Laser Original

Magnetically Insulated Inertial Fusion – A Hybrid Fusion Scheme

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An innovative scheme of inertial fusion is introduced in which the plasma pressure is contained by a metallic shell while its heat is contained by a magnetic field. The scheme utilizes a self-generated, force-free magnetic field of $\geq 10^2$ T in a plasma with a density of $\sim 10^{21}$ cm $^{-3}$ and a temperature of ~ 10 keV which are produced by a laser and/or a particle beam in a hard electrically conducting shell. This new scheme can resolve some fundamental difficulties encountered in both magnetic and inertial fusions.

I. INTRODUCTION

Achieving a sufficiently large energy confinement time has been a major obstacle in the controlled thermonuclear fusion research both in inertial fusion and in magnetic fusion. In order to fulfill the break even condition within a limited confinement time, either an extremely large plasma density (in inertial fusion) or an extremely large magnetic container (in magnetic fusion) are required.

In this paper, a novel method is introduced in which merits of these alternative approaches are combined to overcome difficulties in such a way that the break even condition may be achieved with a relatively low density plasma compared with the conventional inertial fusion yet with a size much smaller than a conventional magnetic fusion by utilizing a laser-or beam-generated intense magnetic field. A typical target plasma has a density of $\sim 10^{21}$ cm $^{-3}$, a temperature of ~ 10 keV contained in a hard electrically conductive shell of several millimeter radius in which a force free magnetic field of $\sim 10^2$ T is inbedded for the purpose of insulating the electron heat conduction from the plasma to the shell.

If the thermal insulation by the magnetic field is perfect, that is, if the heat conduction time is

much longer than the inertial confinement time, the expansion speed of the fuel is given by the piston speed $(p/\rho_s)^{1/2}$ for the shock wave which propagates radially outward through the shell. Here p is the plasma pressure and ρ_s is the mass density of the shell.

Consequently, if the plasma mass density is made much less than the shell mass density by a combination of a smaller number density of the plasma and a larger atomic number of the shell material, the inertial confinement time can be enhanced to $\alpha a(p/\rho_s)^{-1/2}$ from $\alpha a/c_s$ of a regular inertial confinement time where α is a numerical constant derived in the text, a is the plasma radius and c_s is the plasma sound speed and ρ_s is the shell mass density. This new inertial confinement time when combined with the ignition condition of $n\tau = 10^{14}$, presents a new scaling on the plasma radius and number density for the ignition given by $a\sqrt{n} \geq \text{const}$ rather than $an \geq \text{const}$ as in the case of the ordinary inertial fusion. This new scaling changes the condition required on the input energy E_i to $E_i \propto na^3 \propto n^{1/2}$ from $E_i \propto n^2$ in the conventional inertial fusion. Therefore, the need for the fuel implosion which is the major obstacle in the conventional inertial fusion becomes less critical. This opens up a new parameter regime in the

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reactor design.

From the point of view of a magnetic fusion, the present scheme is attractive because 1. the pressure gradient driven instability does not exist (since the pressure is uniform) 2. much less portion of energy is expended in the magnetic field (since plasma beta is much larger than one) 3. problems related to the inductive build-up of the magnetic field which increase with the size of the plasma are eliminated and most importantly as a reactor, 4. the wall loading problem is absent (because the shell is disposable).

A dc-operation of a toroidally confined plasma is practically impossible due to plasma disruptions caused by either the thermal instabilities and/or loss of equilibrium at the ignitions as well as to the α -particle accumulations in the plasma. If a pulse operation is thus unavoidable, a smaller scheme is naturally more desirable, in particular a scheme in which the wall loading problem does not exist. Toroidal magnetic fields of larger than 100T (1 MG) have been generated by laser induced electron motions in a relatively large domain (a domain much larger than the laser spot size).¹⁻⁶⁾ The expansion of the domain of the generated magnetic field is most convincingly demonstrated using a computer simulation.⁷⁾ The domain propagates due to the electron $E \times B$ current along the target surface along which the return current flows. Although such magnetic fields cannot contain the plasma pressure typically generated in an inertial fusion, it may be sufficiently large for the purpose of reducing the electron thermal conductivity. Classically, the electron thermal conductivity in the direction perpendicular to a magnetic field is reduced by a factor of $(\omega_{ce} \tau_c)^2$, compared with that in the absent of a magnetic field where ω_{ce} and τ_c are the electron cyclotron frequency and the collision time respectively. This reduction factor, $(\omega_{ce} \tau_c)^2$ becomes approximately 10^{-6} for an example of a plasma with a density of 10^{21} cm^{-3} ,

magnetic field of 10^3 T (10 MG), temperature of 10 keV and for the effective ion charge Z of 3.

