COMPLETE SPACE-LIKE HYPER SURFACES OF 
A DE SITTER SPACE WITH \( r = kH \)

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Abstract

In this paper, we classify complete space-like hypersurfaces in a de Sitter space with non-negative sectional curvature and \( r = kH \), where \( k \) is a nonnegative constant, and \( r \) and \( H \) are the scalar curvature and the mean curvature respectively.

1. Introduction

It is seen that a complete space-like hypersurface of a Minkowski space \( R^{n+1}_1 \) possesses a remarkable Bernstein property in the maximum by Calabi [4] and Cheng and Yau [7]. The Bernstein-type property was also generalized by Nishikawa from a different point of view. Complete space-like hypersurfaces with constant mean curvature of a de Sitter space \( S^{n+1}_1(c) \) were also studied by many authors [2], [5], [6], [10] and so on. Milnor [8] researched the surface in Minkowski space \( R^3_1 \) on which mean curvature \( H \) and Gaussian curvature \( K \) are linearly related. She showed that a space-like surface \( M \) on which \( \alpha + \beta H + \gamma K + 0 \) with \( \beta^2 \neq 4\alpha \gamma \) is equidistant from at least one surface with \( H \) or \( K \) constant.

In this paper, we shall prove the following facts:

**Theorem 1.** Let \( S^{n+1}_1(c) \) be an \((n+1)\)-dimensional de Sitter space of positive constant curvature \( c \). Let \( M \) be a complete space-like hypersurface with non-negative sectional curvature. If \( r = kH(k = \text{const.} \geq 0) \) and \( H \) obtains its maximum on \( M \), then \( M \) is isometric to an Euclidean space \( R^n \) or a sphere \( S^n(c_1) \), \( 0 < c_1 < c \), where \( r \) and \( H \) are the scalar curvature and the mean curvature of \( M \) respectively.

**Theorem 2.** Let \( S^{n+1}_1(c) \) be an \((n+1)\)-dimensional de Sitter space of
positive constant curvature $c$. Let $M$ be a complete space-like hypersurface with non-negative sectional curvature. If $r = kH (k = \text{const.} > 0)$ and the multiplicity of each principal curvature is greater than one, then $M$ is isometric to an Euclidean space $\mathbb{R}^n$ or a sphere $S^n(c_1)$, $0 < c_1 < c$.

2. Preliminaries.

Let $(M', g')$ be an $m$-dimensional indefinite Riemannian manifold of index $s(>0)$. Throughout this paper, manifolds are always assumed to be of class $C^\infty$. We choose a local field of orthonormal frame $e_1, \ldots, e_m$ adapted to the indefinite Riemannian metric in $M'$ and let $\omega_1, \ldots, \omega_m$ denote the dual coframe. Suppose that we have $g'(e_A, e_B) = \varepsilon_A \delta_{AB}$, $\varepsilon_A = \pm 1$ for $A, B, \ldots = 1, \ldots, m$. The connection forms $\{\omega_{AB}\}$ of $M'$ are characterized by the equations

\begin{align}
&d\omega_A + \sum \varepsilon_B \omega_{AB} \wedge \omega_B = 0, \quad \omega_{AB} + \omega_{BA} = 0, \\
&d\omega_{AB} + \sum \varepsilon_C \omega_{AC} \wedge \omega_{CB} = \Omega_{AB}, \\
&\Omega_{AB} = -1/2 \sum \varepsilon_C \varepsilon_D R'_{ABCD} \omega_C \wedge \omega_D,
\end{align}

where $\Omega_{AB}$ (resp. $R'_{ABCD}$) denotes the indefinite Riemannian curvature form (resp. components of the indefinite Riemannian curvature tensor $R'$) of $M'$. The components of the Ricci curvature tensor $\text{Ric}'$ and the scalar curvature $r'$ are defined respectively by

\begin{align}
&R'_{AB} = \sum \varepsilon_C R'_{CABC}, \\
&r' = \sum \varepsilon_A R'_{AA} = \sum \varepsilon_A \varepsilon_C R'_{CAAC}.
\end{align}

