Laser Original

Ion Emission from Laser Produced Plasmas of Multi-ion Species

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Many laser produced plasmas contain ions with different charge-to-mass ratios ($Z_1/M_1 \neq Z_2/M_2$). We study here the free expansion into a vacuum of a multi-ion species plasma with both single and two electron temperatures. Numerical solutions for CH_2 and SiO_2 targets are obtained. Analytic asymptotic solutions are obtained for some limiting cases using perturbation analysis. These numerical and asymptotic solutions are compared with experimental fast ion spectra. It is shown that from the ion velocity spectrum important plasma parameters such as the absolute hot and cold electron temperatures, the hot to cold electron temperature ratio, and the hot to cold electron density ratio can be estimated. The energy loss associated with the fast ion is also estimated.

1. Introduction

Recently the problem of energy loss has received considerable attention in the study of plasma expansion into a vacuum. In laser-produced plasma experiments high-energy ions have been detected^{1,2)}. Most of the previous simulation works^{1,3,4)} used a single ion species to model the ion expansion.

Many laser-produced plasmas contain ions with different charge-to-mass ratios ($Z_1/M_1 \neq Z_2/M_2$). Some experiments show that it could be two ion components⁵⁾ which result from target disassembly, for instance CH_2 . The two ion components can be distinguished by the difference in their time-of flight to a charge collecting cup. Ions with velocities greater than 10^8 cm/sec have been observed in a variety of high-intensity laser-target interaction experiments^{6,7)}. Most expansion models^{4,8)} show that the energetic ions are the direct results of the presence of high-energy electrons. The processes can be

deciphered by comparing the experimental ion velocity spectra with theoretical predictions⁹⁾ for the plasma expansion with single electron temperature. The fact that electrons have a non-Maxwellian velocity distribution with a high-velocity tail^{10,11)} was established by measurements of x-ray emission. This distribution can be represented as the superposition of a hot Maxwellian velocity distribution and a cold one. It is well characterized by two electron temperatures¹¹⁾. It is thought that it is the hot electrons which accelerate the fast ions^{8,10)}. The possibility of ion acceleration in a rarefied expanding plasma has been theoretically investigated for the case of a Maxwellian electron distribution^{13,14)}.

Little research¹⁵⁻¹⁷⁾ has been conducted with multispecies ions. Some researchers used perturbations in order to obtain an approximate solution. Here, we have solved the problem to obtain exact numerical solutions. We obtained satisfactory agreement between experimental

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results and both numerical results and asymptotic solutions.

Our physical model can be described as follows:

1) multispecies ion plasma with single electron temperature, and 2) multispecies ion plasma with two electron temperatures. We consider CH_2 and SiO_2 targets as examples of multispecies ion plasmas. The ratio between charge-to-mass for CH_2 and SiO_2 plasmas are different. Under these circumstances of a plasma containing ions with more than one charge-to-mass ratio a multi-ion fluid description is required. We expected that for CH_2 target there would be two different types of ions with different velocity and density.

2. Basic Equations

We consider a one dimensional model varying only in x and t . At $t=0$ the region $x>0$ is assumed to be in vacuum and for times $t>0$ this half space becomes occupied by quasineutral plasma freely expanding from the half space $x<0$ as shown in Fig. 1.

Fig. 1. Schematic of initial conditions for expanding plasma.

The plasma is generally modelled by four fluid species:

a) cold ions of species one, b) cold ions of species two, c) hot electrons and d) cold electrons.

The one dimensional collision-free planar moment equations for conservation of ion momentum and continuity are used, namely

$$\partial v_1 / \partial t + v_1 (\partial v_1 / \partial x) = -(Z_1 e / M_1) (\partial \psi / \partial x), \quad (2.1)$$

$$\partial v_2 / \partial t + v_2 (\partial v_2 / \partial x) = -(Z_2 e / M_2) (\partial \psi / \partial x), \quad (2.2)$$

$$\partial n_1 / \partial t + \partial (n_1 v_1) / \partial x = 0, \quad (2.3)$$

$$\partial n_2 / \partial t + \partial (n_2 v_2) / \partial x = 0, \quad (2.4)$$

where v_1 and v_2 are the velocities of species 1 and 2 ions respectively, φ is the electrostatic potential and n_1 and n_2 are the number densities of ions of species 1 and 2 respectively.