Although anomalous effects would reduce this value by a significant factor, the large value of $\omega_{ce} \tau_c$ indicates usefulness of the magnetic field for the purpose of a thermal shield. On the other hand for the same parameter, the plasma beta becomes approximately four hence the magnetic confinement of the plasma pressure may be difficult. However, if the thermal shield of the magnetic field is sufficiently large, and inertial confinement of a plasma by a metallic shell is feasible provided that the explosion time of the shell can be made sufficiently large.

For the magnetic field structure the force-free configuration

$$\nabla \times \mathbf{B} - \lambda \mathbf{B} = 0 \quad (1)$$

is desirable because it represents a minimum energy state⁸⁾ and is known to be stable⁹⁾ for limited ranges of λ . For a spherical shell the magnetic field becomes that of a spheromac¹⁰⁾ where λa is given approximately by 4.5, where a is the inner radius of the conducting shell. The magnetic field configuration given by Eq (1) can be considered as a self-organized state^{11,12)} hence it can be constructed naturally in a plasma through a current driven instability. This property is essential for the present purpose because other complicated field structures are difficult to construct.

A simple scheme of the generation of the toroidal magnetic field inside a conductive shell is shown in Fig.1. Electrons which are knocked out by the laser irradiation drift due to the $E \times B$ drift producing currents which amplify the original field. Ions which are emitted from the wall as well as those produced by the ionization are accelerated by the radial electric field toward the center. However it requires additional process, a process to drive field aligned current, to form a desired

Fig. 1 A simple scheme for generation of the toroidal magnetic field in a conductive shell. Electrons which are knocked out by the laser beam drift due to $E \times B$ drift along the inner surface and produce the current which amplifies and maintains the initial magnetic field. Ions, due to their large Larmor radii, are simply accelerated radially inward producing high temperature region near the core.

force-free configuration. One possibility may be to use a circularly polarized beam. However, we leave this part to future studies.

The required input energy for the ignition depend crucially on the piston speed and influx of cold fuel to the hot core which depend on the amount of preheating of the shell due to bremsstrahlung phonons and/or α -particles. In the absence of the preheating, the required input energy becomes approximately 10 MJ. On the other hand if the piston speed is reduced by a factor of two by the preheating, the input energy needed for the ignition becomes approximately 1 MJ.

We derive the ignition condition in Section II and present examples of the reactor design in

Section III of the proposed scheme.

II. IGNITION CONDITIONS

To achieve the ignition condition for a D-T fusion reaction, the energy confinement time of the plasma should exceed a critical value. This requires that both the magnetic confinement time of the heat and inertial confinement time of the pressure should exceed the critical value. Since the magnetic field is held by the conducting shell, its lifetime is limited by the inertial time of the shell. Hence the ignition condition leads

$$\tau_c < \tau_{in} \leq \tau_H \quad (2)$$

Where τ_c is the critical time of the ignition, τ_{in} is the inertial confinement time and τ_H is the heat confinement time by the magnetic field. We thus discuss the condition on the inertial confinement time and that on the magnetic confinement time separately.

II.a INERTIAL CONFINEMENT TIME

The energy balance equation for a plasma which is expanding radially at a speed of u is given by

$$\begin{aligned} \frac{d}{dt} \int_0^r \left[\frac{p}{\gamma-1} \right] dV + 4\pi r^2 \left(u + \frac{5}{2} u_a \right) p \\ = \int_0^r (E_\alpha n_\alpha \nu_\alpha + \eta J^2 + P_L - P_B) dV \\ + 4\pi r^2 \left[\kappa_\perp \frac{\partial T}{\partial r} \right]. \end{aligned} \quad (3)$$

Here $dV = 4\pi r^2 dr$, $nu_d = -D_n \nabla n$ is the (inward) diffusive particle flux across the expansion boundary, $\kappa_\perp$ is the heat conductivity perpendicular to the magnetic field, p is the plasma pressure, γ ($= 5/3$) is the adiabatic constant, E_α ($= 3.5$ MeV) is the α -particle energy, ν_α is the energy exchange rate from α -particles to plasmas (electrons) η is the plasma electrical resistivity, J is the plasma current density, P_L is the deposited laser and/or beam

power and P_B is the bremsstrahlung radiation loss rate (both in Joule/second/cm³ units). The magnetic field energy is ignored according to the assumption of high beta plasma.

