 - x_3x_4x_5x_8y_1y_2y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7
\end{aligned}$$

$$\begin{aligned}
& +x_2x_3x_5x_8y_1y_4y_6y_7 - x_1x_2x_5x_8y_3y_4y_6y_7 - x_3x_4x_6x_7y_1y_2y_5y_8 \\
& +x_1x_4x_6x_7y_2y_3y_5y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_2x_6x_7y_3y_4y_5y_8 \\
& +x_3x_4x_5x_6y_1y_2y_7y_8 - x_1x_4x_5x_6y_2y_3y_7y_8 - x_2x_3x_5x_6y_1y_4y_7y_8 \\
& +x_1x_2x_5x_6y_3y_4y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_{10} = & x_3x_4x_6x_8y_1y_2y_5y_7 - x_1x_4x_6x_8y_2y_3y_5y_7 - x_2x_3x_6x_8y_1y_4y_5y_7 \\
& +x_1x_2x_6x_8y_3y_4y_5y_7 - x_3x_4x_5x_8y_1y_2y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7 \\
& +x_2x_3x_5x_8y_1y_4y_6y_7 - x_1x_2x_5x_8y_3y_4y_6y_7 - x_3x_4x_6x_7y_1y_2y_5y_8 \\
& +x_1x_4x_6x_7y_2y_3y_5y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_2x_6x_7y_3y_4y_5y_8 \\
& +x_3x_4x_5x_7y_1y_2y_6y_8 - x_1x_4x_5x_7y_2y_3y_6y_8 - x_2x_3x_5x_7y_1y_4y_6y_8 \\
& +x_1x_2x_5x_7y_3y_4y_6y_8
\end{aligned}$$

$$\begin{aligned}
f_{11} = & x_2x_6x_7x_8y_1y_3y_4y_5 - x_1x_6x_7x_8y_2y_3y_4y_5 - x_2x_3x_7x_8y_1y_4y_5y_6 \\
& +x_1x_3x_7x_8y_2y_4y_5y_6 - x_2x_3x_6x_8y_1y_4y_5y_7 + x_1x_3x_6x_8y_2y_4y_5y_7 \\
& -x_2x_4x_5x_8y_1y_3y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7 + x_2x_3x_5x_8y_1y_4y_6y_7 \\
& -x_1x_3x_5x_8y_2y_4y_6y_7 + x_2x_3x_4x_8y_1y_5y_6y_7 - x_1x_3x_4x_8y_2y_5y_6y_7 \\
& -x_2x_5x_6x_7y_1y_3y_4y_8 + x_1x_5x_6x_7y_2y_3y_4y_8 - x_2x_4x_6x_7y_1y_3y_5y_8 \\
& +x_1x_4x_6x_7y_2y_3y_5y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_3x_6x_7y_2y_4y_5y_8 \\
& +x_2x_4x_5x_7y_1y_3y_6y_8 - x_1x_4x_5x_7y_2y_3y_6y_8 + x_2x_4x_5x_6y_1y_3y_7y_8 \\
& -x_1x_4x_5x_6y_2y_3y_7y_8 - x_2x_3x_4x_5y_1y_6y_7y_8 + x_1x_3x_4x_5y_2y_6y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_{12} = & x_2x_5x_7x_8y_1y_3y_4y_6 - x_1x_5x_7x_8y_2y_3y_4y_6 - x_2x_3x_7x_8y_1y_4y_5y_6 \\
& +x_1x_3x_7x_8y_2y_4y_5y_6 - x_2x_4x_5x_8y_1y_3y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7 \\
& +x_2x_3x_4x_8y_1y_5y_6y_7 - x_1x_3x_4x_8y_2y_5y_6y_7 - x_2x_5x_6x_7y_1y_3y_4y_8 \\
& +x_1x_5x_6x_7y_2y_3y_4y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_3x_6x_7y_2y_4y_5y_8 \\
& +x_2x_4x_5x_6y_1y_3y_7y_8 - x_1x_4x_5x_6y_2y_3y_7y_8 - x_2x_3x_4x_6y_1y_5y_7y_8 \\
& +x_1x_3x_4x_6y_2y_5y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_{13} = & x_2x_5x_6x_8y_1y_3y_4y_7 - x_1x_5x_6x_8y_2y_3y_4y_7 - x_2x_3x_6x_8y_1y_4y_5y_7 \\
& +x_1x_3x_6x_8y_2y_4y_5y_7 - x_2x_4x_5x_8y_1y_3y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7
\end{aligned}$$

$$\begin{aligned}
& +x_2x_3x_4x_8y_1y_5y_6y_7 - x_1x_3x_4x_8y_2y_5y_6y_7 - x_2x_5x_6x_7y_1y_3y_4y_8 \\
& +x_1x_5x_6x_7y_2y_3y_4y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_3x_6x_7y_2y_4y_5y_8 \\
& +x_2x_4x_5x_7y_1y_3y_6y_8 - x_1x_4x_5x_7y_2y_3y_6y_8 - x_2x_3x_4x_7y_1y_5y_6y_8 \\
& +x_1x_3x_4x_7y_2y_5y_6y_8
\end{aligned}$$

$$\begin{aligned}
f_{14} = & x_2x_4x_7x_8y_1y_3y_5y_6 - x_1x_4x_7x_8y_2y_3y_5y_6 - x_2x_3x_7x_8y_1y_4y_5y_6 \\
& +x_1x_3x_7x_8y_2y_4y_5y_6 - x_2x_4x_5x_8y_1y_3y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7 \\
& +x_2x_3x_5x_8y_1y_4y_6y_7 - x_1x_3x_5x_8y_2y_4y_6y_7 - x_2x_4x_6x_7y_1y_3y_5y_8 \\
& +x_1x_4x_6x_7y_2y_3y_5y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_3x_6x_7y_2y_4y_5y_8 \\
& +x_2x_4x_5x_6y_1y_3y_7y_8 - x_1x_4x_5x_6y_2y_3y_7y_8 - x_2x_3x_5x_6y_1y_4y_7y_8 \\
& +x_1x_3x_5x_6y_2y_4y_7y_8
\end{aligned}$$

$$\begin{aligned}
f_{15} = & x_2x_4x_6x_8y_1y_3y_5y_7 - x_1x_4x_6x_8y_2y_3y_5y_7 - x_2x_3x_6x_8y_1y_4y_5y_7 \\
& +x_1x_3x_6x_8y_2y_4y_5y_7 - x_2x_4x_5x_8y_1y_3y_6y_7 + x_1x_4x_5x_8y_2y_3y_6y_7 \\
& +x_2x_3x_5x_8y_1y_4y_6y_7 - x_1x_3x_5x_8y_2y_4y_6y_7 - x_2x_4x_6x_7y_1y_3y_5y_8 \\
& +x_1x_4x_6x_7y_2y_3y_5y_8 + x_2x_3x_6x_7y_1y_4y_5y_8 - x_1x_3x_6x_7y_2y_4y_5y_8 \\
& +x_2x_4x_5x_7y_1y_3y_6y_8 - x_1x_4x_5x_7y_2y_3y_6y_8 - x_2x_3x_5x_7y_1y_4y_6y_8 \\
& +x_1x_3x_5x_7y_2y_4y_6y_8
\end{aligned}$$

TABLE 3. $\dim \mathbb{C}[x_1, y_1, \dots, x_l, y_l]_{s_1, s_2, \dots, s_l}^G$ with $n = 8$.

$s_1, s_2, \dots, s_l$								d
1,	1,	1,	1,	1,	1,	1	15	
	1,	1,	1,	1,	1,	2	10	
		1,	1,	1,	1,	3	5	
		1,	1,	1,	2,	2	7	
			1,	1,	1,	4	2	
			1,	1,	2,	3	4	
			1,	1,	2,	2	5	
				1,	1,	5	1	
				1,	1,	2,	4	
				1,	1,	3,	3	
				1,	2,	2,	3	
				2,	2,	2,	4	
					1,	1,	6	
					1,	2,	5	
					1,	3,	4	
					2,	2,	4	
					2,	3,	3	
						1,	7	
						2,	6	
						3,	5	
						4,	4	
							8	
							1	

TABLE 4. An explicit basis of $J_{4,m}^{\natural}$ ($0 \leq m \leq 20$). (Let $t^i = \tau^i + \tau^{-i}$ for $i \geq 1$.)

 $m = 0$
 $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 0, 0, 0, 0) \quad f_1:$

$$1 + 240q + 2160q^2 + 6720q^3 + 17520q^4 + 30240q^5 + 60480q^6 + 82560q^7 + 140400q^8 + 181680q^9 + 272160q^{10} + \dots$$

 $m = 1$
 $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 0, 0, 0, 1) \quad f_1:$

$$1 + q(126 + 56t + t^2) + q^2(756 + 576t + 126t^2) + q^3(2072 + 1512t + 756t^2 + 56t^3) + q^4(4158 + 4032t + 2072t^2 + 576t^3 + t^4) + q^5(7560 + 5544t + 4158t^2 + 1512t^3 + 126t^4) + q^6(11592 + 12096t + 7560t^2 + 4032t^3 + 756t^4) + q^7(16704 + 13664t + 11592t^2 + 5544t^3 + 2072t^4 + 56t^5) + q^8(24948 + 24192t + 16704t^2 + 12096t^3 + 4158t^4 + 576t^5) + q^9(31878 + 27216t + 24948t^2 + 13664t^3 + 7560t^4 + 1512t^5 + t^6) + q^{10}(39816 + 44352t + 31878t^2 + 24192t^3 + 11592t^4 + 4032t^5 + 126t^6) + \dots$$

 $m = 2$
 $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 0, 0, 1, 1) \quad f_1:$

$$1 + q(84 + 64t + 14t^2) + q^2(574 + 448t + 280t^2 + 64t^3 + t^4) + q^3(1288 + 1344t + 840t^2 + 448t^3 + 84t^4) + q^4(3444 + 2688t + 2368t^2 + 1344t^3 + 574t^4 + 64t^5) + q^5(4424 + 4928t + 3542t^2 + 2688t^3 + 1288t^4 + 448t^5 + 14t^6) + q^6(9240 + 8064t + 7560t^2 + 4928t^3 + 3444t^4 + 1344t^5 + 280t^6) + q^7(11088 + 11200t + 8456t^2 + 8064t^3 + 4424t^4 + 2688t^5 + 840t^6 + 64t^7) + q^8(18494 + 16192t + 16576t^2 + 11200t^3 + 9240t^4 + 4928t^5 + 2368t^6 + 448t^7 + t^8) + q^9(19740 + 22848t + 17808t^2 + 16192t^3 + 11088t^4 + 8064t^5 + 3542t^6 + 1344t^7 + 84t^8) + q^{10}(34440 + 27776t + 27720t^2 + 22848t^3 + 18494t^4 + 11200t^5 + 7560t^6 + 2688t^7 + 574t^8) + \dots$$

 $m = 3$
 $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 0, 1, 1, 1) \quad f_1:$

$$1 + q(74 + 54t + 27t^2 + 2t^3) + q^2(414 + 432t + 270t^2 + 144t^3 + 27t^4) + q^3(1260 + 972t + 891t^2 + 542t^3 + 270t^4 + 54t^5 + t^6) + q^4(2378 + 2592t + 1998t^2 + 1584t^3 + 891t^4 + 432t^5 + 74t^6) + q^5(4176 + 3618t + 3564t^2 + 2412t^3 + 1998t^4 + 972t^5 + 414t^6 + 54t^7) + q^6(7074 + 7344t + 5724t^2 + 5760t^3 + 3564t^4 + 2592t^5 + 1260t^6 + 432t^7 + 27t^8) + q^7(9468 + 8370t + 8910t^2 + 6302t^3 + 5724t^4 + 3618t^5 + 2378t^6 + 972t^7 + 270t^8 + 2t^9) + q^8(13662 + 15120t + 12528t^2 + 11664t^3 + 8910t^4 + 7344t^5 + 4176t^6 + 2592t^7 + 891t^8 + 144t^9) + q^9(20054 + 15714t + 16794t^2 + 14148t^3 + 12528t^4 +$$

$$8370t^5 + 7074t^6 + 3618t^7 + 1998t^8 + 542t^9 + 27t^{10} + q^{10}(22860 + 26784t + 21978t^2 + 21744t^3 + 16794t^4 + 15120t^5 + 9468t^6 + 7344t^7 + 3564t^8 + 1584t^9 + 270t^{10}) + \dots$$

$$m = 4$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 1, 1, 1, 1) \quad f_1:$$

$$1 + q(126 + 56t^2 + t^4) + q^2(756 + 576t^2 + 126t^4) + q^3(2072 + 1512t^2 + 756t^4 + 56t^6) + q^4(4158 + 4032t^2 + 2072t^4 + 576t^6 + t^8) + q^5(7560 + 5544t^2 + 4158t^4 + 1512t^6 + 126t^8) + q^6(11592 + 12096t^2 + 7560t^4 + 4032t^6 + 756t^8) + q^7(16704 + 13664t^2 + 11592t^4 + 5544t^6 + 2072t^8 + 56t^{10}) + q^8(24948 + 24192t^2 + 16704t^4 + 12096t^6 + 4158t^8 + 576t^{10}) + q^9(31878 + 27216t^2 + 24948t^4 + 13664t^6 + 7560t^8 + 1512t^{10} + t^{12}) + q^{10}(39816 + 44352t^2 + 31878t^4 + 24192t^6 + 11592t^8 + 4032t^{10} + 126t^{12}) + \dots$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 1, 1, 1, 1) \quad f_2:$$

$$1 + q(56 + 56t + 28t^2 + 8t^3) + q^2(420 + 336t + 288t^2 + 168t^3 + 70t^4 + 8t^5) + q^3(896 + 1008t + 756t^2 + 616t^3 + 336t^4 + 168t^5 + 28t^6) + q^4(2366 + 2024t + 2016t^2 + 1400t^3 + 1176t^4 + 616t^5 + 288t^6 + 56t^7 + t^8) + q^5(3360 + 3472t + 2772t^2 + 2856t^3 + 1792t^4 + 1400t^5 + 756t^6 + 336t^7 + 56t^8) + q^6(6440 + 5880t + 6048t^2 + 4536t^3 + 4200t^4 + 2856t^5 + 2016t^6 + 1008t^7 + 420t^8 + 56t^9) + q^7(7168 + 8456t + 6832t^2 + 6664t^3 + 5152t^4 + 4536t^5 + 2772t^6 + 2024t^7 + 896t^8 + 336t^9 + 28t^{10}) + q^8(14196 + 11200t + 12096t^2 + 10416t^3 + 9536t^4 + 6664t^5 + 6048t^6 + 3472t^7 + 2366t^8 + 1008t^9 + 288t^{10} + 8t^{11}) + q^9(14168 + 16296t + 13608t^2 + 13664t^3 + 10752t^4 + 10416t^5 + 6832t^6 + 5880t^7 + 3360t^8 + 2024t^9 + 756t^{10} + 168t^{11}) + q^{10}(22120 + 20776t + 22176t^2 + 17528t^3 + 17710t^4 + 13664t^5 + 12096t^6 + 8456t^7 + 6440t^8 + 3472t^9 + 2016t^{10} + 616t^{11} + 70t^{12}) + \dots$$

$$m = 5$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 1, 1, 1, 1, 1) \quad f_1:$$

$$1 + q(60 + 44t + 33t^2 + 12t^3 + t^4) + q^2(316 + 352t + 253t^2 + 192t^3 + 92t^4 + 32t^5 + t^6) + q^3(928 + 804t + 792t^2 + 536t^3 + 444t^4 + 216t^5 + 92t^6 + 12t^7) + q^4(1980 + 1984t + 1628t^2 + 1632t^3 + 1057t^4 + 800t^5 + 444t^6 + 192t^7 + 33t^8) + q^5(3226 + 2820t + 3036t^2 + 2200t^3 + 2104t^4 + 1456t^5 + 1057t^6 + 536t^7 + 253t^8 + 44t^9 + t^{10}) + q^6(5080 + 5952t + 4884t^2 + 4832t^3 + 3732t^4 + 3360t^5 + 2104t^6 + 1632t^7 + 792t^8 + 352t^9 + 60t^{10}) + q^7(7992 + 6372t + 6776t^2 + 5916t^3 + 5568t^4 + 3928t^5 + 3732t^6 + 2200t^7 + 1628t^8 + 804t^9 + 316t^{10} + 44t^{11}) + q^8(10428 + 11872t + 10208t^2 + 10016t^3 + 8349t^4 + 7808t^5 + 5568t^6 + 4832t^7 + 3036t^8 + 1984t^9 + 928t^{10} + 352t^{11} + 33t^{12}) + q^9(14100 + 12828t + 14268t^2 + 11132t^3 + 11664t^4 + 8880t^5 + 8349t^6 + 5916t^7 + 4884t^8 + 2820t^9 + 1980t^{10} + 804t^{11} + 253t^{12} + 12t^{13}) + q^{10}(19656 + 20064t +$$