For electrons we assume that each electron species is in equilibrium with the potential φ , which is generated by the expanding plasma. Accordingly they obey the respective Boltzman density distribution, i.e.

$$n_c = n_{c0} \exp(e\varphi / T_c), \quad (2.5)$$

$$n_h = n_{h0} \exp(e\varphi / T_h), \quad (2.6)$$

where n_c and n_h are the number densities of cold and hot electrons respectively, n_{c0} and n_{h0} are the number densities of cold and hot electrons in the source plasma and T_c and T_h are the temperatures for cold and hot electrons in units of energy.

Since the characteristic length for plasma density variations is in general large compared with the Debye-length, the plasma remains quasineutral during the expansions, thus

$$n_e = n_c + n_h = Z_1 n_1 + Z_2 n_2, \quad (2.7)$$

where n_e is the total electron density

3. The Numerical Solutions

The system of the partial differential Eqs. (2.1)~(2.4) can be transformed into a system of ordinary differential equations by assuming that all dependent variables are functions of the self-similar variable $\xi = x/t$ alone. Using Eqs. (2.5) and (2.6), we can then write Eqs. (2.1)~(2.4) as follows

$$(v_1 - \xi)(dv_1/d\xi) + (Z_1 T_c / M_1 n_c)(dn_c/d\xi) = 0, \quad (3.1)$$

$$(v_2 - \xi)(dv_2/d\xi) + (z_c T_c / M_2 n_c)(dn_c/d\xi) = 0, \quad (3.2)$$

$$(v_1 - \xi)(dn_1/d\xi) + n_1(dv_1/d\xi) = 0, \quad (3.3)$$

$$(v_2 - \xi)(dn_2/d\xi) - n_2(dv_2/d\xi) = 0, \quad (3.4)$$

In order to solve these equations numerically, we need one additional differential equation. It can be derived from Eqs. (2.5), (2.6) and (2.7) as follows.

$$\begin{aligned} & Z_1(dn_1/d\xi) + Z_2(dn_2/d\xi) \\ &= (n_c/T_c + n_h/T_h)(T_c/n_c)(dn_c/d\xi). \end{aligned} \quad (3.5)$$

The condition for the non-zero solution of the system of homogeneous Eqs. (3.1)~(3.5) is

$$\begin{aligned} & \left[\frac{(Z_1^2/M_1)n_1}{(v_1 - \xi)^2} + \frac{(Z_2^2/M_2)n_2}{(v_2 - \xi)^2} \right] \\ & \times \frac{1}{(n_c/T_c + n_h/T_h)} = 1. \end{aligned} \quad (3.6)$$

We should mention here that if $Z_1 = Z_2 \equiv Z$ (i.e. one species ion case), and $T_c = T_h \equiv T$ (i.e. single electron temperature case), then $v - \xi = C_s$ where $C_s = (T/M)^{1/2}$ is the ion acoustic speed.

Numerical calculations are performed for the system of Eqs. (3.1)~(3.5). The following boundary conditions are used in the calculations, at $\xi = \xi_0$, $v_1 = v_2 = 0$, $n_1/n_2 = 1/2$ and $n_c = n_{c0}$ ($\varphi = 0$) and $n_h = n_{h0}$ where ξ_0 can be obtained from Eq. (3.6). The numerical results are given in Figs. 2~12. It is interesting that if the electron temperature ratio is equal to or greater than $5 + \sqrt{24}$, then the quasineutral self-similar solution breaks down¹⁸⁾.

On the basis of the numerical results and the discussion pertaining thereto we propose the following inference conclusion. In a CH_2 target the ratio Z/M is twice as large for H^+ as for C^{6+} . This means that the protons are more effectively accelerated than the carbon ions. Thus, for large ξ the proton component should be dominant as shown in Figs. 2a, 3a and 5. For large ξ , the velocity behavior curves for H^+

and C^{6+} are found to be similar, namely, $dv/d\xi$ for H^+ is the same as that for C^{6+} as shown in Figs. 2b, 3b and 7. But the velocity curves are shifted by a constant, and this means that the velocity difference between the two ion species has a constant maximum value which will be shown to be determined by the hot or cold electron temperature.