The energy balance equation for α -particles are given by, assuming $E_\alpha = \text{const}$,

$$\frac{d}{dt} \int_0^r n_\alpha dV = \int_0^r \left[\frac{1}{4} n^2 \langle \sigma v \rangle - n_\alpha \nu_\alpha \right] dV + 4\pi r^2 \left[D_\alpha \frac{\partial n_\alpha}{\partial r} \right] \quad (4)$$

Here n is the plasma electron density, $\langle \sigma v \rangle$ is the fusion cross section and D_α is the radial diffusion constant of α -particles.

As will be shown in II.b, with an appropriate choice of the magnetic field, the heat conduction term in Eq. (3) and the α -particle diffusion term in Eq. (4) can be made much smaller than the term representing the inertial confinement time (second term of LHS of Eq. (3)). In addition the Joule heating term is negligible in most cases. Assuming that these conditions are satisfied, the ignition condition may be obtained by integrating Eqs. (3) and (4) over the expanding plasma volume $4\pi r^3/3$ and substituting the resultant equation of (4) to (3)

$$4\pi r^2 \left(u + \frac{3}{2} u_a \right) p = \frac{4\pi r^3}{3} \times \left[\frac{E_\alpha}{4} n^2 \langle \sigma v \rangle + P_L - P_B \right], \quad (5)$$

where the particle diffusion rate across the boundary is assumed to be the same as the heat loss rate due to the cold plasma influx. If the plasma temperature is made sufficiently high (≥ 10 keV) the bremsstrahlung loss is negligible, then the ignition condition for $u_d=0$ simplifies to

$$\tau_{in} = \frac{r}{4u} \geq \frac{8T}{E_\alpha} \frac{1}{n \langle \sigma v \rangle}. \quad (6)$$

Here, use has been made of $p = 2nT$, with T being the plasma temperature. We note that Eq. (6) gives $n\tau_{in} \geq 1.6 \times 10^{14} \text{ s}\cdot\text{cm}^{-3}$ at $T = 10$ keV.

On the other hand if $u_d = -u$ (thereby if n remains constant during the expansion) the ignition condition is given by the requirement that the α -heating rate is larger than P_B and is given by $T \geq 5$ keV. Furthermore, if $u_d = -u$, condition (6) gives $dp/dt > 0$ and the plasma temperature increases locally and enhances Q .

Now, the radial expansion speed $u(t)$ is given by the piston speed for a shock wave which propagates through the shell and is given by¹³⁾

$$u = \left[\frac{p - p_s}{\rho_s} \right]^{1/2}. \quad (7)$$

where p_s is the pressure of the shell. The shell pressure can arise due to the deposition of bremsstrahlung photons radiated from the plasma before the shock is formed. This can reduce (or even invert) the piston velocity according to this expression. However we here obtain the ignition condition without this effect by assuming $p \ll p_s$. Then Eq. (7) gives

$$u = \left[\frac{p}{\rho_s} \right]^{1/2}. \quad (7')$$

Inequality (6) cannot be satisfied at all times because all the quantities vary in time. We note $\langle \sigma v \rangle \approx T^2$ at $10 \text{ keV} \leq T \leq 20 \text{ keV}$, hence the right hand side of (6) varies as p^{-1} , while u varies as $p^{1/2}$. Hence if p varies slower than r^2 the ignition condition is satisfied for that period of time.

Naturally, in the absence of the α -particle heating and the inward diffusion of the plasma density, the plasma expands adiabatically whence p varies as r^{-5} . However, in the presence of α -heating p can vary slower than r^2 for some extended time and the ignition condition can be maintained.

II.b MAGNETIC CONFINEMENT TIME

For the magnetic field to be effective, not only should the energy confinement time be made longer than τ_{in} but also should the lifetime of the magnetic field itself be made longer than τ_{in} . So let

us first derive the lifetime of the magnetic field. For this purpose, we assume that the conductive shell has an infinite mass.

