CONTINUATION OF HOLOMORPHIC MAPPINGS INTO HYPERBOLIC SPACES

By

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§1. Introduction.

Making use of the results of Shiffman [12], in the previous paper [6] we proved that any holomorphic mapping of a domain (V, ψ) over the complex projective n -space P_n into a complex manifold whose universal covering manifold has a complete hermitian metric with nonpositive holomorphic sectional curvatures, can be continued to the envelope of holomorphy of (V, ψ) .

Fujimoto [3] proved the any holomorphic mapping of a domain (V, ψ) over a Stein manifold into a taut complex space can be continued to the envelope of holomorphy of (V, ψ) .

Recently Kwack [10] generalized the above results of Fujimoto. Namely, any holomorphic mapping of a domain (V, ψ) over a Stein manifold into a hyperbolic space can be continued to the envelope of holomorphy of (V, ψ) .

In the present paper we shall prove that any holomorphic mapping of a domain (V, ψ) over the complex projective n -space P_n into a taut or a hyperbolic complex space can be continued to the envelope of holomorphy of (V, ψ) and extend this result a domain over the cartesian product space of a compact connected manifold and a Stein manifold by using the results of Fujita [5].

§2. Envelopes of holomorphy.

Let M be a complex manifold. If there exists a locally biholomorphic mapping ψ of a complex manifold V into M , the pair (V, ψ) is called an *open set over* M . Moreover, if V is connected, (V, ψ) is called a *domain over* M . Let (V, ψ) and (V', ψ') be open sets over M . A holomorphic mapping λ of V into V' with $\psi = \psi' \circ \lambda$ is called a *mapping of* (V, ψ) *into* (V', ψ') . Consider domains (V, ψ) and (V', ψ') over M with a mapping λ of (V, ψ) into (V', ψ') . Let f be a holomor-

phic mapping of V into a complex manifold X . A holomorphic mapping f' of V' into X with $f=f'\circ\lambda$ is called a *holomorphic continuation* of f to (λ, V', ψ') . Let $\mathfrak{F}$ be a family of holomorphic mappings of V into X . If any holomorphic mapping of $\mathfrak{F}$ has a holomorphic continuation to (λ, V', ψ') , (λ, V', ψ') is called a *holomorphic completion* of (V, ψ) with respect to the family $\mathfrak{F}$. A holomorphic completion $(\tilde{\lambda}_{\mathfrak{F}}, \tilde{V}_{\mathfrak{F}}, \tilde{\psi}_{\mathfrak{F}})$ of (V, ψ) with respect to $\mathfrak{F}$ is called an *envelope of holomorphy* of (V, ψ) with respect to the family $\mathfrak{F}$, if the following conditions are satisfied:

Let (λ', V', ψ') be another holomorphic completion of (V, ψ) with respect to $\mathfrak{F}$. Then there exists a mapping ϕ of (V', ψ') into $(\tilde{V}_{\mathfrak{F}}, \tilde{\psi}_{\mathfrak{F}})$ with $\lambda_{\mathfrak{F}}=\phi\circ\lambda'$ such that $(\phi, \tilde{V}_{\mathfrak{F}}, \tilde{\psi}_{\mathfrak{F}})$ is a holomorphic completion of (V', ψ') with respect to the family $\mathfrak{F}'$ of holomorphic continuations to (λ', V', ψ') of all holomorphic mappings of $\mathfrak{F}$.

If $\mathfrak{F}$ is the family of all holomorphic functions on V , the envelope of holomorphy of (V, ψ) with respect to $\mathfrak{F}$ is called the *envelope of holomorphy* of (V, ψ) . If $\mathfrak{F}$ consists of only one holomorphic mapping f of V into X , the envelope of holomorphy of (V, ψ) with respect to $\mathfrak{F}$ is called the *open set of holomorphy* of f . Considering a connected component of a sheaf, Malgrange [11] proved the unique existence of an envelope of holomorphy. Namely,

LEMMA 1. *Let (V, ψ) be a domain over a complex manifold M , X be a complex manifold and $\mathfrak{F}$ be a family of holomorphic mappings of V into X . Then there exists uniquely an envelope $(\tilde{\lambda}_{\mathfrak{F}}, \tilde{V}_{\mathfrak{F}}, \tilde{\psi}_{\mathfrak{F}})$ of holomorphy of (V, ψ) with respect to $\mathfrak{F}$.*

§3. Continuation of holomorphic mapping.

