SYNTAX-DIRECTED TRANSLATION SCHEME OF M-S TRANSLATION IN A LIST PROCESSING LANGUAGE LISP

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(Received September 14, 1970)

The object of the paper is to prove the following theorem.

THEOREM. *There exists a syntax-directed translation scheme $\mathfrak{G}$ $= (\mathfrak{B}, \Sigma, \Delta, \emptyset, \langle M\text{-expression} \rangle)$ of order 2 such that*

$$T(\mathfrak{G}) = \{(w, \tau(w)) \mid (\langle M\text{-expression} \rangle, \langle M\text{-expression} \rangle) \xrightarrow[\mathfrak{G}]{}^* (w, \tau(w))\},$$

where τ is M-S translation in a list processing language LISP ($[1]$).

PROOF. By the theorem 3.2 in $[2; \text{p. 40}]$ it is sufficient to prove that there exists a syntax directed translation scheme of order 3 having the above property. We construct the grammar as follows:

Let $\mathfrak{B}$, Σ and Δ be finite sets of variables, input symbols, and output symbols, respectively.

$$\begin{aligned} \mathfrak{B} \equiv \{ & \langle M\text{-expression} \rangle, \langle function \rangle, \langle form \rangle, \langle L\text{-form} \rangle, \\ & \langle L.F\text{-head} \rangle, \langle L.F\text{-tail} \rangle, \langle C\text{-form} \rangle, \langle C.F\text{-head} \rangle, \\ & \langle C.F\text{-tail} \rangle, \langle identifier \rangle, \langle var.\text{list} \rangle, \langle var.\text{head} \rangle, \\ & \langle var.\text{tail} \rangle, \langle constant \rangle, \langle variable \rangle, \langle argument \rangle, \langle letter \rangle, \\ & \langle id.\text{part} \rangle, \langle empty \rangle, \langle number \rangle, \langle S\text{-expression} \rangle, \\ & \langle S\text{-exp. head} \rangle, \langle S\text{-exp. tail} \rangle, \langle S\text{-exp. 1} \rangle, \langle S\text{-exp. 1 head} \rangle, \\ & \langle S\text{-exp. 1 tail} \rangle, \langle atomic\ symbol \rangle, \langle atom\ part \rangle, \langle LETTER \rangle \} \end{aligned}$$

$$\Sigma \equiv \{a, b, \dots, z, A, B, \dots, Z, 0, 1, \dots, 9, \lambda, label, [,], ;, \rightarrow, (,), \cdot\}$$

$$\Delta \equiv \{A, B, \dots, Z, 0, 1, \dots, 9, (,), \cdot\}.$$

The set of rules $\emptyset$ is shown below.

$\langle M\text{-expression} \rangle \rightarrow (\langle function \rangle, \langle function \rangle)$
 $\langle M\text{-expression} \rangle \rightarrow (\langle form \rangle, \langle form \rangle)$
 $\langle function \rangle \rightarrow (\langle identifier \rangle, \langle identifier \rangle)$
 $\langle function \rangle \rightarrow (\lambda [\langle var. list \rangle; \langle form \rangle],$
 $\quad (LAMBDA \langle var. list \rangle \langle form \rangle))$
 $\langle function \rangle \rightarrow (label [\langle identifier \rangle; \langle function \rangle],$
 $\quad (LABEL \langle identifier \rangle \langle function \rangle))$
 $\langle form \rangle \rightarrow (\langle constant \rangle, \langle constant \rangle)$
 $\langle form \rangle \rightarrow (\langle variable \rangle, \langle variable \rangle)$
 $\langle form \rangle \rightarrow (\langle L\text{-form} \rangle, \langle L\text{-form} \rangle)$
 $\langle form \rangle \rightarrow (\langle C\text{-form} \rangle, \langle C\text{-form} \rangle)$
 $\langle L\text{-form} \rangle \rightarrow (\langle L. F\text{-head} \rangle \langle L. F\text{-tail} \rangle,$
 $\quad \langle L. F\text{-head} \rangle \langle L. F\text{-tail} \rangle)$
 $\langle L. F. head \rangle \rightarrow (\langle function \rangle [\langle argument \rangle,$
 $\quad (\langle function \rangle \langle argument \rangle)$
 $\langle L. F. head \rangle \rightarrow (\langle L. F. head \rangle; \langle argument \rangle,$
 $\quad \langle L. F. head \rangle \langle argument \rangle)$
 $\langle L. F. tail \rangle \rightarrow ([,))$
 $\langle C\text{-form} \rangle \rightarrow (\langle C. F. head \rangle \langle C. F. tail \rangle,$
 $\quad \langle C. F. head \rangle \langle C. F. tail \rangle)$
 $\langle C. F. head \rangle \rightarrow ([\langle form \rangle \rightarrow \langle form \rangle,$
 $\quad (COND (\langle form \rangle \langle form \rangle)$
 $\langle C. F. head \rangle \rightarrow (\langle C. F. head \rangle; \langle form \rangle \rightarrow \langle form \rangle,$
 $\quad \langle C. F. head \rangle) (\langle form \rangle \langle form \rangle)$
 $\langle C. F. tail \rangle \rightarrow ([,))$
 $\langle identifier \rangle \rightarrow (\langle letter \rangle \langle id part \rangle, \langle letter \rangle \langle id part \rangle)$
 $\langle var. list \rangle \rightarrow (\langle var. head \rangle \langle var. tail \rangle,$
 $\quad \langle var. head \rangle \langle var. tail \rangle)$
 $\langle var. head \rangle \rightarrow ([\langle variable \rangle, \langle variable \rangle)$
 $\langle var. head \rangle \rightarrow (\langle var. head \rangle; \langle variable \rangle,$
 $\quad \langle var. head \rangle \langle variable \rangle)$
 $\langle var. tail \rangle \rightarrow ([,))$
 $\langle constant \rangle \rightarrow (\langle S\text{-expression} \rangle, \langle S\text{-expression} \rangle)$
 $\langle variable \rangle \rightarrow (\langle identifier \rangle, \langle identifier \rangle)$
 $\langle argument \rangle \rightarrow (\langle form \rangle, \langle form \rangle)$
 $\langle letter \rangle \rightarrow (a, A)$
 $\langle letter \rangle \rightarrow (b, B)$