On the basis of Figs. 5~8 it may be inferred that in a certain region of the self-similar solutions the ions are rapidly accelerated (jump) because in this region the electrostatic potential

Fig. 2. (a), (b) Variations of the normalized density and velocity of different types of ions, $\ln(n/n_{e0})$ and v/C_s , with the normalized self-similar variable ξ/C_s for one electron temperature.

Fig. 3. (a), (b) Variations of the normalized density and velocity of different types of ions, $\ln(n/n_{eo})$ and v/C_{sc} , with the normalized self-similar variable ξ/C_{sc} . The following parameters were taken for the source plasma $T_h/T_c=8$ and $n_{co}/n_{ho}=10$.

φ changes rapidly (see Fig. 9) and since the ions are not in equilibrium with the potential. The depth of the velocity dip is sensitive to the hot-to-cold electron temperature ratio in the source plasma (see Fig. 9). Whereas its position provides an estimate of the cold-to-hot electron density ratio in the source plasma (see the following section Eq. (4.8)). Therefore, from the

Fig. 4. Variations of the normalized cold electron density $\ln(n_c/n_{eo})$ and normalized hot electron density $\ln(n_h/n_{eo})$ with the normalized self-similar variable ξ/C_{sc} . The following parameters were taken for the source plasma $T_h/T_c=8$ and $n_{co}/n_{ho}=10$. Solid curves for CH_2 and dashed curves for SiO_2 .

Fig. 5. Variations of the normalized density of different types of ions $\ln(n/n_{eo})$ with the normalized self-similar variable ξ/C_{sc} , for CH_2 . Solid curves for $T_h/T_c=8$; dashed curves for $T_h/T_c=5$. The values of n_{co}/n_{ho} are shown against the curves.

curves we can estimate the electron temperature ratio and electron density ratio in the source plasma.

Fig. 6. Variations of the normalized density of different types of ions $\ln(n/n_{e0})$ with the normalized self-similar variable ξ/C_{sc} for SiO_2 . The following parameters were taken for the source plasma $T_h/T_c=8$. Solid curves for $n_{co}/n_{ho}=30$ and dashed curves for $n_{co}/n_{ho}=10$.

Fig. 8. Variation of the normalized velocity of different types of ions, v/C_{sc} , with the normalized self-similar variable ξ/C_{sc} , for SiO_2 for the source plasma electron temperature T_h/T_c and density n_{co}/n_{ho} ratios. Solid curves for $T_h/T_c=5$, dashed curves for $T_h/T_c=8$ and the values of n_{co}/n_{ho} shown on the curves.

Fig. 7. Variations of the normalized velocity of different types of ions v/C_{sc} with the normalized self-similar variable ξ/C_{sc} , for CH_2 . The following parameters were taken for the source plasma $T_h/T_c=8$, Solid curves for $n_{co}/n_{ho}=30$ and dashed curves for $n_{co}/n_{ho}=100$.

Fig. 9. Variation of the normalized potential $e\phi/T_c$ with the normalized self-similar variable ξ/C_{sc} , for both CH_2 , the dashed line and SiO_2 the solid line. The parameters are shown against the curves.

Fig. 10. Variation of the normalized single ion density $\ln(n/n_{e0})$ with the normalized self-similar variable ξ/C_{sc} . The following parameters were taken for the source plasma: $T_h/T_c=8$, $n_{co}/n_{ho}=1$ and 10. The values of n_{co}/n_{ho} are shown against the curves.