For complex spaces X and Y , we denote the space of all holomorphic mappings Y into X endowed with the compact-open topology by $Hol(Y, X)$. A sequence $\{f_v\}$ in $Hol(Y, X)$ is said to be *compactly divergent* if for any compact subsets K in Y and L in X , there exists some positive integer v_0 such that $f_v(K) \cap L = \emptyset$ for any $v \geq v_0$. A complex space X is said to be *taut* if for any complex manifold Y , any sequence in $Hol(Y, X)$ has a subsequence which is convergent in $Hol(Y, X)$ or compactly divergent. Fujimoto [3] proved the following theorem.

THEOREM 2. *Let (V, ψ) be a domain over a Stein manifold and f be a holomorphic mapping of V into a taut complex space. Then the domain of holo-*

morphology of f is a Stein manifold.

A relatively compact complex subspace X in a complex space is said to be *strictly Levi pseudoconvex* if for every boundary point p of X there exist a neighborhood U_p of p and a biholomorphic mapping Φ_p of U_p onto a subvariety of a domain D_p in some $\mathbb{C}^n$ and a function ϕ defined in U_p such that $\phi \circ \Phi_p^{-1}$ is the restriction to $\Phi_p(U_p)$ of a strictly pluri-subharmonic function $\tilde{\phi}_p$ defined in D_p and $\Phi_p(U_p \cap X) = \{x \in \Phi_p(U_p); \tilde{\phi}_p(x) < 0\}$. Kwack [10] proved the following theorem.

THEOREM 3. *Let X be one of the following:*

- (i) *X is a hyperbolic (defined by S. Kobayashi [9]) and strictly Levi pseudoconvex subspace of a complex space.*
- (ii) *Every holomorphic mapping of the punctured disk in $\mathbb{C}$ into a complex space X extends holomorphically to a mapping of the whole disk into X . Let (V, ψ) be a domain over a Stein manifold and f be a holomorphic mapping of V into X . Then the domain of holomorphy of f is a Stein manifold.*

An open set (V, ψ) over a complex manifold M is said to be p^* -convex if $\psi^{-1}(S)$ is a Stein manifold for any Stein subdomain S of M .

LEMMA 4. *Let X be one of the following:*

- (i) *X is a taut complex space.*
- (ii) *X is a hyperbolic and strictly Levi pseudoconvex subspace of a complex space.*
- (iii) *Every holomorphic mapping of the punctured disk in $\mathbb{C}$ into a complex space X extends holomorphically to a mapping of the whole disk into X . Let (V, ψ) be a domain over a complex manifold M , f be a holomorphic mapping of V into X and $(\tilde{\lambda}_f, \tilde{V}_f, \tilde{\psi}_f)$ be the domain of holomorphy of f . Then $(\tilde{V}_f, \tilde{\psi}_f)$ is p^* -convex.*

PROOF. Let $\tilde{f}$ be the holomorphic continuation to $(\tilde{\lambda}_f, \tilde{V}_f, \tilde{\psi}_f)$ of f . Let S be any Stein subdomain of M . Let W be a connected component of $\tilde{\psi}_f^{-1}(S)$ and ϕ be the restriction of $\tilde{\psi}_f$ to W . (W, ϕ) can be regarded as a domain over S . Let g be the restriction of $\tilde{f}$ to W and $(\tilde{\mu}, \tilde{W}, \tilde{\phi})$ be the domain of holomorphy of g . From Theorems 2 and 3, $\tilde{W}$ is a Stein manifold. By the same manner as in the previous paper [6], it can be proved that $\tilde{\mu}$ is biholomorphic mapping of W onto $\tilde{W}$. Hence W is a Stein manifold. q. e. d.