$$16857t^2 + 19392t^3 + 14360t^4 + 14976t^5 + 11664t^6 + 10016t^7 + 6776t^8 + 5952t^9 + 3226t^{10} + 1984t^{11} + 792t^{12} + 192t^{13} + t^{14}) + \dots$$

$$m = 6$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 1, 1, 1, 1, 1, 1) \quad f_1 : \\ 1 + q(46 + 48t + 30t^2 + 16t^3 + 3t^4) + q^2(330 + 288t + 270t^2 + 176t^3 + 123t^4 + 48t^5 + 10t^6) + q^3(786 + 816t + 636t^2 + 640t^3 + 396t^4 + 288t^5 + 140t^6 + 48t^7 + 3t^8) + q^4(1886 + 1680t + 1776t^2 + 1296t^3 + 1230t^4 + 816t^5 + 592t^6 + 288t^7 + 123t^8 + 16t^9) + q^5(2540 + 2976t + 2442t^2 + 2416t^3 + 1866t^4 + 1680t^5 + 1052t^6 + 816t^7 + 396t^8 + 176t^9 + 30t^{10}) + q^6(5740 + 4656t + 4860t^2 + 4336t^3 + 3963t^4 + 2976t^5 + 2710t^6 + 1680t^7 + 1230t^8 + 640t^9 + 270t^{10} + 48t^{11} + t^{12}) + q^7(5852 + 6864t + 5832t^2 + 5936t^3 + 4746t^4 + 4656t^5 + 3212t^6 + 2976t^7 + 1866t^8 + 1296t^9 + 636t^{10} + 288t^{11} + 46t^{12}) + q^8(10570 + 9696t + 10656t^2 + 8272t^3 + 8910t^4 + 6864t^5 + 6512t^6 + 4656t^7 + 3963t^8 + 2416t^9 + 1776t^{10} + 816t^{11} + 330t^{12} + 48t^{13}) + q^9(12466 + 13008t + 10842t^2 + 12576t^3 + 9453t^4 + 9696t^5 + 7860t^6 + 6864t^7 + 4746t^8 + 4336t^9 + 2442t^{10} + 1680t^{11} + 786t^{12} + 288t^{13} + 30t^{14}) + q^{10}(18512 + 16992t + 18090t^2 + 15072t^3 + 16236t^4 + 13008t^5 + 12060t^6 + 9696t^7 + 8910t^8 + 5936t^9 + 4860t^{10} + 2976t^{11} + 1886t^{12} + 816t^{13} + 270t^{14} + 16t^{15}) + \dots$$

$$m = 7$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 1, 1, 1, 1, 1, 1, 1) \quad f_1 : \\ 1 + q(44 + 42t + 35t^2 + 14t^3 + 7t^4) + q^2(294 + 294t + 217t^2 + 210t^3 + 119t^4 + 70t^5 + 21t^6 + 2t^7) + q^3(770 + 672t + 735t^2 + 518t^3 + 462t^4 + 294t^5 + 189t^6 + 84t^7 + 21t^8) + q^4(1584 + 1764t + 1456t^2 + 1400t^3 + 1099t^4 + 952t^5 + 609t^6 + 422t^7 + 189t^8 + 70t^9 + 7t^{10}) + q^5(2898 + 2338t + 2478t^2 + 2184t^3 + 2044t^4 + 1442t^5 + 1365t^6 + 784t^7 + 609t^8 + 294t^9 + 119t^{10} + 14t^{11}) + q^6(4270 + 5040t + 4284t^2 + 4340t^3 + 3465t^4 + 3402t^5 + 2366t^6 + 2184t^7 + 1365t^8 + 952t^9 + 462t^{10} + 210t^{11} + 35t^{12}) + q^7(6300 + 5614t + 6209t^2 + 4760t^3 + 5187t^4 + 4088t^5 + 3787t^6 + 2800t^7 + 2366t^8 + 1442t^9 + 1099t^{10} + 518t^{11} + 217t^{12} + 42t^{13} + t^{14}) + q^8(9534 + 9604t + 8484t^2 + 9534t^3 + 7343t^4 + 7364t^5 + 6132t^6 + 5266t^7 + 3787t^8 + 3402t^9 + 2044t^{10} + 1400t^{11} + 735t^{12} + 294t^{13} + 44t^{14}) + q^9(12308 + 11004t + 11844t^2 + 9660t^3 + 10899t^4 + 8358t^5 + 8204t^6 + 6384t^7 + 6132t^8 + 4088t^9 + 3465t^{10} + 2184t^{11} + 1456t^{12} + 672t^{13} + 294t^{14} + 42t^{15}) + q^{10}(14966 + 17794t + 15120t^2 + 15904t^3 + 13524t^4 + 14056t^5 + 10703t^6 + 11060t^7 + 8204t^8 + 7364t^9 + 5187t^{10} + 4340t^{11} + 2478t^{12} + 1764t^{13} + 770t^{14} + 294t^{15} + 35t^{16}) + \dots$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 1, 1, 1, 2) \quad f_1 : \\ 1 + q(72 + 28t + 27t^2 + 27t^3 + t^4 + t^5) + q^2(270 + 288t + 243t^2 + 216t^3 + 99t^4 + 72t^5 + 27t^6) + q^3(774 + 729t + 675t^2 + 487t^3 + 486t^4 + 297t^5 + 217t^6 + 54t^7 + 27t^8 + t^9) +$$

$$\begin{aligned}
& q^4(1368 + 1800t + 1540t^2 + 1368t^3 + 1179t^4 + 936t^5 + 531t^6 + 432t^7 + 217t^8 + 72t^9 + t^{10}) \\
& + q^5(3078 + 2313t + 2502t^2 + 2079t^3 + 2016t^4 + 1422t^5 + 1377t^6 + 918t^7 + 531t^8 + 297t^9 \\
& + 99t^{10} + 27t^{11}) + q^6(4374 + 4968t + 4212t^2 + 4392t^3 + 3537t^4 + 3456t^5 + 2286t^6 + 2160t^7 \\
& + 1377t^8 + 936t^9 + 486t^{10} + 216t^{11} + 27t^{12}) + q^7(6300 + 5508t + 6121t^2 + 5094t^3 + 5049t^4 \\
& + 4023t^5 + 3987t^6 + 2702t^7 + 2286t^8 + 1422t^9 + 1179t^{10} + 487t^{11} + 243t^{12} + 28t^{13} + t^{14}) \\
& + q^8(9774 + 9648t + 8568t^2 + 9504t^3 + 7083t^4 + 7272t^5 + 6048t^6 + 5328t^7 + 3987t^8 \\
& + 3456t^9 + 2016t^{10} + 1368t^{11} + 675t^{12} + 288t^{13} + 72t^{14}) + q^9(12168 + 10989t + 11772t^2 + 9613t^3 \\
& + 10881t^4 + 8775t^5 + 8316t^6 + 6156t^7 + 6048t^8 + 4023t^9 + 3537t^{10} + 2079t^{11} + 1540t^{12} + 729t^{13} + 270t^{14} \\
& + 28t^{15}) + q^{10}(14742 + 17784t + 15084t^2 + 15984t^3 + 13572t^4 + 14112t^5 + 10809t^6 + 10944t^7 + 8316t^8 \\
& + 7272t^9 + 5049t^{10} + 4392t^{11} + 2502t^{12} + 1800t^{13} + 774t^{14} + 288t^{15} + 27t^{16}) + \dots
\end{aligned}$$

$$m = 8$$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (1, 1, 1, 1, 1, 1, 1, 1) \quad f_1: \\
& 1 + q(84 + 64t^2 + 14t^4) + q^2(574 + 448t^2 + 280t^4 + 64t^6 + t^8) + q^3(1288 + 1344t^2 + 840t^4 \\
& + 448t^6 + 84t^8) + q^4(3444 + 2688t^2 + 2368t^4 + 1344t^6 + 574t^8 + 64t^{10}) + q^5(4424 + 4928t^2 \\
& + 3542t^4 + 2688t^6 + 1288t^8 + 448t^{10} + 14t^{12}) + q^6(9240 + 8064t^2 + 7560t^4 + 4928t^6 \\
& + 3444t^8 + 1344t^{10} + 280t^{12}) + q^7(11088 + 11200t^2 + 8456t^4 + 8064t^6 + 4424t^8 + 2688t^{10} \\
& + 840t^{12} + 64t^{14}) + q^8(18494 + 16192t^2 + 16576t^4 + 11200t^6 + 9240t^8 + 4928t^{10} + 2368t^{12} \\
& + 448t^{14} + t^{16}) + q^9(19740 + 22848t^2 + 17808t^4 + 16192t^6 + 11088t^8 + 8064t^{10} + 3542t^{12} \\
& + 1344t^{14} + 84t^{16}) + q^{10}(34440 + 27776t^2 + 27720t^4 + 22848t^6 + 18494t^8 + 11200t^{10} + 7560t^{12} + 2688t^{14} \\
& + 574t^{16}) + \dots
\end{aligned}$$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 1, 1, 1, 1, 2) \quad f_1: \\
& 1 + q(42 + 42t + 28t^2 + 21t^3 + 7t^4 + t^5) + q^2(280 + 253t + 252t^2 + 175t^3 + 140t^4 + 77t^5 \\
& + 36t^6 + 7t^7) + q^3(644 + 735t + 588t^2 + 567t^3 + 420t^4 + 357t^5 + 196t^6 + 126t^7 + 42t^8 \\
& + 7t^9) + q^4(1764 + 1400t + 1512t^2 + 1302t^3 + 1184t^4 + 833t^5 + 756t^6 + 434t^7 + 294t^8 \\
& + 126t^9 + 36t^{10} + t^{11}) + q^5(2212 + 2597t + 2156t^2 + 2191t^3 + 1771t^4 + 1708t^5 + 1176t^6 \\
& + 1057t^7 + 644t^8 + 434t^9 + 196t^{10} + 77t^{11} + 7t^{12}) + q^6(4480 + 4242t + 4536t^2 + 3619t^3 \\
& + 3780t^4 + 3003t^5 + 2772t^6 + 2037t^7 + 1680t^8 + 1057t^9 + 756t^{10} + 357t^{11} + 140t^{12} \\
& + 21t^{13}) + q^7(5544 + 5740t + 4900t^2 + 5691t^3 + 4228t^4 + 4389t^5 + 3528t^6 + 3151t^7 \\
& + 2212t^8 + 2037t^9 + 1176t^{10} + 833t^{11} + 420t^{12} + 175t^{13} + 28t^{14}) + q^8(9534 + 8400t \\
& + 9108t^2 + 7371t^3 + 8288t^4 + 6594t^5 + 6300t^6 + 4935t^7 + 4760t^8 + 3151t^9 + 2772t^{10} \\
& + 1708t^{11} + 1184t^{12} + 567t^{13} + 252t^{14} + 42t^{15} + t^{16}) + q^9(9870 + 11592t + 9996t^2 + 10626t^3 \\
& + 8904t^4 + 9303t^5 + 7084t^6 + 7434t^7 + 5544t^8 + 4935t^9 + 3528t^{10} + 3003t^{11} + 1771t^{12} \\
& + 1302t^{13} + 588t^{14} + 253t^{15} + 42t^{16}) + q^{10}(16800 + 14203t + 15624t^2 + 14427t^3 + 13860t^4 \\
& + 11585t^5 + 12852t^6 + 9506t^7 + 8960t^8 + 7434t^9 + \dots
\end{aligned}$$

$$6300t^{10} + 4389t^{11} + 3780t^{12} + 2191t^{13} + 1512t^{14} + 735t^{15} + 280t^{16} + 42t^{17}) + \dots$$

$$m = 9$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 1, 1, 1, 1, 1, 2) \quad f_1:$$

$$\begin{aligned} & 1 + q(42 + 36t + 33t^2 + 18t^3 + 10t^4 + 2t^5) + q^2(234 + 272t + 212t^2 + 192t^3 + \\ & 132t^4 + 96t^5 + 42t^6 + 16t^7 + t^8) + q^3(740 + 582t + 622t^2 + 522t^3 + 464t^4 + 310t^5 + \\ & 261t^6 + 134t^7 + 74t^8 + 20t^9 + t^{10}) + q^4(1386 + 1552t + 1332t^2 + 1296t^3 + 1057t^4 + \\ & 992t^5 + 666t^6 + 560t^7 + 330t^8 + 192t^9 + 74t^{10} + 16t^{11}) + q^5(2340 + 2138t + 2378t^2 + \\ & 1848t^3 + 1944t^4 + 1480t^5 + 1386t^6 + 986t^7 + 814t^8 + 468t^9 + 330t^{10} + 134t^{11} + \\ & 42t^{12} + 2t^{13}) + q^6(4140 + 4336t + 3614t^2 + 4176t^3 + 3151t^4 + 3232t^5 + 2610t^6 + \\ & 2288t^7 + 1582t^8 + 1440t^9 + 814t^{10} + 560t^{11} + 261t^{12} + 96t^{13} + 10t^{14}) + q^7(5568 + \\ & 5006t + 5384t^2 + 4374t^3 + 4958t^4 + 3834t^5 + 3726t^6 + 2888t^7 + 2784t^8 + 1848t^9 + \\ & 1582t^{10} + 986t^{11} + 666t^{12} + 310t^{13} + 132t^{14} + 18t^{15}) + q^8(7722 + 8832t + 7888t^2 + \\ & 8064t^3 + 6996t^4 + 7088t^5 + 5568t^6 + 5664t^7 + 4356t^8 + 3744t^9 + 2784t^{10} + 2288t^{11} + \\ & 1386t^{12} + 992t^{13} + 464t^{14} + 192t^{15} + 33t^{16}) + q^9(11466 + 9192t + 10570t^2 + 9396t^3 + \\ & 9224t^4 + 7760t^5 + 8613t^6 + 6202t^7 + 6072t^8 + 4916t^9 + 4356t^{10} + 2888t^{11} + 2610t^{12} + \\ & 1480t^{13} + 1057t^{14} + 522t^{15} + 212t^{16} + 36t^{17} + t^{18}) + q^{10}(13272 + 15664t + 13484t^2 + \\ & 14256t^3 + 12306t^4 + 13136t^5 + 10206t^6 + 10992t^7 + 8466t^8 + 8064t^9 + 6072t^{10} + \\ & 5664t^{11} + 3726t^{12} + 3232t^{13} + 1944t^{14} + 1296t^{15} + 622t^{16} + 272t^{17} + 42t^{18}) + \dots \end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 0, 0, 0, 3) \quad f_1:$$

$$\begin{aligned} & 1 + q(126 + 56t^3 + t^6) + q^2(756 + 576t^3 + 126t^6) + q^3(2072 + 1512t^3 + 756t^6 + 56t^9) + \\ & q^4(4158 + 4032t^3 + 2072t^6 + 576t^9 + t^{12}) + q^5(7560 + 5544t^3 + 4158t^6 + 1512t^9 + \\ & 126t^{12}) + q^6(11592 + 12096t^3 + 7560t^6 + 4032t^9 + 756t^{12}) + q^7(16704 + 13664t^3 + \\ & 11592t^6 + 5544t^9 + 2072t^{12} + 56t^{15}) + q^8(24948 + 24192t^3 + 16704t^6 + 12096t^9 + \\ & 4158t^{12} + 576t^{15}) + q^9(31878 + 27216t^3 + 24948t^6 + 13664t^9 + 7560t^{12} + 1512t^{15} + \\ & t^{18}) + q^{10}(39816 + 44352t^3 + 31878t^6 + 24192t^9 + 11592t^{12} + 4032t^{15} + 126t^{18}) + \dots \end{aligned}$$

$$m = 10$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 1, 1, 1, 1, 1, 1, 2) \quad f_1:$$

$$\begin{aligned} & 1 + q(32 + 40t + 30t^2 + 20t^3 + 10t^4 + 4t^5) + q^2(276 + 220t + 220t^2 + 180t^3 + \\ & 145t^4 + 92t^5 + 60t^6 + 20t^7 + 5t^8) + q^3(564 + 660t + 540t^2 + 540t^3 + 420t^4 + 372t^5 + \\ & 230t^6 + 180t^7 + 90t^8 + 40t^9 + 6t^{10}) + q^4(1452 + 1320t + 1440t^2 + 1120t^3 + 1145t^4 + \\ & 856t^5 + 800t^6 + 540t^7 + 420t^8 + 220t^9 + 128t^{10} + 40t^{11} + 5t^{12}) + q^5(2184 + 2220t + \\ & 1870t^2 + 2160t^3 + 1610t^4 + 1664t^5 + 1290t^6 + 1120t^7 + 740t^8 + 660t^9 + 364t^{10} + \\ & 220t^{11} + 90t^{12} + 20t^{13}) + q^6(4128 + 3780t + 4020t^2 + 3340t^3 + 3600t^4 + 2892t^5 + \\ & 2680t^6 + 2160t^7 + 1980t^8 + 1320t^9 + 1080t^{10} + 660t^{11} + 420t^{12} + 180t^{13} + 60t^{14} + \dots \end{aligned}$$