According to Fig. 10 for the single ion model with two electron temperatures at the singular points ξ_1 , the cold electron density satisfies the condition: $n_c < n_h < n_c \frac{T_h}{T_c}$. This can be understood as follows;

The sound velocity in the two-electron-temperature plasma¹⁹⁾ is given by

$$C_s = \sqrt{\frac{1}{M} \frac{n_c + n_h}{\frac{n_c}{T_c} + \frac{n_h}{T_h}}} \quad (3.7)$$

so that

$$C_s \sim \sqrt{\frac{T_c}{M}} \quad \text{for } n_c > n_h, \quad (3.8)$$

$$C_s \sim \sqrt{\frac{T_c}{M}} \left(1 + \frac{n_h}{n_c}\right) \quad \text{for } n_c < n_h < n_c \frac{T_h}{T_c}, \quad (3.9)$$

$$C_s \sim \sqrt{\frac{T_h}{M}} \quad \text{for } n_h > n_c \frac{T_h}{T_c}. \quad (3.10)$$

and the jump can occur in the region $n_c < n_h < n_c T_h/T_c$.

Figure 4 shows how the cold and hot electron densities vary through the self-similar expansion region. It is natural that for large ξ the hot electron species is dominant.

Since the number of ions $N=n_1+n_2$ at time t_0 in the range $\xi \rightarrow \xi + \delta\xi$ is proportional to $n(\xi)\delta\xi$, we have

$$\frac{dN}{dv} \propto n \frac{d\xi}{dv}, \quad (3.11)$$

Figure 11 shows the ion velocity spectrum computed from Eq. (3.11). Note again that the dip exists for C^{6+} and H^+ . This dip has been detected in experiments and is due to the difference in electron temperature ratio in the source plasma (Fig. 11a). And its position depends on the electron density ratio in the source plasma (Fig. 11b).

Fig. 11. Variation of the normalized ion velocity distribution with the normalized ion velocity for CH_2 plasma. The following parameters were taken for the source plasma.

- a) $n_{co}/n_{ho}=30$, , $T_h/T_c=6$; — , $T_h/T_c=8$; - - - - , $T_h/T_c=9$.
b) $T_h/T_c=9$, , $n_{co}/n_{ho}=10$; — , $n_{co}/n_{ho}=50$; - - - - , $n_{co}/n_{ho}=100$.

4. Asymptotic Solution for Limiting Cases

In this section we discuss some limiting cases for the two target plasmas (SiO_2 and CH_2) using some approximations. These approximate solutions are in good agreement with our numerical solutions and experimental results.

4.1 SiO_2 Case

For the SiO_2 case with two electron temperatures, Eqs. (3.3), (3.4) and (3.6) become

$$(v - \xi)(dn/d\xi) + n(dv/d\xi) = 0, \quad (4.1)$$

$$v - \xi = S, \quad (4.2)$$

where

$$S^2 = (T_c/2M)(n_c + n_h)/(n_c + n_h T_c/T_h) \quad (4.3)$$

and M is the proton mass

First we consider that ξ is close to ξ_0 , where the hot electron number density is assumed to be much smaller than the cold one. Accordingly,

$$v \simeq \xi + C_{sc}/\sqrt{2}, \quad C_{sc} = (T_c/M)^{1/2} \quad (4.4)$$

Equation (4.1) can be integrated and combined with Eq. (4.2) to give

$$n_e \simeq n_c \simeq n_{co} \exp[-(\sqrt{2} \xi/C_{sc} + 1)] \quad (4.5)$$

Secondly, we consider large ξ , where the hot electron number density is much larger than the cold one. Thus we obtain

$$v \simeq \xi + C_{sh}/\sqrt{2}, \quad C_{sh} = (T_h/M)^{1/2} \quad (4.6)$$

Using Eqs. (2.5), (2.6), (2.7) and (4.6), we also obtain

$$\begin{aligned} n_e \simeq n_h &\simeq n_{hb} \exp\left[-\frac{\sqrt{2}}{C_{sh}}(\xi - \xi_b)\right], \\ &\simeq \frac{T_c n_{ho}^2}{T_h n_{co}} \exp\left[-\frac{\sqrt{2}}{C_{sh}}\xi\right], \end{aligned} \quad (4.7)$$

where n_{hb} and ξ_b satisfy the relations

$$n_{hb} \simeq n_{ho} (n_{cb}/n_{co})^{T_c/T_h}$$

and

$$n_{cb} \simeq n_{co} \exp[-\sqrt{2} \xi_b/C_{sc} - 1].$$

As mentioned in the previous section, the jump can occur between the point ξ_a where $n_h \simeq n_c$ and the point ξ_b where $n_h \simeq n_c T_h/T_c$. The points ξ_a and ξ_b are obtained from Eqs. (4.5) and (4.7) as follows;