An indefinite Riemannian manifold $M'$ of constant sectional curvature is called an indefinite space form of index $s$ if $M'$ is of index $s$. By $M_t^\infty(c)$ an $m$-dimensional indefinite space form of index $s$ and of constant curvature $c$ is denoted. Then the components $R'_{ABCD}$ of the indefinite Riemannian curvature tensor $R'$ for an indefinite space form $M_t^\infty(c)$ are given by

\begin{equation}
R'_{ABCD} = c \varepsilon_A \varepsilon_B (\delta_{AB} \delta_{CD} - \delta_{AC} \delta_{BD}).
\end{equation}

Therefore the Ricci curvature tensor $\text{Ric}'$ and the scalar curvature $r'$ are also given by

\begin{align}
&R'_{AB} = (m-1) c \varepsilon_A \delta_{AB}, \\
r' = m(m-1)c.
\end{align}
In particular, $M^n(c)$ is called a Lorentz space form and it is called a Minkowski space provided that $c = 0$.

Standard models of simply connected Lorentz space forms are given as follows. In an $(n + p)$-dimensional Euclidean space $\mathbb{R}^{n+p}$ with a standard basis, a scalar product $\langle \cdot, \cdot \rangle$ is defined by

$$\langle x, y \rangle = -\sum_{i=1}^{s} x_i y_i + \sum_{s+1}^{n+p} x_i y_i,$$

where $x = (x_1, \ldots, x_{n+p})$ and $y = (y_1, \ldots, y_{n+p})$ are in $\mathbb{R}^{n+p}$. This is a scalar product of index $s$ and the space $(\mathbb{R}^{n+p}, \langle \cdot, \cdot \rangle)$ is an indefinite Euclidean space, which is simply denoted by $\mathbb{R}^{n+p}$. Let $S^{n+1}_{(n+1)}(c)$ be a hypersurface of $\mathbb{R}^{n+2}$ defined by

$$\langle x, x \rangle = 1/c.$$

Then $S^{n+1}_1(c)$ inherits a Lorentz metric from the ambient space $\mathbb{R}^{n+2}$ with constant curvature $c$, which is called a de Sitter space.

From now on, let $M' = M^{n+1}_1(c)$ be an $(n + 1)$-dimensional Lorentz space form of index 1 and of constant curvature $c$. Let $M$ be a positive definite hypersurface of $M^{n+1}_1(c)$ which is said to be space-like. In the sequel, the following convention on the ranges of indices are used, unless otherwise stated: $A, B, \cdots = 0, 1, \ldots, n; i, j, \cdots = 1, 2, \ldots, n$. By restricting the canonical forms $\omega_A$ and the connection forms $\omega_{AB}$ to the hypersurface $M$, they are denoted by the same symbols respectively. Then we have

$$(2.7) \quad \omega_0 = 0.$$

Also $\{e_1, \ldots, e_n\}$ becomes a field of orthonormal frame on $M$. From (2.1) and Cartan's Lemma, it follows that we have

$$(2.8) \quad \omega_{0i} = \sum h_{ij} \omega_j, h_{ij} = h_{ji}.$$

The quadratic form $\sigma = \sum e_i e_j \omega_i \omega_j$ with valued in the normal bundle is called the second fundamental form on $M$, where $e = e_0$, that is.

$$\sigma(e_i, e_j) = e h_{ij} e_0.$$

The scalar $H = \sum h_{ii}/n$ is called the mean curvature of the hypersurface $M$. The connection forms $\{\omega_{ij}\}$ of $M$ are characterized by the structure equations.
\[ d\omega_i + \sum \omega_{ij} \wedge \omega_j = 0, \quad \omega_{ij} + \omega_{ji} = 0. \]  
(2.9) \[ d\omega_{ij} + \sum \omega_k \wedge \omega_{kj} = \Omega_{ij}, \]  
\[ \Omega_{ij} = -(1/2) \sum R_{ijkl} \omega_k \wedge \omega_l, \]  
where \( \Omega_{ij} \) (resp. \( R_{ijkl} \)) denote the Riemannian curvature form (resp. the components of the Riemannian curvature tensor \( R \)) of \( M \). From (2.1) and (2.9) we have the Gauss equations.