$$\begin{aligned}
& 4t^{15} + q^7(4468 + 5420t + 4610t^2 + 4820t^3 + 4100t^4 + 4264t^5 + 3250t^6 + 3340t^7 + \\
& 2500t^8 + 2220t^9 + 1590t^{10} + 1320t^{11} + 740t^{12} + 540t^{13} + 230t^{14} + 92t^{15} + 10t^{16}) + \\
& q^8(8796 + 7200t + 8160t^2 + 7320t^3 + 7260t^4 + 5916t^5 + 6720t^6 + 4820t^7 + 4705t^8 + \\
& 3780t^9 + 3264t^{10} + 2220t^{11} + 1980t^{12} + 1120t^{13} + 800t^{14} + 372t^{15} + 145t^{16} + 20t^{17}) + \\
& q^9(8936 + 10440t + 9000t^2 + 9520t^3 + 8190t^4 + 8772t^5 + 6810t^6 + 7320t^7 + 5640t^8 + \\
& 5420t^9 + 4044t^{10} + 3780t^{11} + 2500t^{12} + 2160t^{13} + 1290t^{14} + 856t^{15} + 420t^{16} + 180t^{17} + \\
& 30t^{18}) + q^{10}(14674 + 13140t + 14100t^2 + 12100t^3 + 13500t^4 + 11548t^5 + 11000t^6 + \\
& 9520t^7 + 9600t^8 + 7200t^9 + 7280t^{10} + 5420t^{11} + 4705t^{12} + 3340t^{13} + 2680t^{14} + \\
& 1664t^{15} + 1145t^{16} + 540t^{17} + 220t^{18} + 40t^{19} + t^{20}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 1, 1, 2, 2) \quad f_1:$$

$$\begin{aligned}
& 1 + q(60 + 32t + 13t^2 + 32t^3 + 12t^4 + t^6) + q^2(252 + 224t + 220t^2 + 192t^3 + 161t^4 + \\
& 64t^5 + 60t^6 + 32t^7 + t^8) + q^3(568 + 672t + 564t^2 + 512t^3 + 372t^4 + 384t^5 + 264t^6 + \\
& 192t^7 + 72t^8 + 32t^9 + 12t^{10}) + q^4(1340 + 1312t + 1504t^2 + 1056t^3 + 1213t^4 + 960t^5 + \\
& 704t^6 + 512t^7 + 412t^8 + 224t^9 + 160t^{10} + 32t^{11} + t^{12}) + q^5(2184 + 2304t + 1897t^2 + \\
& 2112t^3 + 1480t^4 + 1664t^5 + 1344t^6 + 1056t^7 + 856t^8 + 672t^9 + 314t^{10} + 224t^{11} + \\
& 72t^{12} + 32t^{13} + t^{14}) + q^6(4000 + 3744t + 4020t^2 + 3424t^3 + 3672t^4 + 2880t^5 + \\
& 2680t^6 + 2112t^7 + 1980t^8 + 1312t^9 + 1080t^{10} + 672t^{11} + 412t^{12} + 192t^{13} + 60t^{14}) + \\
& q^7(5016 + 5216t + 4384t^2 + 4896t^3 + 4144t^4 + 4288t^5 + 3268t^6 + 3424t^7 + 2448t^8 + \\
& 2304t^9 + 1380t^{10} + 1312t^{11} + 856t^{12} + 512t^{13} + 264t^{14} + 64t^{15} + 12t^{16}) + q^8(8892 + \\
& 7328t + 8128t^2 + 7392t^3 + 7260t^4 + 5632t^5 + 6624t^6 + 4896t^7 + 4641t^8 + 3744t^9 + \\
& 3488t^{10} + 2304t^{11} + 1980t^{12} + 1056t^{13} + 704t^{14} + 384t^{15} + 161t^{16} + 32t^{17}) + q^9(8460 + \\
& 10464t + 8904t^2 + 9728t^3 + 8316t^4 + 8640t^5 + 7009t^6 + 7392t^7 + 5676t^8 + 5216t^9 + \\
& 4080t^{10} + 3744t^{11} + 2448t^{12} + 2112t^{13} + 1344t^{14} + 960t^{15} + 372t^{16} + 192t^{17} + 13t^{18}) + \\
& q^{10}(14874 + 13024t + 14100t^2 + 11936t^3 + 13532t^4 + 11648t^5 + 11000t^6 + 9728t^7 + \\
& 9464t^8 + 7328t^9 + 7280t^{10} + 5216t^{11} + 4641t^{12} + 3424t^{13} + 2680t^{14} + 1664t^{15} + \\
& 1213t^{16} + 512t^{17} + 220t^{18} + 32t^{19} + t^{20}) + \dots
\end{aligned}$$

$$m = 11 \quad (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (1, 1, 1, 1, 1, 1, 1, 2) \quad f_1:$$

$$\begin{aligned}
& 1 + q(42 + 28t + 35t^2 + 21t^3 + 8t^4 + 7t^5) + q^2(224 + 252t + 182t^2 + 182t^3 + 147t^4 + \\
& 106t^5 + 56t^6 + 35t^7 + 7t^8 + t^9) + q^3(560 + 567t + 609t^2 + 469t^3 + 462t^4 + 315t^5 + 294t^6 + \\
& 168t^7 + 126t^8 + 49t^9 + 21t^{10}) + q^4(1386 + 1366t + 1162t^2 + 1281t^3 + 966t^4 + 952t^5 + \\
& 756t^6 + 623t^7 + 400t^8 + 315t^9 + 154t^{10} + 70t^{11} + 21t^{12} + t^{13}) + q^5(2240 + 1883t + \\
& 2149t^2 + 1680t^3 + 1876t^4 + 1505t^5 + 1351t^6 + 1036t^7 + 952t^8 + 609t^9 + 455t^{10} + \\
& 294t^{11} + 154t^{12} + 49t^{13} + 7t^{14}) + q^6(3430 + 4011t + 3486t^2 + 3675t^3 + 3045t^4 + \\
& 3192t^5 + 2366t^6 + 2436t^7 + 1785t^8 + 1582t^9 + 1134t^{10} + 882t^{11} + 455t^{12} + 315t^{13} + \\
& 126t^{14} + 35t^{15}) + q^7(5292 + 4389t + 4858t^2 + 4389t^3 + 4270t^4 + 3444t^5 + 3948t^6 +
\end{aligned}$$

$$\begin{aligned}
& 2905t^7 + 2814t^8 + 2163t^9 + 1890t^{10} + 1190t^{11} + 1134t^{12} + 609t^{13} + 400t^{14} + 168t^{15} + \\
& 56t^{16} + 7t^{17}) + q^8(6930 + 8099t + 7056t^2 + 7274t^3 + 6636t^4 + 6762t^5 + 5488t^6 + \\
& 5537t^7 + 4347t^8 + 4067t^9 + 3200t^{10} + 2814t^{11} + 1890t^{12} + 1582t^{13} + 952t^{14} + 623t^{15} + \\
& 294t^{16} + 106t^{17} + 8t^{18}) + q^9(9870 + 8820t + 9534t^2 + 7966t^3 + 9240t^4 + 7476t^5 + \\
& 7532t^6 + 6237t^7 + 6510t^8 + 4788t^9 + 4809t^{10} + 3570t^{11} + 3200t^{12} + 2163t^{13} + 1785t^{14} + \\
& 1036t^{15} + 756t^{16} + 315t^{17} + 147t^{18} + 21t^{19}) + q^{10}(12810 + 13734t + 11886t^2 + 14049t^3 + \\
& 10878t^4 + 11746t^5 + 10500t^6 + 10199t^7 + 8246t^8 + 8967t^9 + 6118t^{10} + 6244t^{11} + \\
& 4809t^{12} + 4067t^{13} + 2814t^{14} + 2436t^{15} + 1351t^{16} + 952t^{17} + 462t^{18} + 182t^{19} + 35t^{20}) + \dots
\end{aligned}$$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 1, 1, 1, 2, 2) \quad f_1: \\
& 1 + q(42 + 34t + 26t^2 + 20t^3 + 16t^4 + 2t^5 + t^6) + q^2(202 + 240t + 212t^2 + 192t^3 + 123t^4 + \\
& 112t^5 + 58t^6 + 32t^7 + 10t^8) + q^3(644 + 554t + 593t^2 + 436t^3 + 462t^4 + 342t^5 + 282t^6 + \\
& 182t^7 + 116t^8 + 52t^9 + 17t^{10} + 2t^{11}) + q^4(1322 + 1360t + 1162t^2 + 1312t^3 + 1002t^4 + \\
& 944t^5 + 756t^6 + 592t^7 + 400t^8 + 320t^9 + 154t^{10} + 80t^{11} + 17t^{12}) + q^5(2160 + 1976t + \\
& 2104t^2 + 1718t^3 + 1876t^4 + 1400t^5 + 1364t^6 + 1066t^7 + 976t^8 + 598t^9 + 500t^{10} + \\
& 246t^{11} + 154t^{12} + 52t^{13} + 10t^{14}) + q^6(3544 + 4032t + 3416t^2 + 3600t^3 + 3064t^4 + \\
& 3232t^5 + 2372t^6 + 2480t^7 + 1791t^8 + 1552t^9 + 1080t^{10} + 880t^{11} + 500t^{12} + 320t^{13} + \\
& 116t^{14} + 32t^{15} + t^{16}) + q^7(5192 + 4230t + 5018t^2 + 4416t^3 + 4260t^4 + 3558t^5 + 3988t^6 + \\
& 2808t^7 + 2734t^8 + 2186t^9 + 1882t^{10} + 1284t^{11} + 1080t^{12} + 598t^{13} + 400t^{14} + 182t^{15} + \\
& 58t^{16} + 2t^{17}) + q^8(7018 + 8144t + 7152t^2 + 7264t^3 + 6516t^4 + 6672t^5 + 5408t^6 + \\
& 5504t^7 + 4443t^8 + 4144t^9 + 3152t^{10} + 2880t^{11} + 1882t^{12} + 1552t^{13} + 976t^{14} + 592t^{15} + \\
& 282t^{16} + 112t^{17} + 16t^{18}) + q^9(9870 + 8720t + 9593t^2 + 7930t^3 + 9200t^4 + 7568t^5 + \\
& 7586t^6 + 6266t^7 + 6510t^8 + 4812t^9 + 4802t^{10} + 3482t^{11} + 3152t^{12} + 2186t^{13} + 1791t^{14} + \\
& 1066t^{15} + 756t^{16} + 342t^{17} + 123t^{18} + 20t^{19}) + q^{10}(12972 + 13744t + 11630t^2 + 14032t^3 + \\
& 11006t^4 + 11792t^5 + 10534t^6 + 10352t^7 + 8104t^8 + 8832t^9 + 6356t^{10} + 6064t^{11} + \\
& 4802t^{12} + 4144t^{13} + 2734t^{14} + 2480t^{15} + 1364t^{16} + 944t^{17} + 462t^{18} + 192t^{19} + 26t^{20}) + \dots
\end{aligned}$$

$$m = 12$$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 1, 1, 1, 1, 2, 2) \quad f_1: \\
& 1 + q(32 + 36t + 27t^2 + 22t^3 + 12t^4 + 6t^5 + t^6) + q^2(230 + 210t + 216t^2 + 162t^3 + \\
& 150t^4 + 102t^5 + 72t^6 + 36t^7 + 15t^8 + 2t^9) + q^3(560 + 582t + 486t^2 + 542t^3 + 384t^4 + \\
& 372t^5 + 271t^6 + 210t^7 + 120t^8 + 80t^9 + 27t^{10} + 6t^{11}) + q^4(1354 + 1212t + 1296t^2 + \\
& 1034t^3 + 1134t^4 + 858t^5 + 792t^6 + 582t^7 + 507t^8 + 302t^9 + 216t^{10} + 102t^{11} + 42t^{12} + \\
& 6t^{13}) + q^5(1792 + 2124t + 1809t^2 + 1884t^3 + 1584t^4 + 1626t^5 + 1206t^6 + 1212t^7 + \\
& 864t^8 + 742t^9 + 486t^{10} + 372t^{11} + 184t^{12} + 102t^{13} + 27t^{14} + 2t^{15}) + q^6(3930 + 3312t + \\
& 3672t^2 + 3360t^3 + 3180t^4 + 2658t^5 + 2880t^6 + 2124t^7 + 1980t^8 + 1572t^9 + 1296t^{10} + \\
& 858t^{11} + 700t^{12} + 372t^{13} + 216t^{14} + 80t^{15} + 15t^{16}) + q^7(4208 + 4926t + 4185t^2 +
\end{aligned}$$

$$\begin{aligned}
& 4434t^3 + 3840t^4 + 4122t^5 + 3151t^6 + 3312t^7 + 2544t^8 + 2400t^9 + 1809t^{10} + 1626t^{11} + \\
& 1024t^{12} + 858t^{13} + 486t^{14} + 302t^{15} + 120t^{16} + 36t^{17} + t^{18}) + q^8(7774 + 6828t + 7560t^2 + \\
& 6252t^3 + 7152t^4 + 5874t^5 + 5832t^6 + 4926t^7 + 5070t^8 + 3646t^9 + 3672t^{10} + 2658t^{11} + \\
& 2384t^{12} + 1626t^{13} + 1296t^{14} + 742t^{15} + 507t^{16} + 210t^{17} + 72t^{18} + 6t^{19}) + q^9(8672 + \\
& 9276t + 7857t^2 + 9432t^3 + 7464t^4 + 7914t^5 + 7074t^6 + 6828t^7 + 5376t^8 + 5962t^9 + \\
& 4185t^{10} + 4122t^{11} + 3144t^{12} + 2658t^{13} + 1809t^{14} + 1572t^{15} + 864t^{16} + 582t^{17} + 271t^{18} + \\
& 102t^{19} + 12t^{20}) + q^{10}(12700 + 12144t + 13392t^2 + 11146t^3 + 12210t^4 + 10560t^5 + \\
& 10872t^6 + 9276t^7 + 9330t^8 + 7384t^9 + 7560t^{10} + 5874t^{11} + 5260t^{12} + 4122t^{13} + \\
& 3672t^{14} + 2400t^{15} + 1980t^{16} + 1212t^{17} + 792t^{18} + 372t^{19} + 150t^{20} + 22t^{21}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 1, 1, 1, 1, 2, 2) \quad f_4:$$

$$\begin{aligned}
& 1 + q(-10 + 72t + 44t^3 - 3t^4 + 12t^5) + q^2(46 + 420t + 324t^3 + 30t^4 + 204t^5 + 72t^7 + 3t^8 + \\
& 4t^9) + q^3(-140 + 1164t + 1084t^3 - 123t^4 + 744t^5 + 420t^7 - 30t^8 + 160t^9 + 12t^{11} - t^{12}) + \\
& q^4(330 + 2424t + 2068t^3 + 270t^4 + 1716t^5 + 1164t^7 + 123t^8 + 604t^9 + 204t^{11} + 10t^{12} + \\
& 12t^{13}) + q^5(-592 + 4248t + 3768t^3 - 396t^4 + 3252t^5 + 2424t^7 - 270t^8 + 1484t^9 + \\
& 744t^{11} - 46t^{12} + 204t^{13} + 4t^{15}) + q^6(786 + 6624t + 6720t^3 + 636t^4 + 5316t^5 + 4248t^7 + \\
& 396t^8 + 3144t^9 + 1716t^{11} + 140t^{12} + 744t^{13} + 160t^{15} + 3t^{16}) + q^7(-1052 + 9852t + \\
& 8868t^3 - 1230t^4 + 8244t^5 + 6624t^7 - 636t^8 + 4800t^9 + 3252t^{11} - 330t^{12} + 1716t^{13} + \\
& 604t^{15} - 30t^{16} + 72t^{17}) + q^8(1886 + 13656t + 12504t^3 + 1776t^4 + 11748t^5 + 9852t^7 + \\
& 1230t^8 + 7292t^9 + 5316t^{11} + 592t^{12} + 3252t^{13} + 1484t^{15} + 123t^{16} + 420t^{17} + 12t^{19}) + \\
& q^9(-2710 + 18552t + 18864t^3 - 1866t^4 + 15828t^5 + 13656t^7 - 1776t^8 + 11924t^9 + \\
& 8244t^{11} - 786t^{12} + 5316t^{13} + 3144t^{15} - 270t^{16} + 1164t^{17} + 204t^{19} - 3t^{20}) + q^{10}(2540 + \\
& 24288t + 22292t^3 + 2442t^4 + 21120t^5 + 18552t^7 + 1866t^8 + 14768t^9 + 11748t^{11} + \\
& 1052t^{12} + 8244t^{13} + 4800t^{15} + 396t^{16} + 2424t^{17} + 744t^{19} + 30t^{20} + 44t^{21}) + \dots
\end{aligned}$$