$$\begin{aligned} \xi_a &\simeq \frac{C_{sc}}{\sqrt{2}} \left[\frac{\ln(n_{co}/n_{ho})}{1 - T_c/T_h} - 1 \right] \\ &\simeq \frac{C_{sc}}{\sqrt{2}} [\ln(n_{co}/n_{ho}) - 1] \end{aligned} \quad (4.8)$$

$$\begin{aligned} \xi_b &\simeq \frac{C_{sc}}{\sqrt{2}} \left[\frac{\ln(T_h n_{co}/T_c n_{ho})}{1 - T_c/T_h} - 1 \right], \\ &\simeq \frac{C_{sc}}{\sqrt{2}} [\ln(T_h n_{co}/T_c n_{ho}) - 1], \end{aligned}$$

These results indicate that in a laser-produced plasma expansion, the singular point in the rarefaction region is sensitive to the cold-to-hot-electron density ratio. This expression is valid for CH_2 too (see for example Fig. 5).

4.2 CH_2 Case

In this case we will now consider the single and two electron temperature cases.

4.2.1 Single electron temperature case

(i) If ξ is close to ξ_0 , i.e. small expansion distances or large times, we assume that $\xi \simeq \xi_0 + \delta\xi$, $v \simeq \delta v$ and $n \simeq n_0 + \delta n$. To first order from Eqs. (2.1), (2.2), (2.3) and (2.4), the velocities and densities are given by:

$$\delta v_H \simeq 2(2/5)^{1/2} C_{sc} (\xi - \xi_0)/\xi_0, \quad (4.9)$$

$$\delta v_c \simeq (2/5)^{1/2} C_{sc} (\xi - \xi_0)/\xi_0 \simeq v_H/2,$$

$$\delta n_H \simeq 2(2/5)^{1/2} n_{ho} C_{sc} (\xi - \xi_0)/\xi_0^2, \quad (4.10)$$

$$\delta n_c \simeq (2/5)^{1/2} n_{co} C_{sc} (\xi - \xi_0)/\xi_0^2. \quad (4.11)$$

$$(4.12)$$

where v_H is the proton velocity, v_c the carbon velocity, n_H the proton density and n_c the carbon density.

As was expected, the velocity of the proton is two times greater than that of the carbon near ξ_0 .

(ii) For large ξ , we can assume that $d(e\varphi/T_c)/d(\xi/C_{sc}) \simeq -1$ and $dv/d\xi \simeq 1$ as shown in Fig. 2b. With these assumptions, we obtain the following;

$$v_H \simeq \xi + C_{sc}, \quad (4.13)$$

$$v_C \simeq \xi + C_{sc}/2, \quad (4.14)$$

$$n_H \simeq n_{H0} \exp[-(\xi/C_{sc})], \quad (4.15)$$

$$n_C \simeq n_{C0} \exp[-(2\xi/C_{sc})]. \quad (4.16)$$

For large ξ , both the proton and carbon velocities are proportional to ξ and the difference between the velocities is constant, $C_{sh}/2$. This property is quite different from that near ξ_0 and it can not be obtained from the perturbation method used in the previous work.

4.2.2 Two electron temperature case

We rewrite Eq. (3.6) in the following way

$$(v_C - \xi)^2 = [C_{sc}/(2(n_C + n_H T_C/T_H))][n_C + n_H - n_H + 2n_H((v_C - \xi)/(v_H - \xi))^2]. \quad (4.17)$$

We note that $(v_C - \xi)/(v_H - \xi) < 1$ and $n_{H0}/n_{C0} < 1$.