The estimations of the lifetime of the magnetic field is not straightforward when the coupling between the magnetic field and the velocity field is included. The dynamo effects would generate magnetic field, while the magnetic reconnection would dissipate it. However after the minimum energy state is established, the current is everywhere parallel to B hence the reconnection is unlikely to occur. This would lead to a state in which the magnetic field increases due to dynamo effects. Consequently, if we estimate the lifetime of the magnetic field based only on the electromagnetic properties (by ignoring couplings to kinetic properties of the plasma), it would give a most pessimistic lifetime. The lifetime of the magnetic field in this case depends on both the plasma resistivity η and on the resistivity η_c of the conductive shell. If we integrate the energy conservation equations of the magnetic field over the plasma volume, the time rate change of the magnetic field energy W_b becomes

$$\frac{\partial W_b}{\partial t} = -P(x=a) + \int_0^a \eta J^2 dx \quad (8)$$

where $W_b = \int_0^a \frac{B^2}{2\mu_0} dx \simeq \frac{B_t^2 a}{2\mu_0}$ and B_t is

the tangential magnetic field on the conductor surface. P is the Poynting flux given by $E \times H$ on the conductor surface representing the power loss at the wall and

$$\begin{aligned} \int_0^a \eta J^2 dx &= \int_0^a \eta \left[\frac{\lambda}{\mu_0} \right]^2 B^2 dx \\ &= \frac{2 < \eta > \lambda^2}{\mu_0} W_b \end{aligned} \quad (9)$$

represents the Ohmic loss rate in the plasma. Here λ is the eigenvalue of Eq. (1) and $< \eta >$ is the space-averaged plasma resistivity. The Poynting

flux across the conductive shell is given by

$$P(x=0) = \eta_c J_c \frac{B_t}{\mu_0} \quad (10)$$

where J_c is the current density on the conductor surface and B_t is the tangential magnetic field also on the conductor surface. The current density distribution in the conductor is given by the diffusion equation,

$$\frac{\partial J}{\partial t} = \frac{\eta_c}{\mu_0} \frac{\partial^2 J}{\partial x^2} \quad (11)$$

whose solution in response to an initial condition of $J_s \delta(x-a)$ is given by

$$J_0 = \frac{J_s}{\sqrt{2\pi t}} \frac{\mu_0}{\eta_c} \exp \left[-\frac{\mu_0(x-a)^2}{4\eta_c t} \right] \quad (12)$$

where J_s is related to B_t through

$$J_s = \frac{1}{\mu_0} B_t \quad (13)$$

Combining Eqs. (10), (12) and (13), we have

$$P(x=0) = \frac{B_t^2}{\mu_0} \frac{1}{\sqrt{\pi t}} \left[\frac{\eta_c}{\mu_0} \right]^{1/2}. \quad (14)$$

Thus the energy conservation equation becomes,

$$\begin{aligned} \frac{dW_b}{dt} &= -\frac{2}{a\sqrt{\pi t}} \left[\frac{\eta_c}{\mu_0} \right]^{1/2} W_b \\ &\quad - \frac{2 < \eta > \lambda^2}{\mu_0} W_b. \end{aligned} \quad (14')$$

Integrating over t

$$W_b \simeq W_{b0} \exp \left[-\frac{4}{\lambda a \sqrt{\pi}} \left(\frac{\eta_c t}{\eta \tau_M} \right)^{1/2} - \frac{2t}{\tau_M} \right] \quad (15)$$

where $\tau_M (= a^2 \mu_0 / < \eta > (\lambda a)^2)$ is the diffusion time of the bulk magnetic field.

If we assume a copper shell, the conductivity at 10^3 °C is approximately $6 \times 10^{-8} \Omega m$. Hence at $T \simeq 10$ keV, $\eta_c/\eta \simeq 10$ and the e-folding time of the magnetic field energy τ_{M0} is given approximately

from (15) as

$$\tau_{M0} \simeq 0.3 \tau_M \quad (16)$$

This result indicates that the lifetime of the magnetic field is about a third of that of the bulk plasma magnetic field at 10 keV. We naturally require

$$\tau_{in} < \tau_{M0} \quad (17)$$

In addition to the requirement of a longer lifetime of the magnetic field, we must require that the force-free field is set up within a time scale much shorter than τ_{in} .

The time constant τ_R for setting up the force-free field may be estimated to be on the order of or shorter than the reconnection time of the magnetic field, which is given by,⁽¹⁴⁾

$$\tau_R = \frac{\mu_0 a^2}{\eta} S^{-2/5}, \quad (18)$$

where S is the magnetic Reynold's number,

$$S = \frac{a \mu_0 v_A}{\eta} \quad (19)$$

and v_A is the Alfvén speed.