$$m = 13$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 1, 1, 1, 1, 1, 2, 2) \quad f_1:$$

$$\begin{aligned}
& 1 + q(32 + 32t + 31t^2 + 18t^3 + 15t^4 + 6t^5 + 2t^6) + q^2(210 + 220t + 179t^2 + 192t^3 + 128t^4 + \\
& 112t^5 + 75t^6 + 46t^7 + 17t^8 + 6t^9) + q^3(584 + 510t + 546t^2 + 422t^3 + 459t^4 + 342t^5 + \\
& 293t^6 + 198t^7 + 156t^8 + 84t^9 + 45t^{10} + 12t^{11} + t^{12}) + q^4(1152 + 1284t + 1139t^2 + 1160t^3 + \\
& 963t^4 + 946t^5 + 703t^6 + 682t^7 + 479t^8 + 370t^9 + 229t^{10} + 146t^{11} + 62t^{12} + 20t^{13} + t^{14}) + \\
& q^5(2094 + 1726t + 1934t^2 + 1722t^3 + 1674t^4 + 1350t^5 + 1497t^6 + 1026t^7 + 967t^8 + \\
& 732t^9 + 601t^{10} + 364t^{11} + 282t^{12} + 124t^{13} + 62t^{14} + 12t^{15}) + q^6(3162 + 3780t + 3162t^2 + \\
& 3352t^3 + 2907t^4 + 3030t^5 + 2362t^6 + 2442t^7 + 1866t^8 + 1722t^9 + 1242t^{10} + 1110t^{11} + \\
& 665t^{12} + 540t^{13} + 282t^{14} + 146t^{15} + 45t^{16} + 6t^{17}) + q^7(4654 + 4100t + 4586t^2 + 3772t^3 + \\
& 4332t^4 + 3524t^5 + 3434t^6 + 2910t^7 + 2972t^8 + 2110t^9 + 2122t^{10} + 1524t^{11} + 1332t^{12} + \\
& 876t^{13} + 665t^{14} + 364t^{15} + 229t^{16} + 84t^{17} + 17t^{18}) + q^8(7074 + 7136t + 6320t^2 + 7266t^3 +
\end{aligned}$$

$$\begin{aligned}
& 5855t^4 + 6222t^5 + 5592t^6 + 5240t^7 + 4240t^8 + 4512t^9 + 3304t^{10} + 3026t^{11} + 2397t^{12} + \\
& 1936t^{13} + 1332t^{14} + 1110t^{15} + 601t^{16} + 370t^{17} + 156t^{18} + 46t^{19} + 2t^{20} + q^9(8756 + \\
& 8022t + 9234t^2 + 7478t^3 + 8262t^4 + 6954t^5 + 7399t^6 + 6192t^7 + 6372t^8 + 4818t^9 + \\
& 5088t^{10} + 3804t^{11} + 3522t^{12} + 2664t^{13} + 2397t^{14} + 1524t^{15} + 1242t^{16} + 732t^{17} + 479t^{18} + \\
& 198t^{19} + 75t^{20} + 6t^{21}) + q^{10}(10912 + 13040t + 11337t^2 + 12126t^3 + 10692t^4 + 11584t^5 + \\
& 9246t^6 + 10230t^7 + 8234t^8 + 8334t^9 + 6661t^{10} + 6780t^{11} + 4959t^{12} + 4940t^{13} + \\
& 3522t^{14} + 3026t^{15} + 2122t^{16} + 1722t^{17} + 967t^{18} + 682t^{19} + 293t^{20} + 112t^{21} + 15t^{22}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 1, 1, 1, 1, 3) \quad f_5:$$

$$\begin{aligned}
& 1 + q(60 + 12t + 32t^2 + 32t^3 + t^4 + 12t^5 + t^6) + q^2(252 + 192t + 193t^2 + 192t^3 + 92t^4 + \\
& 160t^5 + 60t^6 + 32t^7 + 32t^8 + t^{10}) + q^3(544 + 504t + 540t^2 + 492t^3 + 444t^4 + 312t^5 + \\
& 284t^6 + 204t^7 + 192t^8 + 44t^9 + 60t^{10} + 12t^{11}) + q^4(1084 + 1440t + 1084t^2 + 992t^3 + \\
& 1024t^4 + 992t^5 + 736t^6 + 640t^7 + 481t^8 + 352t^9 + 252t^{10} + 160t^{11} + 33t^{12} + 32t^{13}) + \\
& q^5(1944 + 1708t + 1985t^2 + 1764t^3 + 1852t^4 + 1100t^5 + 1500t^6 + 1144t^7 + 893t^8 + \\
& 792t^9 + 576t^{10} + 344t^{11} + 252t^{12} + 192t^{13} + 33t^{14} + 12t^{15} + t^{16}) + q^6(3040 + 3840t + \\
& 3192t^2 + 3232t^3 + 3000t^4 + 3072t^5 + 2392t^6 + 2400t^7 + 1752t^8 + 1792t^9 + 1212t^{10} + \\
& 1152t^{11} + 732t^{12} + 480t^{13} + 252t^{14} + 160t^{15} + 60t^{16}) + q^7(4928 + 4184t + 4436t^2 + \\
& 3812t^3 + 4192t^4 + 3352t^5 + 3552t^6 + 3000t^7 + 2908t^8 + 2316t^9 + 2072t^{10} + 1356t^{11} + \\
& 1376t^{12} + 856t^{13} + 732t^{14} + 344t^{15} + 252t^{16} + 44t^{17} + 32t^{18}) + q^8(6972 + 6976t + \\
& 6464t^2 + 7200t^3 + 5921t^4 + 6432t^5 + 5568t^6 + 5184t^7 + 4188t^8 + 4512t^9 + 3104t^{10} + \\
& 3264t^{11} + 2460t^{12} + 1760t^{13} + 1376t^{14} + 1152t^{15} + 576t^{16} + 352t^{17} + 192t^{18} + 32t^{19} + \\
& t^{20}) + q^9(8604 + 7812t + 9276t^2 + 7908t^3 + 8184t^4 + 7032t^5 + 7196t^6 + 6192t^7 + \\
& 6420t^8 + 4676t^9 + 5184t^{10} + 3672t^{11} + 3673t^{12} + 2736t^{13} + 2460t^{14} + 1356t^{15} + \\
& 1212t^{16} + 792t^{17} + 481t^{18} + 204t^{19} + 60t^{20} + 12t^{21}) + q^{10}(11800 + 13184t + 11092t^2 + \\
& 12000t^3 + 10104t^4 + 11936t^5 + 9053t^6 + 10112t^7 + 8632t^8 + 8192t^9 + 7036t^{10} + \\
& 6752t^{11} + 4764t^{12} + 4544t^{13} + 3673t^{14} + 3264t^{15} + 2072t^{16} + 1792t^{17} + 893t^{18} + \\
& 640t^{19} + 284t^{20} + 160t^{21} + t^{22}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 1, 2, 2, 2) \quad f_i:$$

$$\begin{aligned}
& 1 + q(72 + 27t + t^2 + 27t^3 + 27t^4 + t^5 + t^7) + q^2(270 + 216t + 99t^2 + 216t^3 + 216t^4 + \\
& 72t^5 + 27t^6 + 72t^7 + 27t^8) + q^3(720 + 486t + 486t^2 + 460t^3 + 459t^4 + 297t^5 + 243t^6 + \\
& 270t^7 + 216t^8 + 28t^9 + 27t^{10} + 27t^{11} + t^{12}) + q^4(936 + 1296t + 1179t^2 + 1152t^3 + \\
& 1081t^4 + 936t^5 + 675t^6 + 720t^7 + 459t^8 + 288t^9 + 217t^{10} + 216t^{11} + 72t^{12} + t^{14}) + \\
& q^5(2214 + 1809t + 2016t^2 + 1620t^3 + 1422t^4 + 1395t^5 + 1539t^6 + 963t^7 + 1107t^8 + \\
& 729t^9 + 531t^{10} + 459t^{11} + 270t^{12} + 54t^{13} + 72t^{14} + 27t^{15}) + q^6(2646 + 3672t + 3510t^2 + \\
& 3312t^3 + 2889t^4 + 3240t^5 + 2430t^6 + 2376t^7 + 1566t^8 + 1800t^9 + 1350t^{10} + 1080t^{11} + \\
& 747t^{12} + 432t^{13} + 270t^{14} + 216t^{15} + 27t^{16}) + q^7(4518 + 4159t + 4806t^2 + 3771t^3 + \\
& 3744t^4 + 3564t^5 + 3942t^6 + 2700t^7 + 3052t^8 + 2286t^9 + 2070t^{10} + 1378t^{11} + 1152t^{12} +
\end{aligned}$$

$$\begin{aligned}
& 918t^{13} + 747t^{14} + 459t^{15} + 217t^{16} + 28t^{17} + 27t^{18} + t^{19} + q^8(6750 + 7416t + 6408t^2 + \\
& 7128t^3 + 6147t^4 + 6192t^5 + 5400t^6 + 4896t^7 + 3960t^8 + 4752t^9 + 3528t^{10} + 2880t^{11} + \\
& 2619t^{12} + 2160t^{13} + 1152t^{14} + 1080t^{15} + 531t^{16} + 288t^{17} + 216t^{18} + 72t^{19}) + q^9(9252 + \\
& 7668t + 9342t^2 + 7263t^3 + 7830t^4 + 7425t^5 + 7561t^6 + 6399t^7 + 6372t^8 + 5022t^9 + \\
& 4968t^{10} + 3537t^{11} + 3294t^{12} + 2700t^{13} + 2619t^{14} + 1378t^{15} + 1350t^{16} + 729t^{17} + 459t^{18} + \\
& 270t^{19} + 27t^{20} + t^{21}) + q^{10}(10368 + 13032t + 11169t^2 + 12240t^3 + 11124t^4 + 11520t^5 + \\
& 9414t^6 + 10224t^7 + 7902t^8 + 8352t^9 + 6993t^{10} + 6552t^{11} + 4977t^{12} + 5184t^{13} + \\
& 3294t^{14} + 2880t^{15} + 2070t^{16} + 1800t^{17} + 1107t^{18} + 720t^{19} + 243t^{20} + 72t^{21} + 27t^{22}) + \dots
\end{aligned}$$

$m = 14$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (1, 1, 1, 1, 1, 1, 2, 2) \quad f_1: \\
& 1 + q(30 + 32t + 27t^2 + 24t^3 + 11t^4 + 8t^5 + 3t^6) + q^2(216 + 200t + 196t^2 + 152t^3 + \\
& 155t^4 + 104t^5 + 83t^6 + 48t^7 + 25t^8 + 8t^9 + t^{10}) + q^3(486 + 552t + 468t^2 + 488t^3 + 393t^4 + \\
& 384t^5 + 254t^6 + 240t^7 + 153t^8 + 104t^9 + 54t^{10} + 24t^{11} + 3t^{12}) + q^4(1326 + 1072t + \\
& 1208t^2 + 1056t^3 + 1003t^4 + 808t^5 + 864t^6 + 592t^7 + 531t^8 + 384t^9 + 288t^{10} + 152t^{11} + \\
& 99t^{12} + 32t^{13} + 8t^{14}) + q^5(1638 + 1976t + 1676t^2 + 1776t^3 + 1508t^4 + 1568t^5 + 1201t^6 + \\
& 1216t^7 + 924t^8 + 808t^9 + 561t^{10} + 488t^{11} + 278t^{12} + 200t^{13} + 86t^{14} + 32t^{15} + 3t^{16}) + \\
& q^6(3510 + 3216t + 3417t^2 + 2872t^3 + 3219t^4 + 2688t^5 + 2559t^6 + 2160t^7 + 2118t^8 + \\
& 1568t^9 + 1473t^{10} + 1056t^{11} + 873t^{12} + 552t^{13} + 390t^{14} + 200t^{15} + 99t^{16} + 24t^{17} + t^{18}) + \\
& q^7(4200 + 4416t + 3766t^2 + 4440t^3 + 3484t^4 + 3712t^5 + 3306t^6 + 3200t^7 + 2414t^8 + \\
& 2688t^9 + 1854t^{10} + 1776t^{11} + 1308t^{12} + 1072t^{13} + 700t^{14} + 552t^{15} + 278t^{16} + 152t^{17} + \\
& 54t^{18} + 8t^{19}) + q^8(6840 + 6296t + 7256t^2 + 5936t^3 + 6580t^4 + 5496t^5 + 5752t^6 + \\
& 4816t^7 + 4955t^8 + 3712t^9 + 3912t^{10} + 2872t^{11} + 2635t^{12} + 1976t^{13} + 1736t^{14} + 1072t^{15} + \\
& 873t^{16} + 488t^{17} + 288t^{18} + 104t^{19} + 25t^{20}) + q^9(7626 + 8832t + 7602t^2 + 8240t^3 + \\
& 7119t^4 + 7896t^5 + 6207t^6 + 6864t^7 + 5562t^8 + 5496t^9 + 4533t^{10} + 4440t^{11} + 3263t^{12} + \\
& 3216t^{13} + 2268t^{14} + 1976t^{15} + 1308t^{16} + 1056t^{17} + 561t^{18} + 384t^{19} + 153t^{20} + 48t^{21} + \\
& 3t^{22}) + q^{10}(13110 + 10784t + 11665t^2 + 11208t^3 + 11302t^4 + 9648t^5 + 10815t^6 + \\
& 8576t^7 + 9040t^8 + 7896t^9 + 7190t^{10} + 5936t^{11} + 6225t^{12} + 4416t^{13} + 4054t^{14} + \\
& 3216t^{15} + 2635t^{16} + 1776t^{17} + 1473t^{18} + 808t^{19} + 531t^{20} + 240t^{21} + 83t^{22} + 8t^{23}) + \dots
\end{aligned}$$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 1, 1, 2, 2, 2) \quad f_1: \\
& 1 + q(42 + 35t + 14t^2 + 21t^3 + 21t^4 + 7t^5 + t^7) + q^2(212 + 182t + 210t^2 + 168t^3 + 147t^4 + \\
& 112t^5 + 70t^6 + 43t^7 + 35t^8 + 7t^9) + q^3(434 + 588t + 504t^2 + 462t^3 + 357t^4 + 357t^5 + \\
& 294t^6 + 252t^7 + 147t^8 + 112t^9 + 42t^{10} + 21t^{11} + 7t^{12}) + q^4(1302 + 1050t + 1190t^2 + \\
& 1071t^3 + 1029t^4 + 854t^5 + 882t^6 + 561t^7 + 511t^8 + 357t^9 + 294t^{10} + 168t^{11} + 105t^{12} + \\
& 35t^{13} + 2t^{14}) + q^5(1750 + 1981t + 1694t^2 + 1736t^3 + 1484t^4 + 1540t^5 + 1148t^6 + \\
& 1274t^7 + 868t^8 + 854t^9 + 630t^{10} + 462t^{11} + 266t^{12} + 182t^{13} + 84t^{14} + 35t^{15} + 7t^{16}) + \dots
\end{aligned}$$