(i) If ξ is small, and $n_C > n_H$, namely in the source region of a laser produced plasma, we can obtain the following:

$$v_H \simeq 2\xi + \sqrt{2}C_{sc}, \quad (4.18)$$

$$v_C \simeq \xi + C_{sc}/\sqrt{2} \simeq v_H/2. \quad (4.19)$$

and using Eqs. (3.3) and (3.4), we find

$$n_H \simeq n_{H0} \exp[-(\sqrt{2}\xi/C_{sc} + 1)], \quad (4.20)$$

$$n_C \simeq n_{C0} \exp[-(\sqrt{2}\xi/C_{sc} + 1)]. \quad (4.21)$$

(ii) When ξ is large, i.e. $n_H \gg n_C$ we can assume that $d(e\varphi/T_h)/d(\xi/C_{sh}) \simeq -1$ and $dv_H/d\xi \simeq dv_C/d\xi$ as shown in Figs. 7, 8 and 9. Using Eqs. (3.1), (3.2) and (4.17), we obtain the following;

$$v_H \simeq \xi + C_{sh}, \quad (4.22)$$

$$v_C \simeq \xi + C_{sh}/2. \quad (4.23)$$

From Eqs. (3.3), (3.4) and by virtue of Eq. (4.17), we arrive at the following expressions for the densities

$$n_H \simeq n_{Hb} \exp[-(\xi - \xi_b)/C_{sh}], \quad (4.24)$$

$$\simeq \frac{T_C n_{Ho}}{T_C n_{Co}} n_{Ho} \exp[-(\xi - \xi_b)/C_{sh}],$$

$$n_C \simeq n_{Cb} \exp[-2(\xi - \xi_b)/C_{sh}],$$

$$\simeq \frac{T_C n_{Ho}}{T_h n_{Co}} n_{Co} \exp[-2(\xi - \xi_b)/C_{sh}]. \quad (4.25)$$

The energy loss of the expanding plasma by cold and hot electrons in the whole system is given by

$$\begin{aligned} E &= \sum_{i=1,2} \left[\int_{-C_{st}}^{\xi_b} dx \left(\frac{1}{2} M n_i v_i^2 + \frac{3}{2} n_C T_C + \frac{3}{2} n_H T_H \right) \right. \\ &\quad \left. + \int_{\xi_b}^{\infty} dx \left(\frac{1}{2} M n_i v_i^2 + \frac{3}{2} n_H T_H \right) \right], \\ &= E_{slw} + E_{fast} \end{aligned} \quad (4.26)$$

where

$$\begin{aligned} \frac{E_{slw}}{n_{Co} T_C C_{sc} t} &\simeq \frac{1}{\sqrt{2}} \left[\left(\frac{n_{Ho}}{n_{Co}} + \frac{1}{4} \frac{m_C n_{Co}}{M n_{Co}} \right) \right. \\ &\quad \left. \left\{ 2 - \frac{T_C n_{Ho}}{T_h n_{Co}} \left(\left(\ln \frac{T_h n_{Co}}{T_C n_{Ho}} \right)^2 \right. \right. \right. \\ &\quad \left. \left. + 2 \ln \frac{T_h n_{Co}}{T_C n_{Ho}} + 2 \right) \right\} + \frac{3}{2} \left(1 - \frac{T_C n_{Ho}}{T_h n_{Co}} \right) \right. \\ &\quad \left. + \frac{3}{2} \frac{n_{Ho}}{n_{Co}} \left(\frac{T_h}{T_C} \right)^2 \left(1 - \left(\frac{T_C n_{Ho}}{T_h n_{Co}} \right)^{T_C/T_h} \right) \right] \\ &\simeq \frac{1}{\sqrt{2}} \left[\frac{3}{2} + \frac{5}{4} \left(1 - \frac{T_C n_{Ho}}{T_h n_{Co}} \left\{ \frac{1}{2} \left(\ln \frac{T_h n_{Co}}{T_C n_{Ho}} \right)^2 \right. \right. \right. \right. \\ &\quad \left. \left. + \ln \frac{T_h n_{Co}}{T_C n_{Ho}} + 1 \right\} \right) + \frac{3}{2} \frac{n_{Ho}}{n_{Co}} \left(\frac{T_h}{T_C} \right)^2 \\ &\quad \left. \times \left(1 - \left(\frac{T_C n_{Ho}}{T_h n_{Co}} \right)^{T_C/T_h} \right) \right]. \end{aligned}$$