THEOREM 5. *Let X be one of the following:*

- (i) X is a taut complex space.
- (ii) X is a hyperbolic and strictly Levi pseudoconvex subspace of a complex space.
- (iii) Every holomorphic mapping of the punctured disk in $\mathbf{C}$ into a complex space X extends holomorphically to a mapping of the whole disk into X . Let (V, ψ) be a domain over the complex projective n -space $\mathbf{P}_n$ and $(\tilde{\lambda}, \tilde{V}, \tilde{\psi})$ be its envelope of holomorphy. Then any holomorphic mapping of V into X can be continued to a holomorphic mapping of $\tilde{V}$ into X .

PROOF. Let f be any holomorphic mapping of V into X and $(\tilde{\lambda}_f, \tilde{V}_f, \tilde{\psi}_f)$ be the domain of holomorphy of f . From Lemma 4 $(\tilde{V}_f, \tilde{\psi}_f)$ is p^* -convex. By the restatement of the theorem of Fujita [4] and Takeuchi [13] due to Kajiwarara [7], either $\tilde{\psi}_f$ is a biholomorphic mapping of $\tilde{V}_f$ onto $\mathbf{P}_n$ or $\tilde{V}_f$ is a Stein manifold. In the former case we have nothing to prove. In the latter case $\tilde{V}_f$ is a domain of holomorphy of a holomorphic function on V by Cartan-Thullen [2]. Since $(\tilde{\lambda}, \tilde{V}, \tilde{\psi})$ is the envelope of holomorphy of (V, ψ) with respect to the family of all holomorphic functions on V , there exists a mapping ϕ of $(\tilde{V}, \tilde{\psi})$ into $(\tilde{V}_f, \tilde{\psi}_f)$ such that $\tilde{\lambda}_f = \phi \circ \tilde{\lambda}$. Let $\tilde{f}$ be the holomorphic continuation of f to $(\tilde{\lambda}_f, \tilde{V}_f, \tilde{\psi}_f)$. Then $\tilde{f} \circ \phi$ is the holomorphic continuation of f to $(\tilde{\lambda}, \tilde{V}, \tilde{\psi})$. q.e.d.

COROLLARY 6. Let (V, ψ) be a domain over $\mathbf{P}_n$ such that ψ is not bijective. V is a Stein manifold if V satisfies one of the following conditions:

- (i) V is taut.
- (ii) V is hyperbolic and strictly Levi pseudoconvex.
- (iii) Every holomorphic mapping of the punctured disk in $\mathbf{C}$ into V extends holomorphically to a mapping of the whole disk into V .

PROOF. The identity mapping $id_V: V \rightarrow V$ has a holomorphic continuation $h: \tilde{V} \rightarrow V$ with $h \circ \tilde{\lambda} = id_V$, where $(\tilde{\lambda}, \tilde{V}, \tilde{\psi})$ is the envelope of holomorphy of (V, ψ) . Since $\tilde{\lambda} \circ h \circ \tilde{\lambda} = \tilde{\lambda}$, $\tilde{\lambda} \circ h$ is the identity mapping of $\tilde{V}$ by the theorem of identity. This shows that V is biholomorphic with $\tilde{V}$, hence V is Stein by the results of Fujita [4] and Takeuchi [13]. q.e.d.

§4. Extension.

We shall state some of the results of Kajiwarara-Kazama [8] concerning a domain over the cartesian product space of a compact connected complex manifold and a connected complex manifold.