(2.10) \[ R_{ijkl} = c(\delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}) - (h_{il} h_{jk} - h_{ik} h_{jl}). \]

The components of the Ricci curvature tensor \( \text{Ric} \) and the scalar curvature \( r \) are given by

(2.11) \[ R_{jk} = c(n - 1)\delta_{jk} - nh_{jk} + \sum h_{jl} h_{lk}. \]

(2.12) \[ r = n(n - 1)c - n^2 H^2 + \sum h_{ij}^2. \]

Now, the covariant derivative \( \nabla \sigma \) of the second fundamental form \( \sigma \) of \( M \) with components \( h_{ijk} \) is given by

(2.13) \[ \sum h_{ijk} \omega_k = dh_{ij} - \sum h_{kj} \omega_{ki} - \sum h_{ik} \omega_{kj}. \]

We have the Codazzi equation

(2.14) \[ h_{ijk} = h_{ikj}. \]

Similarly, the covariant derivative \( \nabla^2 \sigma \) of \( \nabla \sigma \) with components \( h_{ijkl} \) is given by

(2.15) \[ \sum h_{ijkl} \omega_l = dh_{ijk} - \sum h_{jkl} \omega_{li} - \sum h_{ilk} \omega_{lj} - \sum h_{ljk} \omega_{il}. \]

We have the Ricci formula for the second fundamental form:

(2.16) \[ h_{ijkl} - h_{ijlk} = -\sum h_{ri} R_{rjkl} - \sum h_{rj} R_{rik}. \]

Accordingly,

(2.17) \[ \sum h_{ij} \triangle h_{ij} = \sum h_{ij} h_{ijkk} + \sum h_{ij} H_{ij} - \sum h_{ij}(\sum h_{rk} R_{rjik} + \sum h_{rj} R_{rkj}). \]

**Lemma** (see [3] p. 98). Let \( M \) be a complete Riemannian manifold whose Ricci curvature is bounded from below. Let \( f \) be a \( C^2 \)-function which is bounded from above. Then there exists a sequence \( \{q_k\} \) in \( M \) such that

(2.18) \[ \lim f(q_k) = \sup f, \quad \lim \| \nabla f(q_k) \| = 0 \text{ and } \limsup \nabla_i \nabla_j f(q_k) \leq 0, \]
3. The proofs of Theorem 1 and Theorem 2.

First, we give two propositions.

**Proposition 1.** Let $M$ be a space-like hypersurface with non-negative sectional curvature in a de Sitter space $S_1^{n+1}(c)$. If $r = kH (k = \text{const.} > 0)$, the operator

\begin{equation}
L = \Box + (k/2n) \Delta
\end{equation}

is elliptic, $r > 0$ and $H > 0$, where $\Box f = \sum (nH \delta_{ij} - h_{ij})f_{ij}$, $r$ and $H$ are the scalar curvature and the mean curvature respectively.

**Proof.** Because the sectional curvature of $M$ is non-negative, we have the scalar curvature $r \geq 0$. We choose an orthonormal frame field $\{e_j\}$ at each point in $M$ so that

\begin{equation}
h_{ij} = \lambda_i \delta_{ij}.
\end{equation}

Then,

\begin{equation}
r = n(n - 1)c - n^2 H^2 + \sum \lambda_j^2.
\end{equation}

\begin{equation}
\sum \lambda_j^2 = kH + n^2 H^2 - n(n - 1)c.
\end{equation}

Hence, $r > 0$. In fact, if there exists a point $p$ so that $r = 0$, we have $H = 0$ (from $k > 0$). (3.3) implies

\[ \sum \lambda_j^2 + n(n - 1)c = 0. \]

This is impossible. That is, $r > 0$ and $H > 0$.

For any $i$,

\begin{equation}
(nH - \lambda_i + k/2n) = \sum \lambda_j - \lambda_i + (1/2) [\sum \lambda_j^2 - n^2 H^2 + n(n - 1)c]/(nH)
\end{equation}

\[ = [\sum \lambda_j^2 - \lambda_i \sum \lambda_j - (1/2) \sum_{l \neq j} \lambda_l \lambda_j + (1/2)n(n - 1)c]/(nH)^{-1} \]

\[ = [\sum \lambda_j^2 + (1/2) \sum_{l \neq j} \lambda_l \lambda_j - \lambda_i \sum \lambda_j + (1/2)n(n - 1)c]/(nH)^{-1} \]

\[ = [\sum \lambda_j^2 + (1/2) \sum_{l \neq j} \lambda_l \lambda_j + (1/2)n(n - 1)c]/(nH)^{-1} \]

\[ = [\sum \lambda_j^2 + (1/2) \sum_{l \neq j} \lambda_l \lambda_j + (1/2)n(n - 1)c]/(nH)^{-1} \]
Thus $L$ is an elliptic operator.