$$\begin{aligned}
& q^6(3514 + 3171t + 3360t^2 + 2954t^3 + 3255t^4 + 2667t^5 + 2590t^6 + 2142t^7 + 2142t^8 + \\
& 1540t^9 + 1470t^{10} + 1071t^{11} + 805t^{12} + 588t^{13} + 420t^{14} + 182t^{15} + 105t^{16} + 21t^{17}) + \\
& q^7(4200 + 4466t + 3570t^2 + 4473t^3 + 3640t^4 + 3710t^5 + 3234t^6 + 3151t^7 + 2618t^8 + \\
& 2667t^9 + 1750t^{10} + 1736t^{11} + 1344t^{12} + 1050t^{13} + 700t^{14} + 588t^{15} + 266t^{16} + 168t^{17} + \\
& 42t^{18} + 7t^{19}) + q^8(7072 + 6356t + 7364t^2 + 5803t^3 + 6440t^4 + 5565t^5 + 5656t^6 + \\
& 4774t^7 + 4907t^8 + 3710t^9 + 3864t^{10} + 2954t^{11} + 2765t^{12} + 1981t^{13} + 1766t^{14} + \\
& 1050t^{15} + 805t^{16} + 462t^{17} + 294t^{18} + 112t^{19} + 35t^{20} + t^{21}) + q^9(7350 + 8673t + 7770t^2 + \\
& 8225t^3 + 7161t^4 + 7959t^5 + 6272t^6 + 6930t^7 + 5502t^8 + 5565t^9 + 4494t^{10} + 4473t^{11} + \\
& 3213t^{12} + 3171t^{13} + 2184t^{14} + 1981t^{15} + 1344t^{16} + 1071t^{17} + 630t^{18} + 357t^{19} + 147t^{20} + \\
& 43t^{21}) + q^{10}(12810 + 10871t + 11690t^2 + 11214t^3 + 11326t^4 + 9443t^5 + 10920t^6 + \\
& 8736t^7 + 9044t^8 + 7959t^9 + 7210t^{10} + 5803t^{11} + 6237t^{12} + 4466t^{13} + 3920t^{14} + \\
& 3171t^{15} + 2765t^{16} + 1736t^{17} + 1470t^{18} + 854t^{19} + 511t^{20} + 252t^{21} + 70t^{22} + 7t^{23}) + \dots
\end{aligned}$$

$$m = 15$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 1, 1, 1, 1, 1, 1, 3) \quad f_1:$$

$$\begin{aligned}
& 1 + q(28 + 30t + 30t^2 + 20t^3 + 15t^4 + 6t^5 + 5t^6) + q^2(202 + 210t + 165t^2 + 170t^3 + \\
& 135t^4 + 126t^5 + 70t^6 + 60t^7 + 30t^8 + 10t^9 + 3t^{10}) + q^3(546 + 450t + 525t^2 + 440t^3 + \\
& 420t^4 + 318t^5 + 330t^6 + 210t^7 + 165t^8 + 120t^9 + 69t^{10} + 30t^{11} + 10t^{12}) + q^4(1040 + \\
& 1230t + 1065t^2 + 1080t^3 + 930t^4 + 906t^5 + 710t^6 + 690t^7 + 525t^8 + 430t^9 + 282t^{10} + \\
& 210t^{11} + 105t^{12} + 60t^{13} + 15t^{14} + 2t^{15}) + q^5(1924 + 1680t + 1785t^2 + 1480t^3 + \\
& 1680t^4 + 1404t^5 + 1340t^6 + 1080t^7 + 1065t^8 + 700t^9 + 702t^{10} + 450t^{11} + 380t^{12} + \\
& 210t^{13} + 135t^{14} + 52t^{15} + 15t^{16}) + q^6(3160 + 3330t + 2910t^2 + 3430t^3 + 2625t^4 + \\
& 2826t^5 + 2440t^6 + 2340t^7 + 1785t^8 + 1920t^9 + 1332t^{10} + 1230t^{11} + 875t^{12} + 690t^{13} + \\
& 420t^{14} + 302t^{15} + 135t^{16} + 60t^{17} + 10t^{18}) + q^7(4244 + 3840t + 4395t^2 + 3540t^3 + \\
& 4005t^4 + 3270t^5 + 3425t^6 + 2850t^7 + 2910t^8 + 2220t^9 + 2241t^{10} + 1680t^{11} + 1460t^{12} + \\
& 1080t^{13} + 930t^{14} + 544t^{15} + 420t^{16} + 210t^{17} + 105t^{18} + 30t^{19} + 3t^{20}) + q^8(5946 + \\
& 6930t + 6180t^2 + 6310t^3 + 5775t^4 + 6072t^5 + 5020t^6 + 5340t^7 + 4395t^8 + 4240t^9 + \\
& 3444t^{10} + 3330t^{11} + 2490t^{12} + 2340t^{13} + 1680t^{14} + 1392t^{15} + 930t^{16} + 690t^{17} + \\
& 380t^{18} + 210t^{19} + 69t^{20} + 10t^{21}) + q^9(9028 + 7260t + 7995t^2 + 7500t^3 + 7815t^4 + \\
& 6390t^5 + 7380t^6 + 5670t^7 + 6180t^8 + 5180t^9 + 4902t^{10} + 3840t^{11} + 4170t^{12} + 2850t^{13} + \\
& 2625t^{14} + 2052t^{15} + 1680t^{16} + 1080t^{17} + 875t^{18} + 450t^{19} + 282t^{20} + 120t^{21} + 30t^{22}) + \\
& q^{10}(10514 + 12090t + 10425t^2 + 11350t^3 + 9960t^4 + 11232t^5 + 8780t^6 + 9810t^7 + \\
& 7995t^8 + 8070t^9 + 6945t^{10} + 6930t^{11} + 5180t^{12} + 5340t^{13} + 4005t^{14} + 3694t^{15} + \\
& 2625t^{16} + 2340t^{17} + 1460t^{18} + 1230t^{19} + 702t^{20} + 430t^{21} + 165t^{22} + 60t^{23} + 5t^{24}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 1, 1, 1, 2, 2, 2) \quad f_1:$$

$$1 + q(40 + 27t + 26t^2 + 18t^3 + 17t^4 + 10t^5 + t^6 + t^7) + q^2(154 + 216t + 187t^2 +$$

$$\begin{aligned}
& 176t^3 + 131t^4 + 112t^5 + 82t^6 + 56t^7 + 26t^8 + 16t^9 + t^{10}) + q^3(582 + 469t + 489t^2 + \\
& 440t^3 + 412t^4 + 316t^5 + 330t^6 + 213t^7 + 187t^8 + 102t^9 + 73t^{10} + 27t^{11} + 10t^{12} + t^{13}) + \\
& q^4(1080 + 1192t + 1059t^2 + 1072t^3 + 938t^4 + 960t^5 + 706t^6 + 696t^7 + 489t^8 + 416t^9 + \\
& 290t^{10} + 216t^{11} + 113t^{12} + 56t^{13} + 17t^{14}) + q^5(1924 + 1640t + 1853t^2 + 1492t^3 + \\
& 1704t^4 + 1354t^5 + 1292t^6 + 1063t^7 + 1059t^8 + 772t^9 + 702t^{10} + 469t^{11} + 344t^{12} + \\
& 213t^{13} + 131t^{14} + 52t^{15} + 17t^{16} + t^{17}) + q^6(3160 + 3392t + 2862t^2 + 3376t^3 + 2653t^4 + \\
& 2800t^5 + 2440t^6 + 2376t^7 + 1853t^8 + 1920t^9 + 1260t^{10} + 1192t^{11} + 911t^{12} + 696t^{13} + \\
& 412t^{14} + 320t^{15} + 131t^{16} + 56t^{17} + 10t^{18}) + q^7(4068 + 3889t + 4403t^2 + 3542t^3 + \\
& 3927t^4 + 3360t^5 + 3537t^6 + 2832t^7 + 2862t^8 + 2150t^9 + 2297t^{10} + 1640t^{11} + 1492t^{12} + \\
& 1063t^{13} + 938t^{14} + 548t^{15} + 412t^{16} + 213t^{17} + 113t^{18} + 27t^{19} + t^{20}) + q^8(6042 + 6952t + \\
& 6144t^2 + 6448t^3 + 5731t^4 + 5984t^5 + 4984t^6 + 5248t^7 + 4403t^8 + 4192t^9 + 3528t^{10} + \\
& 3392t^{11} + 2466t^{12} + 2376t^{13} + 1704t^{14} + 1344t^{15} + 938t^{16} + 696t^{17} + 344t^{18} + 216t^{19} + \\
& 73t^{20} + 16t^{21}) + q^9(8920 + 7177t + 8023t^2 + 7554t^3 + 7821t^4 + 6350t^5 + 7452t^6 + \\
& 5777t^7 + 6144t^8 + 5198t^9 + 4830t^{10} + 3889t^{11} + 4134t^{12} + 2832t^{13} + 2653t^{14} + \\
& 1980t^{15} + 1704t^{16} + 1063t^{17} + 911t^{18} + 469t^{19} + 290t^{20} + 102t^{21} + 26t^{22} + t^{23}) + \\
& q^{10}(10714 + 12032t + 10439t^2 + 11248t^3 + 9968t^4 + 11232t^5 + 8676t^6 + 9872t^7 + \\
& 8023t^8 + 8208t^9 + 6995t^{10} + 6952t^{11} + 5084t^{12} + 5248t^{13} + 3927t^{14} + 3744t^{15} + \\
& 2653t^{16} + 2376t^{17} + 1492t^{18} + 1192t^{19} + 702t^{20} + 416t^{21} + 187t^{22} + 56t^{23} + t^{24}) + \dots
\end{aligned}$$

$m = 16$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (1, 1, 1, 1, 1, 1, 1, 3) \quad f_1:$$

$$\begin{aligned}
& 1 + q(56 + 56t^2 + 28t^4 + 8t^6) + q^2(420 + 336t^2 + 288t^4 + 168t^6 + 70t^8 + 8t^{10}) + \\
& q^3(896 + 1008t^2 + 756t^4 + 616t^6 + 336t^8 + 168t^{10} + 28t^{12}) + q^4(2366 + 2024t^2 + \\
& 2016t^4 + 1400t^6 + 1176t^8 + 616t^{10} + 288t^{12} + 56t^{14} + t^{16}) + q^5(3360 + 3472t^2 + \\
& 2772t^4 + 2856t^6 + 1792t^8 + 1400t^{10} + 756t^{12} + 336t^{14} + 56t^{16}) + q^6(6440 + 5880t^2 + \\
& 6048t^4 + 4536t^6 + 4200t^8 + 2856t^{10} + 2016t^{12} + 1008t^{14} + 420t^{16} + 56t^{18}) + q^7(7168 + \\
& 8456t^2 + 6832t^4 + 6664t^6 + 5152t^8 + 4536t^{10} + 2772t^{12} + 2024t^{14} + 896t^{16} + 336t^{18} + \\
& 28t^{20}) + q^8(14196 + 11200t^2 + 12096t^4 + 10416t^6 + 9536t^8 + 6664t^{10} + 6048t^{12} + \\
& 3472t^{14} + 2366t^{16} + 1008t^{18} + 288t^{20} + 8t^{22}) + q^9(14168 + 16296t^2 + 13608t^4 + \\
& 13664t^6 + 10752t^8 + 10416t^{10} + 6832t^{12} + 5880t^{14} + 3360t^{16} + 2024t^{18} + 756t^{20} + \\
& 168t^{22}) + q^{10}(22120 + 20776t^2 + 22176t^4 + 17528t^6 + 17710t^8 + 13664t^{10} + 12096t^{12} + \\
& 8456t^{14} + 6440t^{16} + 3472t^{18} + 2016t^{20} + 616t^{22} + 70t^{24}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 1, 1, 1, 1, 2, 2, 2) \quad f_1:$$

$$\begin{aligned}
& 1 + q(26 + 33t + 25t^2 + 20t^3 + 15t^4 + 10t^5 + 3t^6 + t^7) + q^2(210 + 173t + 190t^2 + 165t^3 + \\
& 140t^4 + 103t^5 + 93t^6 + 56t^7 + 35t^8 + 15t^9 + 5t^{10}) + q^3(456 + 531t + 438t^2 + 450t^3 + \\
& 375t^4 + 378t^5 + 271t^6 + 255t^7 + 168t^8 + 130t^9 + 75t^{10} + 45t^{11} + 13t^{12} + 3t^{13}) + q^4(1176 +
\end{aligned}$$

$$\begin{aligned}
& 1045t + 1133t^2 + 935t^3 + 1036t^4 + 821t^5 + 795t^6 + 606t^7 + 588t^8 + 400t^9 + 345t^{10} + \\
& 211t^{11} + 148t^{12} + 73t^{13} + 31t^{14} + 5t^{15} + q^5(1680 + 1776t + 1526t^2 + 1800t^3 + 1381t^4 + \\
& 1445t^5 + 1251t^6 + 1191t^7 + 896t^8 + 930t^9 + 605t^{10} + 556t^{11} + 381t^{12} + 271t^{13} + 146t^{14} + \\
& 90t^{15} + 30t^{16} + 5t^{17}) + q^6(3220 + 2988t + 3315t^2 + 2726t^3 + 2940t^4 + 2571t^5 + 2543t^6 + \\
& 2160t^7 + 2100t^8 + 1608t^9 + 1605t^{10} + 1155t^{11} + 980t^{12} + 705t^{13} + 570t^{14} + 330t^{15} + \\
& 210t^{16} + 90t^{17} + 31t^{18} + 3t^{19}) + q^7(3552 + 4251t + 3691t^2 + 3956t^3 + 3436t^4 + 3723t^5 + \\
& 2911t^6 + 3235t^7 + 2576t^8 + 2463t^9 + 1993t^{10} + 1941t^{11} + 1391t^{12} + 1331t^{13} + 891t^{14} + \\
& 713t^{15} + 440t^{16} + 330t^{17} + 146t^{18} + 73t^{19} + 13t^{20}) + q^8(7140 + 5795t + 6308t^2 + \\
& 5913t^3 + 6216t^4 + 4996t^5 + 5850t^6 + 4442t^7 + 4768t^8 + 4056t^9 + 3753t^{10} + 2981t^{11} + \\
& 3108t^{12} + 2098t^{13} + 1946t^{14} + 1458t^{15} + 1190t^{16} + 713t^{17} + 570t^{18} + 271t^{19} + 148t^{20} + \\
& 45t^{21} + 5t^{22}) + q^9(7106 + 8253t + 7131t^2 + 7689t^3 + 6810t^4 + 7530t^5 + 5968t^6 + \\
& 6600t^7 + 5376t^8 + 5555t^9 + 4566t^{10} + 4620t^{11} + 3396t^{12} + 3471t^{13} + 2565t^{14} + 2341t^{15} + \\
& 1680t^{16} + 1458t^{17} + 891t^{18} + 705t^{19} + 381t^{20} + 211t^{21} + 75t^{22} + 15t^{23}) + q^{10}(11060 + \\
& 10476t + 11665t^2 + 9767t^3 + 10780t^4 + 9528t^5 + 9871t^6 + 8651t^7 + 8855t^8 + 7236t^9 + \\
& 7696t^{10} + 6200t^{11} + 5880t^{12} + 4890t^{13} + 4765t^{14} + 3405t^{15} + 3220t^{16} + 2341t^{17} + \\
& 1946t^{18} + 1331t^{19} + 980t^{20} + 556t^{21} + 345t^{22} + 130t^{23} + 35t^{24} + t^{25}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 1, 1, 1, 2, 3) \quad f_5:$$

$$\begin{aligned}
& 1 + q(42 + 22t + 21t^2 + 35t^3 + 7t^4 + 7t^5 + 7t^6) + q^2(210 + 189t + 182t^2 + 147t^3 + 140t^4 + \\
& 112t^5 + 105t^6 + 42t^7 + 35t^8 + 21t^9 + t^{10} + t^{11}) + q^3(392 + 525t + 462t^2 + 469t^3 + 399t^4 + \\
& 357t^5 + 259t^6 + 252t^7 + 168t^8 + 147t^9 + 63t^{10} + 42t^{11} + 21t^{12}) + q^4(1176 + 1085t + \\
& 1177t^2 + 826t^3 + 1036t^4 + 847t^5 + 735t^6 + 693t^7 + 588t^8 + 351t^9 + 357t^{10} + 231t^{11} + \\
& 148t^{12} + 56t^{13} + 35t^{14} + 7t^{15}) + q^5(1680 + 1771t + 1470t^2 + 1827t^3 + 1421t^4 + 1470t^5 + \\
& 1239t^6 + 1169t^7 + 896t^8 + 924t^9 + 665t^{10} + 497t^{11} + 357t^{12} + 294t^{13} + 154t^{14} + 105t^{15} + \\
& 14t^{16} + 7t^{17}) + q^6(3220 + 2982t + 3255t^2 + 2800t^3 + 2940t^4 + 2562t^5 + 2611t^6 + \\
& 2121t^7 + 2100t^8 + 1505t^9 + 1617t^{10} + 1218t^{11} + 980t^{12} + 756t^{13} + 546t^{14} + 287t^{15} + \\
& 210t^{16} + 105t^{17} + 35t^{18}) + q^7(3808 + 4144t + 3759t^2 + 4081t^3 + 3276t^4 + 3549t^5 + \\
& 2947t^6 + 3199t^7 + 2576t^8 + 2709t^9 + 1925t^{10} + 1932t^{11} + 1351t^{12} + 1309t^{13} + 847t^{14} + \\
& 749t^{15} + 504t^{16} + 287t^{17} + 154t^{18} + 56t^{19} + 21t^{20} + t^{21}) + q^8(7140 + 5691t + 6244t^2 + \\
& 5922t^3 + 6216t^4 + 5229t^5 + 5922t^6 + 4340t^7 + 4768t^8 + 4074t^9 + 3717t^{10} + 2919t^{11} + \\
& 3108t^{12} + 2122t^{13} + 2002t^{14} + 1386t^{15} + 1190t^{16} + 749t^{17} + 546t^{18} + 294t^{19} + 148t^{20} + \\
& 42t^{21} + t^{22}) + q^9(6930 + 8400t + 7119t^2 + 7666t^3 + 6762t^4 + 7539t^5 + 6048t^6 + \\
& 6636t^7 + 5376t^8 + 5390t^9 + 4494t^{10} + 4620t^{11} + 3556t^{12} + 3402t^{13} + 2625t^{14} + \\
& 2401t^{15} + 1680t^{16} + 1386t^{17} + 847t^{18} + 756t^{19} + 357t^{20} + 231t^{21} + 63t^{22} + 21t^{23}) + \\
& q^{10}(11060 + 10563t + 11837t^2 + 9716t^3 + 10780t^4 + 9226t^5 + 9779t^6 + 8694t^7 + \\
& 8855t^8 + 7399t^9 + 7616t^{10} + 6391t^{11} + 5880t^{12} + 4781t^{13} + 4697t^{14} + 3388t^{15} + \\
& 3220t^{16} + 2401t^{17} + 2002t^{18} + 1309t^{19} + 980t^{20} + 497t^{21} + 357t^{22} + 147t^{23} + 35t^{24}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 2, 2, 2, 2) \quad f_1:$$