PROPOSITION 7. *Let P be a compact connected complex manifold and M be a connected complex manifold. Let (Ω, ψ) be a domain over $Y = P \times M$. Let Q be a submanifold of Ω such that $\psi|_Q$ is a biholomorphic mapping of Q onto $P \times \{x\}$ for a point x of M . Then there exist a neighborhood U of Q in Ω and a neighborhood W of x in M such that $\psi|_U$ is a biholomorphic mapping of U onto $P \times W$.*

PROPOSITION 8. *Let P_1 and P_2 be compact connected complex manifolds and M be a connected complex manifold. Let (Ω, ψ) be a domain over $Y = P_1 \times P_2 \times M$. Let Q_1 be a submanifold of Ω such that $\psi|_{Q_1}$ is a biholomorphic mapping of Q_1 onto $P_1 \times \{p_2\} \times \{x\}$ for a point p_2 of P_2 and a point x of M and that for any point ω of Q_1 there exists a submanifold $Q_2(\omega)$ of Ω containing ω such that $\psi|_{Q_2(\omega)}$ is a biholomorphic mapping of $Q_2(\omega)$ onto $\{\eta(\psi(\omega))\} \times P_2 \times \{x\}$ where $\eta: Y \rightarrow P_1$ is the canonical projection. Then there exists a submanifold Q of Ω such that $\psi|_Q$ is a biholomorphic mapping of Q onto $P_1 \times P_2 \times \{x\}$.*

PROPOSITION 9. *Let $\mathbf{P}_m$ be the projective m -space. Let a and b be positive number with $a < b$. Let (Ω, ψ) be a p^* -convex domain over $\mathbf{P}_m \times \mathbf{C}^n$ such that ψ maps an open subset E of Ω biholomorphically onto $\mathbf{P}_m \times W_a \cup U \times W_b$ where*

$$W_a = \{(z_1, z_2, \dots, z_n) \in \mathbf{C}^n; |z_j| < a \quad (j = 1, 2, \dots, n)\}$$

and U is a non-empty subdomain of $\mathbf{P}_m$. Then there exists a subdomain Q of Ω such that ψ maps Q biholomorphically onto $\mathbf{P}_m \times W_b$.

PROPOSITION 10. *Let M and N be complex manifolds. Let (Ω, ψ) and (Δ, ϕ) be, respectively, domains over $M \times N$ and M such that there exists a biholomorphic mapping λ of Ω onto $\Delta \times N$ with $\psi = (\phi \times 1) \circ \lambda$. If (Ω, ψ) is a p^* -convex domain over $M \times N$, then (Δ, ϕ) is a p^* -convex domain over M .*

Let n be a positive integer. Let $m_1, m_2, \dots, m_n$ be positive integers. Let

$$P = \mathbf{P}_{m_1} \times \mathbf{P}_{m_2} \times \dots \times \mathbf{P}_{m_n}$$

be the cartesian product space of n complex projective spaces. Let S be a connected Stein manifold. Consider the cartesian product space

$$Y = P \times S.$$

Let (Ω, ψ) be a domain over Y . A submanifold Q of Ω is called a *univalent set*

equivalent to $\mathbf{P}_{m_j}$ if $\psi|Q$ is a biholomorphic mapping of Q onto a submanifold

$$\{p_1\} \times \{p_2\} \times \cdots \times \{p_{j-1}\} \times \mathbf{P}_{m_j} \times \{p_{j+1}\} \times \cdots \times \{p_n\} \times \{x\}$$

of Y of dimension m_j for some point $p_k \in \mathbf{P}_{m_k}$ ($k \neq j$) and $x \in S$. Moreover Kajiwar-Kazama [8] restated the theorem of Fujita [5] as follows.

THEOREM 11. *Let (Ω, ψ) be a p^* -convex domain over Y such that Ω does not contain a univalent set equivalent to $\mathbf{P}_{m_j}$ for $j=1, 2, \dots, n$. Then Ω is a Stein manifold.*

LEMMA 12. *Let X be one of the following:*

- (i) *X is a taut complex space.*
- (ii) *X is a hyperbolic and strictly Levi pseudoconvex subspace of a complex space.*
- (iii) *Every holomorphic mapping of the punctured disk in $\mathbf{C}$ into a complex space X extends holomorphically to a mapping of the whole disk into X .*
- (iv) *X is a complex manifold whose universal covering manifold has a complete hermitian metric with nonpositive holomorphic sectional curvatures.*
- (v) *X is a complex Lie group.*

Let (Ω, ψ) be a domain over Y defined as the above. Let f be a holomorphic mapping of Ω into X and $(\tilde{\Omega}_f, \tilde{\Omega}_f, \tilde{\psi}_f)$ be the domain of holomorphy of f . If $\tilde{\Omega}_f$ contains a univalent set equivalent to $\mathbf{P}_{m_j}$, then any point of $\tilde{\Omega}_f$ is contained in a univalent set equivalent to $\mathbf{P}_{m_j}$.