**Proposition 2.** Let $M$ be a complete Riemannian manifold whose Ricci curvature is bounded from below. Let $f$ be a $C^2$-function which is bounded from above. Then there exists a sequence $\{q_k\}$ such that

\[
\text{(3.6)} \quad \lim f(q_k) = \sup f, \quad \lim \| \nabla f(q_k) \| = 0,
\]

\[
\text{(3.7)} \quad \lim \sup Lf(q_k) \leq 0,
\]

where $Lf = \sum b_{ij}f_{ij}$, $b_j \geq 0$ is bounded.

**Proof.** According to Lemma, we have that there exists a sequence $\{q_k\}$ such that (3.6) and

\[
\lim \sup [\nabla_i \nabla_j f(q_k)] = 0.
\]

are satisfied for all direction $i$. Because $b_j \geq 0$ is bounded, we have that $b_j(q_k)$ is convergent, if necessary, we can take subsequence. Hence

\[
\lim \sup b_j(q_k) \nabla_i \nabla_j f(q_k) = \lim b_j(q_k) \lim \sup \nabla_i \nabla_j f(q_k) \leq 0.
\]

Thus,

\[
\lim \sup Lf(q_k) = \lim \sup \sum b_j(q_k) \nabla_i \nabla_j f(q_k)
\]

\[
= \sum \lim \sup b_j(q_k) \nabla_i \nabla_j f(q_k) \leq 0.
\]

Hence, we complete the proof of Proposition 2.

**Proof of Theorem 1.** Because the sectional curvature is non-negative on $M$, we get $r \geq 0$.

1. when $k = 0$, we have $r \equiv 0$. Hence $M$ is flat and $h_{ij} = \sqrt{c} \delta_{ij}$.

2. when $k > 0$, according to Proposition 1,

\[
L = \Box + (k/2n) \triangle
\]

is an elliptic operator and $r > 0$, $H > 0$.

\[
\text{(3.8)} \quad \frac{1}{2} n^2 \triangle H^2 = n^2 \| \nabla H \|^2 + n^2 H \triangle H.
\]

On the other hand,
(3.9) \[ \frac{1}{2} n^2 \triangle H^2 = \frac{1}{2} \triangle \sum h_{ij}^2 - \frac{1}{2} \triangle r \]
\[ = \sum h_{ijk}^2 + \sum h_{ij} \triangle h_{ij} - \frac{1}{2} \triangle r \]
\[ = \sum h_{ijk}^2 + n \sum \lambda_i H_{ii} + \frac{1}{2} \sum (\lambda_i - \lambda_j)^2 (c - \lambda_i \lambda_j) - \frac{1}{2} \triangle r. \]
(from (2.10) and (2.17))

(3.10) \[ nLH = n[\Box H + (k/2n) \triangle H] \]
\[ = n \Box H + (1/2) \triangle r \]
\[ = n^2 H \triangle H - n \sum \lambda_i H_{ii} + \frac{1}{2} \triangle r \]
\[ = \frac{1}{2} n^2 H \triangle H^2 - n^2 \| \nabla H \|^2 - n \sum \lambda_i H_{ii} + \frac{1}{2} \triangle r \] (from (3.8))
\[ = \sum h_{ijk}^2 - n^2 \| \nabla H \|^2 + \frac{1}{2} \sum (\lambda_i - \lambda_j)^2 (c - \lambda_i \lambda_j). \] (from (3.9)).

According to \( r = kH \) and (2.12), we have

(3.11) \[ k \nabla_i H = - 2 n^2 H \nabla_i H + 2 \sum h_{kj} h_{kij}. \]

(3.12) \[ \left( \frac{k}{2} + n^2 H \right) \| \nabla H \|^2 = \sum (\sum h_{kj} h_{kij})^2 \leq \sum h_{ij}^2 \sum h_{ijk}^2. \]
(from Schwartz inequality).