$$1 + q(126 + 56t^4 + t^8) + q^2(756 + 576t^4 + 126t^8) + q^3(2072 + 1512t^4 + 756t^8 + 56t^{12}) +$$

$$q^4(4158 + 4032t^4 + 2072t^8 + 576t^{12} + t^{16}) + q^5(7560 + 5544t^4 + 4158t^8 + 1512t^{12} +$$

$$126t^{16}) + q^6(11592 + 12096t^4 + 7560t^8 + 4032t^{12} + 756t^{16}) + q^7(16704 + 13664t^4 +$$

$$11592t^8 + 5544t^{12} + 2072t^{16} + 56t^{20}) + q^8(24948 + 24192t^4 + 16704t^8 + 12096t^{12} +$$

$$4158t^{16} + 576t^{20}) + q^9(31878 + 27216t^4 + 24948t^8 + 13664t^{12} + 7560t^{16} + 1512t^{20} +$$

$$t^{24}) + q^{10}(39816 + 44352t^4 + 31878t^8 + 24192t^{12} + 11592t^{16} + 4032t^{20} + 126t^{24}) + \dots$$

$$m = 17$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (1, 1, 1, 1, 1, 2, 2, 2) \quad f_1:$$

$$1 + q(30 + 27t + 27t^2 + 21t^3 + 16t^4 + 7t^5 + 6t^6 + t^7) + q^2(172 + 202t + 166t^2 + 162t^3 +$$

$$128t^4 + 125t^5 + 82t^6 + 65t^7 + 36t^8 + 21t^9 + 6t^{10} + t^{11}) + q^3(512 + 432t + 492t^2 +$$

$$400t^3 + 426t^4 + 327t^5 + 296t^6 + 234t^7 + 204t^8 + 118t^9 + 96t^{10} + 51t^{11} + 22t^{12} +$$

$$6t^{13}) + q^4(1092 + 1117t + 950t^2 + 1080t^3 + 863t^4 + 872t^5 + 762t^6 + 671t^7 + 508t^8 +$$

$$501t^9 + 318t^{10} + 256t^{11} + 162t^{12} + 96t^{13} + 42t^{14} + 15t^{15} + t^{16}) + q^5(1688 + 1578t +$$

$$1768t^2 + 1425t^3 + 1578t^4 + 1302t^5 + 1344t^6 + 1074t^7 + 1056t^8 + 764t^9 + 764t^{10} +$$

$$518t^{11} + 436t^{12} + 283t^{13} + 217t^{14} + 100t^{15} + 56t^{16} + 12t^{17} + t^{18}) + q^6(2712 + 3255t +$$

$$2856t^2 + 3030t^3 + 2616t^4 + 2796t^5 + 2168t^6 + 2397t^7 + 1896t^8 + 1831t^9 + 1416t^{10} +$$

$$1356t^{11} + 956t^{12} + 867t^{13} + 552t^{14} + 431t^{15} + 240t^{16} + 150t^{17} + 56t^{18} + 15t^{19}) +$$

$$q^7(4452 + 3511t + 3832t^2 + 3567t^3 + 3728t^4 + 3090t^5 + 3504t^6 + 2630t^7 + 2848t^8 +$$

$$2376t^9 + 2208t^{10} + 1694t^{11} + 1758t^{12} + 1175t^{13} + 1040t^{14} + 771t^{15} + 590t^{16} + 344t^{17} +$$

$$240t^{18} + 100t^{19} + 42t^{20} + 6t^{21}) + q^8(5744 + 6427t + 5788t^2 + 6045t^3 + 5428t^4 +$$

$$5837t^5 + 4780t^6 + 5199t^7 + 4312t^8 + 4156t^9 + 3582t^{10} + 3471t^{11} + 2609t^{12} + 2552t^{13} +$$

$$1942t^{14} + 1667t^{15} + 1194t^{16} + 982t^{17} + 590t^{18} + 431t^{19} + 217t^{20} + 96t^{21} + 22t^{22} + t^{23}) +$$

$$q^9(7662 + 7050t + 8052t^2 + 6537t^3 + 7512t^4 + 6333t^5 + 6855t^6 + 5703t^7 + 5946t^8 +$$

$$4845t^9 + 5178t^{10} + 4119t^{11} + 3928t^{12} + 3153t^{13} + 3156t^{14} + 2125t^{15} + 2076t^{16} +$$

$$1422t^{17} + 1194t^{18} + 771t^{19} + 552t^{20} + 283t^{21} + 162t^{22} + 51t^{23} + 6t^{24}) + q^{10}(10344 +$$

$$11052t + 9524t^2 + 11571t^3 + 9168t^4 + 9951t^5 + 9096t^6 + 9163t^7 + 7560t^8 + 8676t^9 +$$

$$6498t^{10} + 6806t^{11} + 5736t^{12} + 5426t^{13} + 4118t^{14} + 4338t^{15} + 2984t^{16} + 2782t^{17} +$$

$$2076t^{18} + 1667t^{19} + 1040t^{20} + 867t^{21} + 436t^{22} + 256t^{23} + 96t^{24} + 21t^{25}) + \dots$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 1, 1, 1, 1, 2, 3) \quad f_4:$$

$$1 + q(18 + 36t + 22t^2 + 22t^3 + 22t^4 - 2t^5 + 11t^6) + q^2(174 + 212t + 162t^2 + 148t^3 +$$

$$124t^4 + 138t^5 + 90t^6 + 50t^7 + 39t^8 + 26t^9 + 2t^{10} + 2t^{11}) + q^3(540 + 362t + 531t^2 +$$

$$444t^3 + 420t^4 + 312t^5 + 262t^6 + 270t^7 + 206t^8 + 104t^9 + 92t^{10} + 66t^{11} + 12t^{12} + 10t^{13} -$$

$$t^{14}) + q^4(1182 + 1114t + 946t^2 + 1024t^3 + 830t^4 + 944t^5 + 752t^6 + 654t^7 + 494t^8 +$$

$$522t^9 + 332t^{10} + 240t^{11} + 163t^{12} + 96t^{13} + 42t^{14} + 14t^{15} + 2t^{16}) + q^5(1632 + 1644t +$$

$$\begin{aligned}
& 1702t^2 + 1454t^3 + 1574t^4 + 1266t^5 + 1366t^6 + 1136t^7 + 1042t^8 + 722t^9 + 792t^{10} + \\
& 472t^{11} + 468t^{12} + 272t^{13} + 232t^{14} + 86t^{15} + 70t^{16} + 4t^{17} + 2t^{18} + q^6(2444 + 3278t + \\
& 2910t^2 + 3068t^3 + 2731t^4 + 2776t^5 + 2134t^6 + 2314t^7 + 1890t^8 + 1870t^9 + 1410t^{10} + \\
& 1352t^{11} + 1002t^{12} + 854t^{13} + 524t^{14} + 462t^{15} + 220t^{16} + 140t^{17} + 70t^{18} + 14t^{19} - t^{20}) + \\
& q^7(4584 + 3592t + 3810t^2 + 3476t^3 + 3686t^4 + 3004t^5 + 3576t^6 + 2586t^7 + 2912t^8 + \\
& 2390t^9 + 2204t^{10} + 1734t^{11} + 1704t^{12} + 1194t^{13} + 1014t^{14} + 780t^{15} + 556t^{16} + 412t^{17} + \\
& 220t^{18} + 86t^{19} + 42t^{20} + 10t^{21}) + q^8(5814 + 6454t + 5688t^2 + 6026t^3 + 5374t^4 + \\
& 5834t^5 + 4904t^6 + 5230t^7 + 4396t^8 + 4104t^9 + 3500t^{10} + 3534t^{11} + 2520t^{12} + 2480t^{13} + \\
& 2044t^{14} + 1702t^{15} + 1203t^{16} + 940t^{17} + 556t^{18} + 462t^{19} + 232t^{20} + 96t^{21} + 12t^{22} + 2t^{23}) + \\
& q^9(7578 + 6856t + 8164t^2 + 6652t^3 + 7672t^4 + 6288t^5 + 6884t^6 + 5618t^7 + 5830t^8 + \\
& 4878t^9 + 5043t^{10} + 4224t^{11} + 3954t^{12} + 3308t^{13} + 3140t^{14} + 1970t^{15} + 2080t^{16} + \\
& 1480t^{17} + 1203t^{18} + 780t^{19} + 524t^{20} + 272t^{21} + 163t^{22} + 66t^{23} + 2t^{24}) + q^{10}(10552 + \\
& 10968t + 9556t^2 + 11590t^3 + 9202t^4 + 9870t^5 + 8980t^6 + 9254t^7 + 7458t^8 + 8712t^9 + \\
& 6630t^{10} + 6812t^{11} + 5640t^{12} + 5460t^{13} + 4034t^{14} + 4404t^{15} + 3074t^{16} + 2684t^{17} + \\
& 2080t^{18} + 1702t^{19} + 1014t^{20} + 854t^{21} + 468t^{22} + 240t^{23} + 92t^{24} + 26t^{25}) + \dots
\end{aligned}$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 1, 2, 2, 2, 2) \quad f_1:$$

$$\begin{aligned}
& 1 + q(60 + 32t + t^2 + 12t^3 + 32t^4 + 12t^5 + t^8) + q^2(252 + 192t + 92t^2 + 160t^3 + 192t^4 + \\
& 160t^5 + 33t^6 + 32t^7 + 60t^8 + 32t^9 + t^{10}) + q^3(544 + 492t + 444t^2 + 344t^3 + 480t^4 + \\
& 312t^5 + 252t^6 + 204t^7 + 252t^8 + 192t^9 + 60t^{10} + 12t^{11} + 32t^{12} + 12t^{13}) + q^4(1020 + \\
& 992t + 1024t^2 + 1152t^3 + 833t^4 + 992t^5 + 732t^6 + 640t^7 + 544t^8 + 480t^9 + 284t^{10} + \\
& 192t^{11} + 192t^{12} + 160t^{13} + 32t^{14} + t^{16}) + q^5(1624 + 1752t + 1852t^2 + 1356t^3 + \\
& 1501t^4 + 1068t^5 + 1376t^6 + 1144t^7 + 1052t^8 + 844t^9 + 736t^{10} + 504t^{11} + 481t^{12} + \\
& 312t^{13} + 193t^{14} + 44t^{15} + 60t^{16} + 32t^{17} + t^{18}) + q^6(2464 + 3072t + 3000t^2 + 3232t^3 + \\
& 2424t^4 + 2880t^5 + 2460t^6 + 2400t^7 + 1752t^8 + 1600t^9 + 1500t^{10} + 1440t^{11} + 892t^{12} + \\
& 960t^{13} + 540t^{14} + 352t^{15} + 252t^{16} + 192t^{17} + 60t^{18}) + q^7(4224 + 3512t + 4192t^2 + \\
& 3480t^3 + 3420t^4 + 2904t^5 + 3672t^6 + 2988t^7 + 2592t^8 + 2436t^9 + 2392t^{10} + 1708t^{11} + \\
& 1692t^{12} + 908t^{13} + 1084t^{14} + 792t^{15} + 576t^{16} + 480t^{17} + 252t^{18} + 44t^{19} + 32t^{20} + \\
& 12t^{21}) + q^8(5756 + 6400t + 5888t^2 + 6304t^3 + 5404t^4 + 5792t^5 + 4704t^6 + 5024t^7 + \\
& 4288t^8 + 3744t^9 + 3520t^{10} + 3840t^{11} + 2657t^{12} + 2624t^{13} + 1984t^{14} + 1792t^{15} + \\
& 1212t^{16} + 832t^{17} + 576t^{18} + 352t^{19} + 193t^{20} + 160t^{21} + 32t^{22}) + q^9(7260 + 7344t + \\
& 7932t^2 + 6296t^3 + 7572t^4 + 6072t^5 + 7105t^6 + 5892t^7 + 6012t^8 + 5028t^9 + 5376t^{10} + \\
& 4152t^{11} + 3704t^{12} + 2712t^{13} + 3132t^{14} + 2316t^{15} + 2040t^{16} + 1464t^{17} + 1212t^{18} + \\
& 792t^{19} + 540t^{20} + 312t^{21} + 192t^{22} + 12t^{23} + t^{24}) + q^{10}(10776 + 10560t + 9405t^2 + \\
& 11520t^3 + 9080t^4 + 10656t^5 + 9144t^6 + 9312t^7 + 7324t^8 + 8352t^9 + 6748t^{10} + \\
& 6784t^{11} + 5652t^{12} + 5472t^{13} + 4185t^{14} + 4512t^{15} + 2912t^{16} + 2432t^{17} + 2040t^{18} + \\
& 1792t^{19} + 1084t^{20} + 960t^{21} + 481t^{22} + 192t^{23} + 60t^{24} + 32t^{25} + t^{26}) + \dots
\end{aligned}$$

$m = 18$

$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 1, 1, 1, 1, 1, 2, 3)$ f_1 :

$$\begin{aligned}
 &1 + q(24 + 30t + 25t^2 + 22t^3 + 14t^4 + 10t^5 + 5t^6 + 2t^7) + q^2(194 + 172t + 180t^2 + 144t^3 + \\
 &145t^4 + 110t^5 + 90t^6 + 64t^7 + 46t^8 + 20t^9 + 10t^{10} + 2t^{11}) + q^3(460 + 484t + 395t^2 + \\
 &462t^3 + 358t^4 + 360t^5 + 291t^6 + 252t^7 + 174t^8 + 160t^9 + 89t^{10} + 64t^{11} + 30t^{12} + 10t^{13} + \\
 &t^{14}) + q^4(1074 + 982t + 1120t^2 + 894t^3 + 960t^4 + 792t^5 + 784t^6 + 624t^7 + 604t^8 + 420t^9 + \\
 &400t^{10} + 252t^{11} + 190t^{12} + 110t^{13} + 64t^{14} + 22t^{15} + 5t^{16}) + q^5(1468 + 1734t + 1499t^2 + \\
 &1584t^3 + 1374t^4 + 1464t^5 + 1134t^6 + 1224t^7 + 940t^8 + 900t^9 + 668t^{10} + 624t^{11} + 414t^{12} + \\
 &360t^{13} + 219t^{14} + 144t^{15} + 70t^{16} + 30t^{17} + 4t^{18}) + q^6(3300 + 2754t + 2910t^2 + 2790t^3 + \\
 &2830t^4 + 2332t^5 + 2610t^6 + 2020t^7 + 2064t^8 + 1760t^9 + 1550t^{10} + 1224t^{11} + 1185t^{12} + \\
 &792t^{13} + 670t^{14} + 462t^{15} + 330t^{16} + 172t^{17} + 100t^{18} + 30t^{19} + 5t^{20}) + q^7(3420 + 4010t + \\
 &3484t^2 + 3736t^3 + 3228t^4 + 3594t^5 + 2834t^6 + 3136t^7 + 2508t^8 + 2484t^9 + 2069t^{10} + \\
 &2020t^{11} + 1478t^{12} + 1464t^{13} + 1034t^{14} + 894t^{15} + 590t^{16} + 484t^{17} + 258t^{18} + 172t^{19} + \\
 &70t^{20} + 22t^{21} + t^{22}) + q^8(6154 + 5586t + 6384t^2 + 5184t^3 + 5940t^4 + 5104t^5 + 5328t^6 + \\
 &4494t^7 + 4705t^8 + 3728t^9 + 4080t^{10} + 3136t^{11} + 2970t^{12} + 2332t^{13} + 2288t^{14} + 1584t^{15} + \\
 &1466t^{16} + 982t^{17} + 800t^{18} + 484t^{19} + 330t^{20} + 144t^{21} + 64t^{22} + 10t^{23}) + q^9(7104 + \\
 &7514t + 6598t^2 + 7890t^3 + 6158t^4 + 6802t^5 + 6135t^6 + 6250t^7 + 5170t^8 + 5824t^9 + \\
 &4283t^{10} + 4494t^{11} + 3834t^{12} + 3594t^{13} + 2665t^{14} + 2790t^{15} + 1884t^{16} + 1734t^{17} + \\
 &1274t^{18} + 982t^{19} + 590t^{20} + 462t^{21} + 219t^{22} + 110t^{23} + 30t^{24} + 2t^{25}) + q^{10}(10668 + \\
 &9930t + 10690t^2 + 9254t^3 + 10570t^4 + 9020t^5 + 9240t^6 + 8344t^7 + 8844t^8 + 7068t^9 + \\
 &7400t^{10} + 6250t^{11} + 6170t^{12} + 5104t^{13} + 4930t^{14} + 3736t^{15} + 3714t^{16} + 2754t^{17} + \\
 &2340t^{18} + 1734t^{19} + 1466t^{20} + 894t^{21} + 670t^{22} + 360t^{23} + 190t^{24} + 64t^{25} + 10t^{26}) + \dots
 \end{aligned}$$