$$\begin{aligned}
\frac{E_{\text{fast}}}{n_{ho} T_h C_{sh} t} &\simeq \frac{3}{2} \left(\frac{T_c n_{ho}}{T_h n_{co}} \right)^{T_c/T_h} + \frac{5}{8} \frac{T_c}{T_h} \\
&\times \left[1 + \frac{2\sqrt{2}}{5} \sqrt{\frac{T_c}{T_h}} \left(\ln \frac{T_h n_{co}}{T_c n_{ho}} - 1 \right) \right. \\
&+ \frac{1}{10} \frac{T_c}{T_h} \left(\ln \frac{T_h n_{co}}{T_c n_{ho}} - 1 \right)^2 \left. \right] \\
&+ 15 \frac{T_c}{T_h} \left[1 + \frac{4\sqrt{2}}{5} \sqrt{\frac{T_c}{T_h}} \left(\ln \frac{T_h}{T_c} \frac{n_{co}}{n_{no}} - 1 \right) \right. \\
&+ \frac{2}{5} \frac{T_c}{T_h} \left(\ln \frac{T_h n_{co}}{T_c n_{ho}} - 1 \right)^2 \left. \right]
\end{aligned}$$

This energy is required as an external energy supply, so that the isothermal expansion will occur.

Finally, we note that by using the asymptotic solutions for small ξ we can estimate the ion velocity spectrum from the following formulae:

$$\begin{aligned}
f(v_c) &= -\frac{dN}{dv_c} \propto \frac{v_c}{(v_c - C_{sc}/\sqrt{2})^2} \\
&\times \exp[-(\sqrt{2} v_c / C_{sc})], \\
&\simeq \frac{\exp[-(\sqrt{2} v_c / C_{sc})]}{v_c}, \quad (4.27)
\end{aligned}$$

and

$$\begin{aligned}
f(v_H) &= -\frac{dN_H}{dv_H} \propto \frac{v_H}{(v_H - \sqrt{2} C_{sc})^2} \\
&\times \exp[-(v_H / \sqrt{2} C_{sc})], \\
&\simeq \frac{\exp[-(v_H / \sqrt{2} C_{sc})]}{v_H} \quad (4.28)
\end{aligned}$$

and for large ξ

$$\begin{aligned}
f(v_c) &= -\frac{dN_c}{dv_c} \propto \frac{v_c}{(v_c - C_{sh}/2)^2} \exp[-2v_c / C_{sh}] \\
&\simeq \frac{\exp[-2v_c / C_{sh}]}{v_c}
\end{aligned}$$

and

$$\begin{aligned}
f(v_H) &= -\frac{dN_H}{dv_H} \propto \frac{v_H}{(v_H - C_{sh})^2} \exp[-v_H / C_{sh}] \\
&\simeq \frac{\exp[-v_H / C_{sh}]}{v_H} \quad (4.30)
\end{aligned}$$

5. Comparison with Experiment and Discussions

In this section we compare both the numerical and asymptotic solutions with the experimental results of ion emission from laser produced plasma. We found agreement between both numerical and asymptotic solutions and experimental results.

From the experimental results of the fast ions⁷⁾ for a CH₂ target, the two electron temperature case is used (since the intensity of the laser was $1 \sim 5.7 \times 10^{15}$ w/cm²). If $v \sim 2 \times 10^8$ cm/sec and the flight distance, L_{TP} is 70 cm, the time of flight $t_f \simeq L_{TP} / v \simeq 3.5 \times 10^{-2}$ nsec, which is much greater than the pulse duration τ_L .

5.1 Comparison between Numerical Results and Experimental Results

Figure 12 shows plots of ion velocity distribution for two values of density ratio n_{co}/n_{ho} . The black circles represent the summation of the values of C⁶⁺, C⁵⁺, C⁴⁺ and C³⁺ observed by the

Fig. 12. Ion velocity distribution for CH₂ plasma for various values of the cold-to-hot-electron density ratio n_{co}/n_{ho} . The circles represent the experimental results. Solid curves for $n_{co}/n_{ho} = 30$, dashed curves for $n_{co}/n_{ho} = 50$.

experiments. The white circles represent H^+ . Although there exists discrepancy between the numerical distribution and the experimental results in the low velocity region of the proton, the overall spectrum including the dip does coincide with each other.