Hence, (3.10) and (3.12) yield

(3.13) \[ n \sum h_{ij}^2 LH \geq \left[ \left( \frac{k}{2} + n^2 H \right)^2 - n^2 \sum h_{ij}^2 \right] \| \nabla H \|^2 \]
\[ + \frac{1}{2} \sum h_{ij}^2 \sum (\lambda_i - \lambda_j)^2 (c - \lambda_i \lambda_j) \]
\[ = \left[ \frac{k^2}{4} + n^3 (n - 1)c \right] \| \nabla H \|^2 + \frac{1}{2} \sum h_{ij}^2 \sum (\lambda_i - \lambda_j)^2 (c - \lambda_i \lambda_j) \geq 0, \]
here we make use of \( c - \lambda_i \lambda_j \geq 0 \) which is the sectional curvature of \( e_i \) and \( e_j \) direction.

Because \( L \) is elliptic and \( H \) obtain its maximum on \( M \), we know \( H = \text{constant from (3.13).} \) Thus

\[ \sum (\lambda_i - \lambda_j)^2 (c - \lambda_i \lambda_j) = 0, \]
that is, \( \lambda_i - \lambda_j = 0 \) or \( c - \lambda_i \lambda_j = 0. \)

Let \( \lambda_1 = \cdots = \lambda_{n_1} > \lambda_{n_1+1} = \cdots = \lambda_{n_2} > \cdots > \lambda_{n_1+1} = \cdots = \lambda_n. \) We can
prove \( t = 1 \) or \( 2 \). In fact, if \( t > 2 \), we have \( \lambda_{n_1} > \lambda_{n_2} > \lambda_{n_3} \) and \( c - \lambda_{n_1} \lambda_{n_3} = c - \lambda_{n_1} \lambda_{n_3} = 0 \).

\[
\lambda_{n_1} \lambda_{n_2} = \lambda_{n_2} \lambda_{n_3}.
\]

Because \( c \neq 0 \), we have \( \lambda_{n_1} \neq 0 \). Thus \( \lambda_{n_2} = \lambda_{n_3} \). This is a contradiction. Hence, \( t = 1 \) or \( 2 \).

The number of distinct principal curvature is at most 2. By means of the congruence Theorem of Abe, Koike and Yamaguchi [1], it complete the proof of Theorem 1.

**Proof of Theorem 2.** Because the sectional curvature is non-negative on \( M \), we get

\[
c - \lambda_j \lambda_k \geq 0 \quad \text{for} \quad j \neq k,
\]

and the Ricci curvature is bounded from below. Because the multiplicity of each principal curvature is greater than one, we have \( c - \lambda_j^2 \geq 0 \). Hence \( H \) and \( \sum \lambda_j^2 \) are bounded.

1. When \( k = 0 \), \( r = 0 \) and \( M \) is flat. Thus the number of distinct principal curvature is only one.

2. When \( k > 0 \), operator \( L \) is elliptic from Proposition 1. Because \( LH = \sum (nH - \lambda_j + (k/2n))H_{jj} \), we have that \( nH - \lambda_j + (k/2n) > 0 \) is bounded. Accordingly Proposition 2 and (3.13) yield that there exists a sequence \( \{q_k\} \) such that

\[
\lim \sum (\lambda_i - \lambda_j)^2 (c - \lambda_i \lambda_j)(q_k) = 0.
\]

\[
H(q_k) \longrightarrow \sup H.
\]

Thus

\[
\lim (\lambda_i - \lambda_j)^2 (c - \lambda_i \lambda_j)(q_k) = 0 \quad \text{for any} \quad i \quad \text{and} \quad j.
\]

Therefore, we have that there exists a subsequence \( \{q_m\} \), such that

(a) \( (\lambda_i - \lambda_j)(q_m) \longrightarrow 0 \) or \( c - \lambda_i \lambda_j(q_m) \longrightarrow 0 \);

(b) \( (\lambda_i - \lambda_j)(q_m) \longrightarrow 0 \).

In fact, Let \( a_m = (c - \lambda_i \lambda_j)(q_m) \), \( b_m = (\lambda_i - \lambda_j)^2(q_m) \). Two sequence \( \{a_m\} \) and \( \{b_m\} \) are bounded because that \( \lambda_j \) is bounded. (3.15) means that the sequence \( \{a_m b_m\} \) converges to 0 as \( m \) tends to infinity. Suppose that there is a subsequence \( \{a_{m_k}\} \) such that \( a_{m_k} \rightarrow a \neq 0 \). Because of \( a_{m_k} b_{m_k} = a b_{m_k} + b_{m_k} a_{m_k} \)
Thus we obtain either
\[(c - \lambda_i \lambda_j)(q_{m_k}) \to 0 \quad \text{or} \quad (\lambda_i - \lambda_j)(q_{m_k}) \to 0.\]

That is, (a) is true.