$$\begin{aligned}
& \vdots \\
\langle letter \rangle & \rightarrow (z, Z) \\
\langle id. part \rangle & \rightarrow (\langle empty \rangle, \langle empty \rangle) \\
\langle id. part \rangle & \rightarrow (\langle letter \rangle \langle id. part \rangle, \langle letter \rangle \langle id. part \rangle) \\
\langle id. part \rangle & \rightarrow (\langle number \rangle \langle id. part \rangle, \langle number \rangle \langle id. part \rangle) \\
\langle empty \rangle & \rightarrow (,) \\
\langle number \rangle & \rightarrow (0, 0) \\
\langle number \rangle & \rightarrow (1, 1) \\
& \vdots \\
\langle number \rangle & \rightarrow (9, 9) \\
\langle S-expression \rangle & \rightarrow (\langle atomic symbol \rangle, \\
& \quad (QUOTE \langle atomic symbol \rangle), \\
\langle S-expression \rangle & \rightarrow ((\langle S-exp. 1 \rangle \cdot \langle S-exp. 1 \rangle), \\
& \quad (QUOTE (\langle S-exp. 1 \rangle \cdot \langle S-exp. 1 \rangle))) \\
\langle S-expression \rangle & \rightarrow (\langle S-exp. head \rangle \langle S-exp. tail \rangle, \\
& \quad \langle S-exp. head \rangle \langle S-exp. tail \rangle) \\
\langle S-exp. head \rangle & \rightarrow ((\langle S-exp. 1 \rangle, (QUOTE (\langle S-exp. 1 \rangle) \\
\langle S-exp. head \rangle & \rightarrow (\langle S-exp. head \rangle \langle S-exp. 1 \rangle, \\
& \quad \langle S-exp. head \rangle \langle S-exp. 1 \rangle) \\
\langle S-exp. tail \rangle & \rightarrow (,) \\
\langle S-exp. 1 \rangle & \rightarrow (\langle atomic symbol \rangle, \langle atomic symbol \rangle) \\
\langle S-exp. 1 \rangle & \rightarrow ((\langle S-exp. 1 \rangle \cdot \langle S-exp. 1 \rangle), \\
& \quad (\langle S-exp. 1 \rangle \cdot \langle S-exp. 1 \rangle)) \\
\langle S-exp. 1 \rangle & \rightarrow (\langle S-exp. 1 head \rangle \langle S-exp. 1 tail \rangle, \\
& \quad \langle S-exp. 1 head \rangle \langle S-exp. 1 tail \rangle) \\
\langle S-exp. 1 head \rangle & \rightarrow ((\langle S-exp. 1 \rangle, (\langle S-exp. 1 \rangle) \\
\langle S-exp. 1 head \rangle & \rightarrow (\langle S-exp. 1 head \rangle \langle S-exp. 1 \rangle, \\
& \quad \langle S-exp. 1 head \rangle \langle S-exp. 1 \rangle) \\
\langle S-exp. 1 tail \rangle & \rightarrow (,) \\
\langle atomic symbol \rangle & \rightarrow (\langle LETTER \rangle \langle atom part \rangle, \\
& \quad \langle LETTER \rangle \langle atom part \rangle) \\
\langle LETTER \rangle & \rightarrow (A, A) \\
\langle LETTER \rangle & \rightarrow (B, B) \\
& \vdots \\
\langle LETTER \rangle & \rightarrow (Z, Z) \\
\langle atom part \rangle & \rightarrow (\langle empty \rangle, \langle empty \rangle) \\
\langle atom part \rangle & \rightarrow (\langle LETTER \rangle \langle atom part \rangle, \\
& \quad \langle LETTER \rangle \langle atom part \rangle)
\end{aligned}$$

$$\langle atom\ part \rangle \rightarrow (\langle number \rangle \langle atom\ part \rangle, \\ \langle number \rangle \langle atom\ part \rangle)$$

By definition of the translation τ it is obvious that the above grammar has the property described in the theorem except the order of the scheme, which, however, is avoided by the aforementioned theorem in [2; p. 40, theorem 3.2].

References

- [1] S. HUZINO and A. TOGASI: *M-S translation in a list processing language LISP*, Mem. Fac. Sci. Kyushu Univ. Ser. A. Math. **24** (1970), 274-290.
- [2] A. V. AHO and J. D. ULLMAN: *Syntax directed translation and the pushdown assembler*, J. Computer and System Sciences, **3** (1969), 37-56.