The numerical solution predicts that the cold electron temperature is $T_c \simeq 1.6$ keV, the hot electron temperature is $T_h \simeq 12.8$ keV and the electron density ratio is $n_{co}/n_{ho} = 30$.

5.2 Comparison between Asymptotic

Solutions and Experimental Results

Using Eqs. (4.22) and (4.23) for large ξ at the jump velocity, we have

$$[(v_H - v_C)]_{\text{jump}} \simeq C_{sh}/2 \quad (5.1)$$

According to this experimental data the absolute hot electron temperature is $T_h \simeq 10$ KeV. And we can estimate the absolute cold-electron temperature from the difference between the carbon velocities at large and small ξ which is obtained by using Eqs. (4.19) and (4.23)

$$[v_C(\xi: \text{large}) - v_C(\xi: \text{small})]_{\text{jump}} \simeq \frac{C_{sh}}{2} - \frac{C_{sc}}{\sqrt{2}}, \quad (5.2)$$

so that, $T_c \simeq 1.3$ KeV.

If we put $\xi = \xi_b$ in Eq. (4.23), using Eq. (4.8), the electron density ratio (n_{co}/n_{ho}) in the source plasma is given by:

$$\ln(n_{co}/n_{ho}) \simeq 1 + \ln(T_c/T_h) + \frac{\sqrt{2}}{C_{sc}} (\xi_0 + v_c - C_{sh}/2) \quad (5.3)$$

If we use $\xi_0 = -C_{sc}$ and $v_c = 2.2 \times 10^8$ cm/sec, then $n_{co}/n_{ho} = 28$.

For large ξ we can get the proton and carbon trajectories as a function of time t by integrating Eqs. (4.22) and (4.23) with respect to time as follows:

$$x_H \simeq t[\xi_0 + C_{sh} \ln(t/t_0)], \quad (5.4)$$

$$x_C \simeq t[\xi_0 + (C_{sh}/2) \ln(t/t_0)] \quad (5.5)$$

The maximum velocities for proton and carbon are given by:

$$v_{H \text{ max.}} \simeq C_{sh} \ln(\tau_L/t_0), \quad (5.6)$$

$$v_{C \text{ max.}} \simeq (C_{sh}/2) \ln(\tau_L/t_0). \quad (5.7)$$

therefore, $v_{H \text{ max.}}/v_{C \text{ max.}} \simeq 2$, and this agrees with the experimental results.

For large ξ the ratio between proton density at maximum velocity to carbon density at maximum velocity is of the same order. By using Eq. (4.26) with $T_h = 10$ keV, $T_c = 1.3$ keV and $n_{co}/n_{ho} = 28$ (or $T_h = 12.8$ keV, $T_c = 1.6$ keV and $n_{co}/n_{ho} = 30$), the energy loss required to sustain the isothermal expansion is estimated to be 6.8×10^{13} W/cm² (or 7.8×10^{13} W/cm²), which is about 5% of the absorbed energy flux.

6. Conclusion

The isothermal expansions of a laser produced plasmas composed of two ions species with different charge-to-mass ratios and with one or two electron temperatures is studied in this paper. When different ion components are present in the expanding plasma the velocity and density essentially depend on the charge-to-mass ratio Z/M . The velocity difference between these two ion types has a maximum value. The numerical and asymptotic solutions show reasonable agreement with the experimental results of ion emission from laser-produced plasmas. It is shown that from asymptotic solutions useful parameters such as the absolute hot electron temperature, the absolute cold electron temperature, and the hot-to-cold electron density ratio can be obtained. Comparison of the theory with the experimental fast ion spectrum shows that the hot electron temperature is about 12 keV, the cold electron temperature is about 1.5 keV, and the cold-to-hot-electron density ratio is about 30. The energy loss associated with the

fast ion is also estimated to be about 5% of the absorbed energy flux. Also we find that the maximum velocity for protons is about two times greater than that for carbons.

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