PROOF. We may assume that $j=1$. We put $Z = \mathbf{P}_{m_2} \times \mathbf{P}_{m_3} \times \cdots \times \mathbf{P}_{m_n} \times S$. Let E be the set of all points ω of $\tilde{\Omega}_f$ such that ω is contained in a univalent set equivalent to $\mathbf{P}_{m_1}$. From Proposition 7, E is a non-empty open subset of $\tilde{\Omega}_f$. Let ω be any point of the closure of E . Let $\eta_1: Y \rightarrow \mathbf{P}_{m_1}$ and $\eta': Y \rightarrow Z$ be canonical projections. There exist, respectively, open neighborhoods W, U and V of $\omega, \eta_1(\tilde{\psi}_f(\omega))$ and $\eta'(\tilde{\psi}_f(\omega))$ such that $\tilde{\psi}_f|W$ is a biholomorphic mapping of W onto $U \times V$ and that U and V are coordinate neighborhoods. There exist a point z of V and a univalent set Q equivalent to $\mathbf{P}_{m_1}$ such that $\tilde{\psi}_f|Q$ is a biholomorphic mapping of Q onto $\mathbf{P}_{m_1} \times \{z\}$. From Proposition 7, there exist, respectively, open neighborhoods Q^* and V_0 of Q and z such that $\tilde{\psi}_f|Q^*$ is a biholomorphic mapping of Q^* onto $\mathbf{P}_{m_1} \times V_0$. We may assume that there exists a biholomorphic mapping μ of V onto a polydisk V' in some $\mathbf{C}^N$ such that $\mu(V_0)$ and V' is a polydisk with center the origin. We put $D = Q^* \cup W$ and $\phi = (1 \times \mu^{-1}) \circ$

$(\tilde{\psi}_f|D)$. From $\tilde{\psi}_f(Q^* \cap W) = U \times V_0$, (D, ϕ) can be regarded as a domain over $\mathbf{P}_{m_1} \times \mathbf{C}^N$ such that ϕ is a biholomorphic mapping of D onto $\mathbf{P}_{m_1} \times V_0 \cup U \times V$. Let $\tilde{f}$ be the holomorphic continuation to $(\tilde{\lambda}_f, \tilde{\Omega}_f, \tilde{\psi}_f)$ of f and g be the restriction to D of $\tilde{f}$. Let $(\tilde{\zeta}, \tilde{D}, \tilde{\phi})$ be the envelope of holomorphy of (D, ϕ) with respect to g . If X is one of the hypotheses (i), (ii) and (iii) and if X is one of the hypotheses (iv) and (v), respectively, from Lemma 4 and from [6], $(\tilde{D}, \tilde{\phi})$ is a p^* -convex domain over $\mathbf{P}_{m_1} \times \mathbf{C}^N$. Since ϕ is a biholomorphic mapping of D into $\mathbf{P}_{m_1} \times \mathbf{C}^N$, $\tilde{\zeta}$ is a biholomorphic mapping of D into $\tilde{D}$. From Proposition 9, $\tilde{f} \circ (\tilde{\psi}_f|W)^{-1} \circ (1 \times \mu^{-1})$ is holomorphically continued to $\mathbf{P}_{m_1} \times V$. Since $(\tilde{\lambda}_f, \tilde{\Omega}_f, \tilde{\psi}_f)$ is the domain of holomorphy of f , there exists a biholomorphic mapping τ of $\mathbf{P}_{m_1} \times V$ into $\tilde{\Omega}_f$ such that $\tilde{\psi}_f \circ \tau$ is the identity mapping of $\mathbf{P}_{m_1} \times V$. Any point of $\tau(\mathbf{P}_{m_1} \times V)$ is contained in a univalent set equivalent to $\mathbf{P}_{m_1}$. Since $\tau(\mathbf{P}_{m_1} \times V) \subset W$, $\omega \in E$. Therefore E is a non-empty open and closed set in the connected set $\tilde{\Omega}_f$, and so we have $E = \tilde{\Omega}_f$. q.e.d.