Now, for principal curvature \(\lambda_1\) and \(\lambda_2\), we can regard the subsequence \(\{q_{m_k}\}\) in the assertion (a) as a sequence \(\{q_m\}\). Suppose anew \(\lambda_1 \lambda_2(q_m) \to c\). Since \(\{\lambda_1(q_m)\}\) is bounded, it converges to \(\lambda_1^{10}\) by taking the subsequence \(\{q_{m_k}\}\) if necessary. Suppose that \(\lambda_1^{10} = 0\). Then we have
\[|\lambda_1 \lambda_2(q_{m_k})| \leq \lambda_2 |\lambda_1(q_{m_k})|,\]
where \(\wedge_2\) denotes the upper bound of \(\{\lambda_2(q_{m_k})\}\), which implies that \(\lambda_1 \lambda_2(q_{m_k}) \to 0 \neq c\). This is a contradiction. Thus \(\lambda_1^{10} \neq 0\) and hence we have

\[|\lambda_1^{10}[\lambda_2(q_{m_k}) - (c/\lambda_1^{10})]|\]
\[\leq |\lambda_2(q_{m_k})| |\lambda_1(q_{m_k}) - \lambda_1^{10}| + |\lambda_1 \lambda_2(q_{m_k}) - c|.

From which it follows that \(\lambda_2(q_{m_k}) \to c/\lambda_1^{10} = \lambda_20\). Consequently two limits have the same sign, which shows that, without loss of generality, we may suppose that they are positive. The assumption that the multiplicity of all principal curvature is greater than one and the condition of the sectional curvature give us the fact \(c - \lambda_1^2(q_{m_k}) \geq 0\) and \(c - \lambda_2^2(q_{m_k}) \geq 0\). Therefore, \(0 < \lambda_1, \lambda_2 \leq \sqrt{c}\). This coupled with (a) yields that \(\lambda_1(q_m) \to \sqrt{c}, \lambda_2(q_m) \to \sqrt{c}\).

\[
\text{we have } n \sum h_{ij}^2(q_m) - n^2 H^2(q_m) = 2 \sum (\lambda_i - \lambda_j)^2(q_m) \to 0.
\]

That is, \(\sup \sum h_{ij}^2 = n \sup H^2\) from (3.14) and (3.16).
From (3.17) and (b), we have

\[(3.18) \quad \inf(h_{ij}^2 - nH^2) = \frac{2}{n} \inf \sum (\lambda_i - \lambda_j)^2 = 0.\]

According to (3.4), we get

\[\sum h_{ij}^2 - nH^2 = kH + n(n - 1)H^2 - n(n - 1)c.\]

Hence,

\[\inf(\sum h_{ij}^2 - nH^2) = \inf[kH + n(n - 1)H^2 - n(n - 1)c] = 0. \quad \text{(from (3.18))}\]

Thus,

\[k \inf H + n(n - 1)(\inf H)^2 - n(n - 1)c = 0,\]

\[\inf H = \left[ -k \pm \sqrt{k^2 + 4n^2(n - 1)^2 c} \right] / [2n(n - 1)].\]

Because the sectional curvature is non-negative, we have \(r = kH \geq 0\). Thus \(H \geq 0\). Hence,

\[\inf H = \left[ -k + \sqrt{k^2 + 4n^2(n - 1)^2 c} \right] / [2n(n - 1)].\]

On the other hand, according to (3.16) and \(\sup \sum h_{ij}^2 = n \sup H^2\), we have

\[k \sup H + n(n - 1)(\sup H)^2 - n(n - 1)c = 0.\]

Hence,

\[\inf H = \sup H = \left[ -k + \sqrt{k^2 + 4n^2(n - 1)^2 c} \right] / [2n(n - 1)].\]

We obtain that \(H\) is constant. According to Theorem 1, we know that Theorem 2 is true.

References


Complete space-like hypersurfaces


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