$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 1, 1, 2, 2, 2, 2)$ f_1 :

$$\begin{aligned}
 &1 + q(42 + 32t + 12t^2 + 16t^3 + 20t^4 + 16t^5 + 2t^6 + t^8) + q^2(170 + 176t + 180t^2 + 144t^3 + \\
 &161t^4 + 112t^5 + 90t^6 + 48t^7 + 42t^8 + 32t^9 + 10t^{10}) + q^3(460 + 464t + 454t^2 + 480t^3 + \\
 &287t^4 + 352t^5 + 282t^6 + 272t^7 + 190t^8 + 160t^9 + 102t^{10} + 48t^{11} + 21t^{12} + 16t^{13} + 2t^{14}) + \\
 &q^4(1002 + 976t + 1104t^2 + 912t^3 + 1012t^4 + 752t^5 + 832t^6 + 672t^7 + 580t^8 + 384t^9 + \\
 &352t^{10} + 272t^{11} + 202t^{12} + 112t^{13} + 80t^{14} + 16t^{15} + t^{16}) + q^5(1528 + 1792t + 1492t^2 + \\
 &1584t^3 + 1314t^4 + 1424t^5 + 1134t^6 + 1200t^7 + 946t^8 + 864t^9 + 728t^{10} + 672t^{11} + \\
 &414t^{12} + 352t^{13} + 192t^{14} + 144t^{15} + 62t^{16} + 32t^{17} + 10t^{18}) + q^6(3300 + 2704t + 2910t^2 + \\
 &2736t^3 + 2862t^4 + 2448t^5 + 2610t^6 + 2048t^7 + 2000t^8 + 1760t^9 + 1550t^{10} + 1200t^{11} + \\
 &1221t^{12} + 752t^{13} + 670t^{14} + 480t^{15} + 330t^{16} + 176t^{17} + 100t^{18} + 32t^{19} + t^{20}) + q^7(3528 + \\
 &3968t + 3388t^2 + 3712t^3 + 3362t^4 + 3632t^5 + 2696t^6 + 3072t^7 + 2668t^8 + 2592t^9 + \\
 &1982t^{10} + 2048t^{11} + 1448t^{12} + 1424t^{13} + 952t^{14} + 912t^{15} + 664t^{16} + 464t^{17} + 276t^{18} + \\
 &176t^{19} + 62t^{20} + 16t^{21} + 2t^{22}) + q^8(6250 + 5728t + 6352t^2 + 5184t^3 + 5940t^4 + 5072t^5 + \\
 &5328t^6 + 4304t^7 + 4641t^8 + 3776t^9 + 4064t^{10} + 3072t^{11} + 2970t^{12} + 2448t^{13} + 2416t^{14} +
 \end{aligned}$$

$$\begin{aligned}
& 1584t^{15} + 1482t^{16} + 976t^{17} + 704t^{18} + 464t^{19} + 330t^{20} + 144t^{21} + 80t^{22} + 16t^{23} + \\
& q^9(7104 + 7456t + 6464t^2 + 7872t^3 + 6104t^4 + 6960t^5 + 6252t^6 + 6304t^7 + 5136t^8 + \\
& 5824t^9 + 4512t^{10} + 4304t^{11} + 3780t^{12} + 3632t^{13} + 2656t^{14} + 2736t^{15} + 1785t^{16} + \\
& 1792t^{17} + 1274t^{18} + 976t^{19} + 664t^{20} + 480t^{21} + 192t^{22} + 112t^{23} + 21t^{24}) + q^{10}(10596 + \\
& 9904t + 10690t^2 + 9296t^3 + 10570t^4 + 8880t^5 + 9240t^6 + 8432t^7 + 8952t^8 + 7104t^9 + \\
& 7400t^{10} + 6304t^{11} + 6122t^{12} + 5072t^{13} + 4930t^{14} + 3712t^{15} + 3662t^{16} + 2704t^{17} + \\
& 2340t^{18} + 1792t^{19} + 1482t^{20} + 912t^{21} + 670t^{22} + 352t^{23} + 202t^{24} + 48t^{25} + 10t^{26}) + \dots
\end{aligned}$$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 0, 0, 3, 3) \quad f_1: \\
& 1 + q(84 + 64t^3 + 14t^6) + q^2(574 + 448t^3 + 280t^6 + 64t^9 + t^{12}) + q^3(1288 + 1344t^3 + \\
& 840t^6 + 448t^9 + 84t^{12}) + q^4(3444 + 2688t^3 + 2368t^6 + 1344t^9 + 574t^{12} + 64t^{15}) + \\
& q^5(4424 + 4928t^3 + 3542t^6 + 2688t^9 + 1288t^{12} + 448t^{15} + 14t^{18}) + q^6(9240 + 8064t^3 + \\
& 7560t^6 + 4928t^9 + 3444t^{12} + 1344t^{15} + 280t^{18}) + q^7(11088 + 11200t^3 + 8456t^6 + \\
& 8064t^9 + 4424t^{12} + 2688t^{15} + 840t^{18} + 64t^{21}) + q^8(18494 + 16192t^3 + 16576t^6 + \\
& 11200t^9 + 9240t^{12} + 4928t^{15} + 2368t^{18} + 448t^{21} + t^{24}) + q^9(19740 + 22848t^3 + \\
& 17808t^6 + 16192t^9 + 11088t^{12} + 8064t^{15} + 3542t^{18} + 1344t^{21} + 84t^{24}) + q^{10}(34440 + \\
& 27776t^3 + 27720t^6 + 22848t^9 + 18494t^{12} + 11200t^{15} + 7560t^{18} + 2688t^{21} + 574t^{24}) + \dots
\end{aligned}$$

$$m = 19$$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (1, 1, 1, 1, 1, 1, 2, 3) \quad f_1: \\
& 1 + q(26 + 25t + 30t^2 + 18t^3 + 16t^4 + 10t^5 + 5t^6 + 3t^7) + q^2(180 + 190t + 140t^2 + 168t^3 + \\
& 131t^4 + 115t^5 + 90t^6 + 68t^7 + 45t^8 + 30t^9 + 8t^{10} + 5t^{11}) + q^3(456 + 423t + 480t^2 + \\
& 376t^3 + 390t^4 + 330t^5 + 318t^6 + 231t^7 + 210t^8 + 130t^9 + 123t^{10} + 63t^{11} + 40t^{12} + \\
& 15t^{13} + 3t^{14}) + q^4(966 + 1058t + 960t^2 + 990t^3 + 861t^4 + 875t^5 + 668t^6 + 705t^7 + \\
& 536t^8 + 505t^9 + 366t^{10} + 303t^{11} + 190t^{12} + 136t^{13} + 78t^{14} + 35t^{15} + 10t^{16} + t^{17}) + \\
& q^5(1800 + 1421t + 1585t^2 + 1446t^3 + 1506t^4 + 1190t^5 + 1371t^6 + 1046t^7 + 1056t^8 + \\
& 855t^9 + 725t^{10} + 551t^{11} + 540t^{12} + 341t^{13} + 266t^{14} + 165t^{15} + 100t^{16} + 41t^{17} + 15t^{18}) + \\
& q^6(2620 + 3120t + 2658t^2 + 2843t^3 + 2475t^4 + 2730t^5 + 2170t^6 + 2313t^7 + 1845t^8 + \\
& 1821t^9 + 1506t^{10} + 1455t^{11} + 1010t^{12} + 975t^{13} + 660t^{14} + 555t^{15} + 345t^{16} + 255t^{17} + \\
& 118t^{18} + 60t^{19} + 15t^{20} + t^{21}) + q^7(3672 + 3486t + 3868t^2 + 3181t^3 + 3650t^4 + 3063t^5 + \\
& 3216t^6 + 2651t^7 + 2846t^8 + 2191t^9 + 2368t^{10} + 1816t^{11} + 1700t^{12} + 1321t^{13} + 1276t^{14} + \\
& 833t^{15} + 770t^{16} + 471t^{17} + 358t^{18} + 210t^{19} + 118t^{20} + 41t^{21} + 10t^{22}) + q^8(5730 + \\
& 5898t + 5336t^2 + 6195t^3 + 4942t^4 + 5323t^5 + 4968t^6 + 4870t^7 + 4136t^8 + 4455t^9 + \\
& 3396t^{10} + 3378t^{11} + 2970t^{12} + 2641t^{13} + 2028t^{14} + 1983t^{15} + 1358t^{16} + 1191t^{17} + \\
& 876t^{18} + 640t^{19} + 358t^{20} + 255t^{21} + 100t^{22} + 35t^{23} + 3t^{24}) + q^9(7646 + 6606t + \\
& 7395t^2 + 6253t^3 + 7320t^4 + 6111t^5 + 6246t^6 + 5583t^7 + 6000t^8 + 4701t^9 + 5145t^{10} + \\
& 4101t^{11} + 4156t^{12} + 3270t^{13} + 3201t^{14} + 2401t^{15} + 2388t^{16} + 1725t^{17} + 1475t^{18} +
\end{aligned}$$

$$990t^{19} + 876t^{20} + 471t^{21} + 345t^{22} + 165t^{23} + 78t^{24} + 15t^{25} + q^{10}(9130 + 10790t + 9406t^2 + 10126t^3 + 9046t^4 + 9966t^5 + 8170t^6 + 9251t^7 + 7600t^8 + 7876t^9 + 6598t^{10} + 7020t^{11} + 5361t^{12} + 5850t^{13} + 4530t^{14} + 4400t^{15} + 3411t^{16} + 3280t^{17} + 2266t^{18} + 2130t^{19} + 1475t^{20} + 1191t^{21} + 770t^{22} + 555t^{23} + 266t^{24} + 136t^{25} + 40t^{26} + 5t^{27}) + \dots$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 1, 1, 1, 2, 2, 2, 2) \quad f_1:$$

$$1 + q(32 + 30t + 21t^2 + 14t^3 + 20t^4 + 12t^5 + 6t^6 + t^8) + q^2(162 + 172t + 168t^2 + 180t^3 + 115t^4 + 122t^5 + 87t^6 + 70t^7 + 38t^8 + 30t^9 + 15t^{10} + 2t^{11}) + q^3(464 + 432t + 477t^2 + 374t^3 + 411t^4 + 282t^5 + 307t^6 + 252t^7 + 222t^8 + 144t^9 + 102t^{10} + 72t^{11} + 35t^{12} + 12t^{13} + 6t^{14}) + q^4(972 + 1072t + 921t^2 + 970t^3 + 855t^4 + 914t^5 + 719t^6 + 700t^7 + 536t^8 + 460t^9 + 359t^{10} + 302t^{11} + 196t^{12} + 150t^{13} + 73t^{14} + 40t^{15} + 7t^{16}) + q^5(1800 + 1446t + 1584t^2 + 1464t^3 + 1471t^4 + 1240t^5 + 1332t^6 + 978t^7 + 1044t^8 + 864t^9 + 790t^{10} + 544t^{11} + 567t^{12} + 326t^{13} + 257t^{14} + 150t^{15} + 104t^{16} + 42t^{17} + 15t^{18} + 2t^{19}) + q^6(2634 + 3090t + 2622t^2 + 2860t^3 + 2556t^4 + 2712t^5 + 2110t^6 + 2322t^7 + 1887t^8 + 1870t^9 + 1470t^{10} + 1440t^{11} + 993t^{12} + 960t^{13} + 666t^{14} + 562t^{15} + 360t^{16} + 252t^{17} + 116t^{18} + 60t^{19} + 15t^{20}) + q^7(3670 + 3438t + 3968t^2 + 3098t^3 + 3614t^4 + 3034t^5 + 3304t^6 + 2736t^7 + 2766t^8 + 2246t^9 + 2347t^{10} + 1822t^{11} + 1642t^{12} + 1334t^{13} + 1289t^{14} + 816t^{15} + 755t^{16} + 492t^{17} + 373t^{18} + 206t^{19} + 116t^{20} + 42t^{21} + 7t^{22}) + q^8(5862 + 5914t + 5368t^2 + 6210t^3 + 4948t^4 + 5262t^5 + 4896t^6 + 4874t^7 + 4069t^8 + 4422t^9 + 3432t^{10} + 3374t^{11} + 2949t^{12} + 2690t^{13} + 2052t^{14} + 2052t^{15} + 1356t^{16} + 1194t^{17} + 852t^{18} + 580t^{19} + 373t^{20} + 252t^{21} + 104t^{22} + 40t^{23} + 6t^{24}) + q^9(7406 + 6546t + 7380t^2 + 6360t^3 + 7260t^4 + 6174t^5 + 6390t^6 + 5562t^7 + 6150t^8 + 4520t^9 + 5097t^{10} + 4152t^{11} + 4196t^{12} + 3252t^{13} + 3216t^{14} + 2418t^{15} + 2319t^{16} + 1650t^{17} + 1452t^{18} + 1104t^{19} + 852t^{20} + 492t^{21} + 360t^{22} + 150t^{23} + 73t^{24} + 12t^{25}) + q^{10}(8940 + 10932t + 9437t^2 + 10142t^3 + 8974t^4 + 9832t^5 + 8138t^6 + 9202t^7 + 7727t^8 + 7974t^9 + 6586t^{10} + 7092t^{11} + 5488t^{12} + 5762t^{13} + 4454t^{14} + 4250t^{15} + 3420t^{16} + 3282t^{17} + 2311t^{18} + 2200t^{19} + 1452t^{20} + 1194t^{21} + 755t^{22} + 562t^{23} + 257t^{24} + 150t^{25} + 35t^{26} + 2t^{27}) + \dots$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 1, 1, 2, 2, 3) \quad f_5:$$

$$1 + q(42 + 20t + 17t^2 + 32t^3 + 16t^4 + 2t^5 + 10t^6 + 2t^7) + q^2(170 + 192t + 154t^2 + 160t^3 + 122t^4 + 112t^5 + 100t^6 + 80t^7 + 26t^8 + 32t^9 + 16t^{10} + t^{12}) + q^3(452 + 404t + 490t^2 + 374t^3 + 420t^4 + 342t^5 + 288t^6 + 214t^7 + 212t^8 + 162t^9 + 113t^{10} + 52t^{11} + 42t^{12} + 20t^{13} + t^{14}) + q^4(906 + 1152t + 980t^2 + 880t^3 + 833t^4 + 912t^5 + 672t^6 + 720t^7 + 592t^8 + 432t^9 + 362t^{10} + 320t^{11} + 186t^{12} + 160t^{13} + 58t^{14} + 32t^{15} + 16t^{16}) + q^5(1680 + 1396t + 1604t^2 + 1546t^3 + 1498t^4 + 1220t^5 + 1342t^6 + 964t^7 + 1104t^8 + 842t^9 + 740t^{10} + 596t^{11} + 532t^{12} + 276t^{13} + 282t^{14} + 180t^{15} + 112t^{16} + 34t^{17} + 10t^{18} + 2t^{19}) + q^6(2820 + 3040t + 2596t^2 + 2832t^3 + 2536t^4 + 2752t^5 + 2100t^6 + 2400t^7 + 1823t^8 + 1840t^9 + 1432t^{10} + 1440t^{11} + 1018t^{12} + 992t^{13} + 740t^{14} + 512t^{15} + 331t^{16} +$$