LEMMA 13. *Let X be one of the following:*

- (i) *X is a taut complex space.*
- (ii) *X is a hyperbolic strictly Levi pseudoconvex subspace of a complex space.*
- (iii) *Every holomorphic mapping of the punctured disk in $\mathbf{C}$ into a complex space X extends holomorphically to a mapping of the whole disk into X .*
- (iv) *X is a complex manifold whose universal covering manifold has a complete hermitian metric with nonpositive holomorphic sectional curvatures.*
- (v) *X is a complex Lie group.*

Let (Ω, ψ) be a domain over Y . Let f be a holomorphic mapping of Ω into X and $(\tilde{\lambda}_f, \tilde{\Omega}_f, \tilde{\psi}_f)$ be the domain of holomorphy of f . Assume that $\tilde{\Omega}_f$ contains univalent sets equivalent to $\mathbf{P}_{m_1}, \mathbf{P}_{m_2}, \dots$ and $\mathbf{P}_{m_k}$ and that $\tilde{\Omega}_f$ does not contain univalent sets equivalent to $\mathbf{P}_{m_j}$ for any j with $k+1 \leq j \leq n$. We put $P = \mathbf{P}_{m_1} \times \mathbf{P}_{m_2} \times \dots \times \mathbf{P}_{m_k}$ and $Z = \mathbf{P}_{m_{k+1}} \times \dots \times \mathbf{P}_{m_n} \times X$. Then there exist a Stein manifold (Δ, ϕ) over Z and a biholomorphic mapping ζ of $\tilde{\Omega}_f$ onto $P \times \Delta$ such that $\tilde{\psi}_f = (1 \times \phi) \circ \zeta$.

PROOF. Let ω be a point of $\tilde{\Omega}_f$. We put $(p, z) = \tilde{\psi}_f(\omega)$. From Proposition 8 and Lemma 13 $\tilde{\psi}_f^{-1}(P \times \{z\})$ is a covering manifold of a simply connected manifold $P \times \{z\}$. Hence each $\tilde{\psi}_f$ maps each connected component of $\tilde{\psi}_f^{-1}(P \times \{z\})$ biholomorphically onto $P \times \{z\}$. We shall induce in $\tilde{\Omega}_f$ an equivalence relation R as follows: $\omega_1 \sim \omega_2$ if and only if ω_1 and ω_2 belong to the same connect-

ed component of $\tilde{\psi}_f^{-1}(P \times \{z\})$ for some $z \in Z$. Then $\Delta = \tilde{\Omega}_f/R$ is a complex manifold such that (Δ, ϕ) is a domain over Z where μ is the canonical mapping of $\tilde{\Omega}_f$ onto Δ , ξ is the canonical projection of Y onto Z and ϕ is the canonical mapping of Δ into Z such that $\xi \circ \tilde{\psi}_f = \phi \circ \mu$. Then the mapping ζ defined by

$$\zeta(\omega) = (\eta(\tilde{\psi}_f(\omega)), \mu(\omega))$$

is a biholomorphic mapping of $\tilde{\Omega}_f$ onto $P \times \Delta$ such that $\tilde{\psi}_f = (1 \times \phi) \circ \zeta$ where η is the canonical projection of Y onto P . Since $\tilde{\Omega}_f$ is p^* -convex, Δ is a p^* -convex domain over Z which does not contain a univalent set equivalent to $\mathbf{P}_{m_j}$ for any j with $k+1 \leq j \leq n$ from Proposition 10. Hence from Theorem 11 Δ is a Stein manifold. q.e.d.

MAIN THEOREM. *Let X be one of the following:*

- (i) *X is a taut complex space.*
- (ii) *X is a hyperbolic and strictly Levi pseudoconvex subspace of a complex space.*
- (iii) *Every holomorphic mapping of the punctured disk in $\mathbf{C}$ into a complex space X extends holomorphically to a mapping of the whole disk into X .*
- (iv) *X is a complex manifold whose universal covering manifold has a complete hermitian metric with nonpositive holomorphic sectional curvatures.*
- (v) *X is a complex Lie group.*

Let Y be the cartesian product of a finite number of complex projective spaces $\mathbf{P}_{m_1}, \mathbf{P}_{m_2}, \dots, \mathbf{P}_{m_n}$ and a Stein manifold S . Let (Ω, ψ) be a domain over Y and $(\tilde{\lambda}, \tilde{\Omega}, \tilde{\psi})$ be its envelope of holomorphy. Then any holomorphic mapping of Ω into X can be continued to a holomorphic mapping of $\tilde{\Omega}$ into X .

PROOF. Let f be any holomorphic mapping of Ω into X and $(\tilde{\lambda}_f, \tilde{\Omega}_f, \tilde{\psi}_f)$ be the domain of holomorphy of f . From Lemma 13, after a suitable permutation of $(m_1, m_2, \dots, m_n)$ either $\tilde{\Omega}_f$ is a Stein manifold or there exist an integer k with $1 \leq k \leq n$, a Stein manifold (Δ, ϕ) over $Z = \mathbf{P}_{m_{k+1}} \times \dots \times \mathbf{P}_{m_n} \times X$ and a biholomorphic mapping ζ of $\tilde{\Omega}_f$ onto $P \times \Delta$ such that $\tilde{\psi}_f = (1 \times \phi) \circ \zeta$ where $P = \mathbf{P}_{m_1} \times \mathbf{P}_{m_2} \times \dots \times \mathbf{P}_{m_k}$. In the former case $\tilde{\Omega}_f$ is a domain of holomorphy of a holomorphic function on Ω from Cartan-Thullen [2]. Since $(\tilde{\lambda}, \tilde{\Omega}, \tilde{\psi})$ is the envelope of holomorphy of (Ω, ψ) with respect to the family of all holomorphic functions on Ω , there exists a mapping μ of $(\tilde{\Omega}, \tilde{\psi})$ into $(\tilde{\Omega}_f, \tilde{\psi}_f)$ such that $\tilde{\lambda}_f = \mu \circ \tilde{\lambda}$. Let $\tilde{f}$ be the holomorphic continuation to $(\tilde{\lambda}_f, \tilde{\Omega}_f, \tilde{\psi}_f)$ of f . Then $\tilde{f} \circ \mu$ is the holomorphic continuation to $(\tilde{\lambda}, \tilde{\Omega}, \tilde{\psi})$ of f . In the latter case, $P \times \Delta$ is a domain of

holomorphy from Cartan-Thullen [2], and so is $\tilde{\Omega}_f$. Therefore by the same argument as the preceding case, we obtain a holomorphic continuation to $(\tilde{\lambda}, \tilde{\Omega}, \tilde{\psi})$ of f . q.e.d.

COROLLARY. *Let (Ω, ψ) be a domain over Y . Ω is a Stein if Ω satisfies one of the following conditions:*

- (i) Ω is a taut.
- (ii) Ω is a hyperbolic and strictly Levi pseudoconvex.
- (iii) Every holomorphic mapping of the punctured disk in $\mathbb{C}$ into Ω extends holomorphically to a mapping of the whole disk into Ω .
- (iv) Ω is a complex manifold whose universal covering manifold has a complete hermitian metric with nonpositive holomorphic sectional curvatures.

The proof of Corollary follows from Main Theorem by the argument as Corollary 6.

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