$$\begin{aligned}
& 240t^{17} + 116t^{18} + 80t^{19} + 10t^{20}) + q^7(3904 + 3466t + 3860t^2 + 3256t^3 + 3512t^4 + \\
& 2854t^5 + 3376t^6 + 2674t^7 + 2734t^8 + 2296t^9 + 2410t^{10} + 1846t^{11} + 1712t^{12} + 1236t^{13} + \\
& 1242t^{14} + 886t^{15} + 702t^{16} + 534t^{17} + 384t^{18} + 182t^{19} + 116t^{20} + 34t^{21} + 16t^{22}) + \\
& q^8(5642 + 5872t + 5216t^2 + 6272t^3 + 5082t^4 + 5264t^5 + 5088t^6 + 4912t^7 + 4058t^8 + \\
& 4352t^9 + 3456t^{10} + 3344t^{11} + 2932t^{12} + 2752t^{13} + 1968t^{14} + 2000t^{15} + 1354t^{16} + \\
& 1200t^{17} + 864t^{18} + 624t^{19} + 384t^{20} + 240t^{21} + 112t^{22} + 32t^{23} + t^{24}) + q^9(7566 + 6518t + \\
& 7480t^2 + 6202t^3 + 7392t^4 + 6236t^5 + 6229t^6 + 5532t^7 + 5940t^8 + 4838t^9 + 5090t^{10} + \\
& 3934t^{11} + 4096t^{12} + 3400t^{13} + 3278t^{14} + 2378t^{15} + 2420t^{16} + 1720t^{17} + 1486t^{18} + \\
& 900t^{19} + 864t^{20} + 534t^{21} + 331t^{22} + 180t^{23} + 58t^{24} + 20t^{25} + t^{26}) + q^{10}(9092 + 10992t + \\
& 9506t^2 + 10192t^3 + 8824t^4 + 9840t^5 + 8014t^6 + 9104t^7 + 7662t^8 + 7952t^9 + 6836t^{10} + \\
& 7024t^{11} + 5524t^{12} + 5792t^{13} + 4372t^{14} + 4272t^{15} + 3464t^{16} + 3232t^{17} + 2244t^{18} + \\
& 2304t^{19} + 1486t^{20} + 1200t^{21} + 702t^{22} + 512t^{23} + 282t^{24} + 160t^{25} + 42t^{26}) + \dots
\end{aligned}$$

$$\begin{aligned}
& (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0, 0, 0, 1, 1, 1, 4) \quad f_1: \\
& 1 + q(72 + t + 27t^2 + 27t^3 + 27t^5 + t^7 + t^8) + q^2(270 + 72t + 216t^2 + 216t^3 + 27t^4 + 216t^5 + \\
& 27t^6 + 72t^7 + 72t^8 + 27t^{10}) + q^3(720 + 297t + 459t^2 + 459t^3 + 243t^4 + 459t^5 + 217t^6 + \\
& 270t^7 + 270t^8 + 28t^9 + 216t^{10} + 27t^{11} + 27t^{12} + 27t^{13} + t^{15}) + q^4(936 + 936t + 1081t^2 + \\
& 1080t^3 + 675t^4 + 1080t^5 + 531t^6 + 720t^7 + 720t^8 + 288t^9 + 459t^{10} + 216t^{11} + 216t^{12} + \\
& 216t^{13} + t^{14} + 72t^{15} + t^{16}) + q^5(2160 + 1395t + 1422t^2 + 1377t^3 + 1539t^4 + 1350t^5 + \\
& 1350t^6 + 963t^7 + 936t^8 + 729t^9 + 1080t^{10} + 486t^{11} + 459t^{12} + 459t^{13} + 99t^{14} + 270t^{15} + \\
& 72t^{16} + 27t^{17} + 27t^{18}) + q^6(2214 + 3240t + 2862t^2 + 2808t^3 + 2430t^4 + 2592t^5 + \\
& 2070t^6 + 2376t^7 + 2187t^8 + 1800t^9 + 1350t^{10} + 1296t^{11} + 1107t^{12} + 1080t^{13} + 486t^{14} + \\
& 720t^{15} + 270t^{16} + 216t^{17} + 216t^{18} + 27t^{20}) + q^7(3654 + 3564t + 3528t^2 + 3267t^3 + \\
& 3942t^4 + 2836t^5 + 3528t^6 + 2673t^7 + 2430t^8 + 2286t^9 + 2593t^{10} + 1809t^{11} + 1566t^{12} + \\
& 1351t^{13} + 1179t^{14} + 936t^{15} + 747t^{16} + 460t^{17} + 459t^{18} + 54t^{19} + 216t^{20} + 27t^{21} + t^{22} + \\
& t^{23}) + q^8(5022 + 6192t + 5688t^2 + 5832t^3 + 5400t^4 + 5040t^5 + 4968t^6 + 4680t^7 + \\
& 4059t^8 + 4752t^9 + 2880t^{10} + 3672t^{11} + 3051t^{12} + 2664t^{13} + 2016t^{14} + 2160t^{15} + \\
& 1152t^{16} + 1152t^{17} + 1080t^{18} + 432t^{19} + 459t^{20} + 216t^{21} + 72t^{22} + 72t^{23}) + q^9(7470 + \\
& 7398t + 6750t^2 + 5941t^3 + 7560t^4 + 5319t^5 + 6966t^6 + 5940t^7 + 5670t^8 + 5022t^9 + \\
& 5049t^{10} + 4158t^{11} + 3888t^{12} + 3078t^{13} + 3510t^{14} + 2241t^{15} + 2619t^{16} + 1620t^{17} + \\
& 1350t^{18} + 918t^{19} + 1080t^{20} + 460t^{21} + 270t^{22} + 270t^{23} + t^{24} + 27t^{25}) + q^{10}(7344 + \\
& 11304t + 9801t^2 + 10224t^3 + 9342t^4 + 9360t^5 + 8190t^6 + 9144t^7 + 7929t^8 + 8352t^9 + \\
& 5526t^{10} + 7344t^{11} + 6102t^{12} + 5472t^{13} + 4806t^{14} + 3816t^{15} + 3294t^{16} + 3312t^{17} + \\
& 2619t^{18} + 2160t^{19} + 1350t^{20} + 1152t^{21} + 747t^{22} + 720t^{23} + 99t^{24} + 216t^{25} + 27t^{26}) + \dots
\end{aligned}$$

$m = 20$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (1, 1, 1, 1, 2, 2, 2, 2) \quad f_1:$$

$$1 + q(60 + 44t^2 + 33t^4 + 12t^6 + t^8) + q^2(316 + 352t^2 + 253t^4 + 192t^6 + 92t^8 + 32t^{10} + t^{12}) + q^3(928 + 804t^2 + 792t^4 + 536t^6 + 444t^8 + 216t^{10} + 92t^{12} + 12t^{14}) + q^4(1980 + 1984t^2 + 1628t^4 + 1632t^6 + 1057t^8 + 800t^{10} + 444t^{12} + 192t^{14} + 33t^{16}) + q^5(3226 + 2820t^2 + 3036t^4 + 2200t^6 + 2104t^8 + 1456t^{10} + 1057t^{12} + 536t^{14} + 253t^{16} + 44t^{18} + t^{20}) + q^6(5080 + 5952t^2 + 4884t^4 + 4832t^6 + 3732t^8 + 3360t^{10} + 2104t^{12} + 1632t^{14} + 792t^{16} + 352t^{18} + 60t^{20}) + q^7(7992 + 6372t^2 + 6776t^4 + 5916t^6 + 5568t^8 + 3928t^{10} + 3732t^{12} + 2200t^{14} + 1628t^{16} + 804t^{18} + 316t^{20} + 44t^{22}) + q^8(10428 + 11872t^2 + 10208t^4 + 10016t^6 + 8349t^8 + 7808t^{10} + 5568t^{12} + 4832t^{14} + 3036t^{16} + 1984t^{18} + 928t^{20} + 352t^{22} + 33t^{24}) + q^9(14100 + 12828t^2 + 14268t^4 + 11132t^6 + 11664t^8 + 8880t^{10} + 8349t^{12} + 5916t^{14} + 4884t^{16} + 2820t^{18} + 1980t^{20} + 804t^{22} + 253t^{24} + 12t^{26}) + q^{10}(19656 + 20064t^2 + 16857t^4 + 19392t^6 + 14360t^8 + 14976t^{10} + 11664t^{12} + 10016t^{14} + 6776t^{16} + 5952t^{18} + 3226t^{20} + 1984t^{22} + 792t^{24} + 192t^{26} + t^{28}) + \dots$$

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (1, 1, 1, 1, 2, 2, 2, 2) \quad f_2:$$

$$1 + q(24 + 28t + 22t^2 + 24t^3 + 16t^4 + 8t^5 + 6t^6 + 4t^7) + q^2(188 + 164t + 176t^2 + 132t^3 + 133t^4 + 120t^5 + 96t^6 + 64t^7 + 52t^8 + 28t^9 + 16t^{10} + 4t^{11} + t^{12}) + q^3(384 + 468t + 402t^2 + 428t^3 + 360t^4 + 360t^5 + 268t^6 + 264t^7 + 192t^8 + 164t^9 + 108t^{10} + 84t^{11} + 40t^{12} + 24t^{13} + 6t^{14}) + q^4(1116 + 916t + 992t^2 + 924t^3 + 924t^4 + 704t^5 + 816t^6 + 612t^7 + 609t^8 + 468t^9 + 400t^{10} + 288t^{11} + 252t^{12} + 132t^{13} + 96t^{14} + 48t^{15} + 17t^{16} + 4t^{17}) + q^5(1456 + 1628t + 1410t^2 + 1492t^3 + 1312t^4 + 1456t^5 + 1100t^6 + 1216t^7 + 920t^8 + 916t^9 + 728t^{10} + 652t^{11} + 448t^{12} + 428t^{13} + 268t^{14} + 208t^{15} + 120t^{16} + 64t^{17} + 22t^{18} + 4t^{19}) + q^6(2824 + 2688t + 2976t^2 + 2472t^3 + 2724t^4 + 2376t^5 + 2416t^6 + 2052t^7 + 2052t^8 + 1628t^9 + 1680t^{10} + 1308t^{11} + 1184t^{12} + 924t^{13} + 816t^{14} + 536t^{15} + 432t^{16} + 264t^{17} + 176t^{18} + 84t^{19} + 36t^{20} + 4t^{21}) + q^7(3456 + 3732t + 3186t^2 + 3828t^3 + 2960t^4 + 3280t^5 + 2958t^6 + 2892t^7 + 2432t^8 + 2688t^9 + 1964t^{10} + 2036t^{11} + 1680t^{12} + 1492t^{13} + 1100t^{14} + 1080t^{15} + 704t^{16} + 612t^{17} + 402t^{18} + 288t^{19} + 128t^{20} + 84t^{21} + 22t^{22} + 4t^{23}) + q^8(5884 + 5280t + 5936t^2 + 4980t^3 + 5856t^4 + 4840t^5 + 5008t^6 + 4372t^7 + 4781t^8 + 3732t^9 + 3904t^{10} + 3152t^{11} + 3136t^{12} + 2472t^{13} + 2416t^{14} + 1720t^{15} + 1724t^{16} + 1216t^{17} + 992t^{18} + 652t^{19} + 544t^{20} + 288t^{21} + 176t^{22} + 64t^{23} + 17t^{24}) + q^9(6360 + 7428t + 6414t^2 + 6964t^3 + 6144t^4 + 6744t^5 + 5566t^6 + 6288t^7 + 5160t^8 + 5280t^9 + 4440t^{10} + 4728t^{11} + 3568t^{12} + 3828t^{13} + 2958t^{14} + 2864t^{15} + 2160t^{16} + 2052t^{17} + 1410t^{18} + 1308t^{19} + 864t^{20} + 652t^{21} + 402t^{22} + 264t^{23} + 120t^{24} + 48t^{25} + 6t^{26}) + q^{10}(10920 + 8988t + 10032t^2 + 9432t^3 + 9377t^4 + 8536t^5 + 9696t^6 + 7760t^7 + 7920t^8 + 7428t^9 + 7488t^{10} + 5940t^{11} + 6504t^{12} + 4980t^{13} + 5008t^{14} + 4368t^{15} + 3816t^{16} + 2892t^{17} + 2976t^{18} + 2036t^{19} + 1770t^{20} + 1308t^{21} + 992t^{22} + 612t^{23} + 432t^{24} + 208t^{25} + 96t^{26} + 24t^{27} + t^{28}) + \dots$$

$$\begin{aligned}
(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) &= (0, 0, 1, 1, 1, 2, 2, 3) \quad f_3: \\
&1 + q(54 + t + 44t^2 + 3t^3 + 37t^4 - 7t^5 + 12t^6 + 3t^7) + q^2(344 - 2t + 352t^2 - 16t^3 + 236t^4 + \\
&26t^5 + 192t^6 - 7t^7 + 94t^8 + t^9 + 32t^{10} - 2t^{11} + 2t^{12}) + q^3(896 - 9t + 804t^2 + 21t^3 + 810t^4 - \\
&3t^5 + 536t^6 - 12t^7 + 444t^8 - 2t^9 + 216t^{10} + 3t^{11} + 90t^{12} + 3t^{13} + 12t^{14} - t^{15}) + q^4(1956 + \\
&32t + 1984t^2 + 18t^3 + 1628t^4 - 71t^5 + 1632t^6 + 19t^7 + 1073t^8 - 9t^9 + 800t^{10} + 21t^{11} + \\
&444t^{12} - 16t^{13} + 192t^{14} + 3t^{15} + 29t^{16} + 3t^{17}) + q^5(3276 - 29t + 2820t^2 - 41t^3 + 3044t^4 + \\
&25t^5 + 2200t^6 + 52t^7 + 2070t^8 + 32t^9 + 1456t^{10} - 51t^{11} + 1041t^{12} + 21t^{13} + 536t^{14} + \\
&270t^{16} - 7t^{17} + 44t^{18} - 2t^{19} + t^{20}) + q^6(5084 - 24t + 5952t^2 - 11t^3 + 4908t^4 + 66t^5 + \\
&4832t^6 - 21t^7 + 3684t^8 - 29t^9 + 3360t^{10} + 6t^{11} + 2138t^{12} + 18t^{13} + 1632t^{14} + 6t^{15} + \\
&774t^{16} - 12t^{17} + 352t^{18} + 3t^{19} + 66t^{20} - 2t^{21}) + q^7(7992 + 104t + 6372t^2 + 6t^3 + 6660t^4 + \\
&t^5 + 5916t^6 - 101t^7 + 5664t^8 - 24t^9 + 3928t^{10} + 42t^{11} + 3780t^{12} - 41t^{13} + 2200t^{14} - \\
&33t^{15} + 1628t^{16} + 19t^{17} + 804t^{18} + 21t^{19} + 288t^{20} + 3t^{21} + 44t^{22} + 3t^{23}) + q^8(10316 - \\
&105t + 11872t^2 + 50t^3 + 10272t^4 + 44t^5 + 10016t^6 - 56t^7 + 8417t^8 + 104t^9 + 7808t^{10} - \\
&21t^{11} + 5472t^{12} - 11t^{13} + 4832t^{14} - 19t^{15} + 3028t^{16} + 52t^{17} + 1984t^{18} - 51t^{19} + 960t^{20} + \\
&21t^{21} + 352t^{22} - 7t^{23} + 29t^{24} - t^{25}) + q^9(14310 - 39t + 12828t^2 - 17t^3 + 14268t^4 - \\
&21t^5 + 11132t^6 + 51t^7 + 11610t^8 - 105t^9 + 8880t^{10} + 81t^{11} + 8281t^{12} + 6t^{13} + 5916t^{14} + \\
&119t^{15} + 4860t^{16} - 21t^{17} + 2820t^{18} + 6t^{19} + 2004t^{20} - 51t^{21} + 804t^{22} - 12t^{23} + 270t^{24} + \\
&3t^{25} + 12t^{26}) + q^{10}(19656 + 71t + 20064t^2 - 6t^3 + 16884t^4 - 150t^5 + 19392t^6 + 210t^7 + \\
&14230t^8 - 39t^9 + 14976t^{10} - 105t^{11} + 11718t^{12} + 50t^{13} + 10016t^{14} + 6892t^{16} - 101t^{17} + \\
&5952t^{18} + 42t^{19} + 3176t^{20} + 6t^{21} + 1984t^{22} + 19t^{23} + 774t^{24} + 192t^{26} + 3t^{27} + 2t^{28}) + \dots
\end{aligned}$$

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