Being satisfied the condition of setting up the magnetic field with sufficiently short start up time and sufficiently long lifetime, the next requirement is to have a sufficiently long heat confinement time of plasma as well as of α -particles. The classical heat conductivities of electrons (κ_{Le}) and ions (κ_{Li}) perpendicular to the magnetic field are given by

$$\chi_{Le} = \frac{4.7 n \rho_e^2}{\tau_e} \quad (20)$$

and

$$\chi_{Li} = \frac{2 n \rho_i^2}{\tau_i} \quad (21)$$

where $\rho_e (= v_{Te}/\omega_{ce})$ and $\rho_i (= v_{Ti}/\omega_{ci})$ are the electron and ion Larmor radius, v_{Tj} and ω_{cj} ($j = i, e$) are the thermal speeds and the cyclotron frequencies. τ_e and τ_i are the electron and ion collision time given by

$$\tau_e = 3.44 \times 10^5 \frac{T_e^{3/2}}{n \ln \theta_e} \quad (22)$$

$$\tau_i = 2.09 \times 10^7 \frac{T_i^{3/2}}{n \ln \theta_i}, \quad (23)$$

where $\ln \theta_j$ and T_j are the Coulomb log (≈ 7) and the temperatures of each species. We note that $\kappa_{Le}/\kappa_{Li} \approx (m_e/m_i)^{1/2}$. The classic heat confinement time τ_H is then given by

$$\tau_{Hj} = \frac{n a^2}{\chi_{\perp j}}, \quad j = i, e. \quad (24)$$

Experiments in magnetic fusion, however, have revealed that the energy confinement time of electrons is much shorter than the classic value and is more or less given by the ion confinement time in magnitude. The scaling, however, varies differently depending on different machines. Some of the well-cited scalings are, the Bohm scaling which is given by

$$\tau_B = \frac{16 a^2 e B_0}{T_e} \quad (25)$$

or the Alcator scaling given by

$$\tau_A = 1.3 \times 10^{-21} n R^2 a \quad (26)$$

where R is the major radius.

The confinement time of the α -particles $\tau_{H\alpha}$ is given by

$$\tau_{H\alpha} = \frac{a^2}{D_\alpha} = \left[\frac{a}{\rho_\alpha} \right]^2 \tau_\alpha \quad (27)$$

where ρ_α is the Larmor radius of the α -particle and

τ_α is the α -collision time given by Eq. (23) with $T_i = E_\alpha$. For the magnetic field to be effective, we require

$$\tau_{in} < \tau_{M0} < \tau_\alpha \quad (28)$$

as well as

$$\tau_{in} < \tau_{M0} < \tau_{Hm} \quad (29)$$

where τ_{Hm} is the minimum of various heat confinement time among Eqs. (24), (25) and (26) and possibly other unknown scaling.

III. REACTOR DESIGN

We are now ready to design a reactor based on the present scheme. For this purpose we first list various plasma parameters in terms of the reduced units of the density n in 10^{21} cm^{-3} (the cut off density of a glass laser), temperature $\bar{T}$ in keV, magnetic field $\bar{B}$ in 100 T (1 MG) and the plasma radius $\bar{a}$ in mm,

1. plasma to magnetic field energy ratio, β :

$$\beta = 80 \frac{\bar{n} \bar{T}}{\bar{B}^2}, \quad (30)$$

2. Proton cyclotron frequency, ω_{ci} :

$$\omega_{ci} = 10^{10} \bar{B} (\text{sec}^{-1}), \quad (31)$$

3. proton Larmor radius, ρ_i :

$$\frac{\rho_i}{\bar{a}} = 3.2 \times 10^{-2} \frac{\bar{T}^{1/2}}{\bar{a} \bar{B}} \quad (32)$$

4. α -particle Larmor radius at 3.5 MeV, ρ_α :

$$\frac{\rho_\alpha}{\bar{a}} = 2.7 \frac{1}{\bar{B} \bar{a}} \quad (33)$$

5. electron skin depth, $\lambda_s = c/\omega_{pe}$:

$$\frac{\lambda_s}{\bar{a}} = 1.7 \times 10^{-4} \frac{1}{\bar{n}^{1/2} \bar{a}}, \quad (34)$$

6. electron collision time, τ_e :

$$\tau_e = 3.7 \times 10^{-12} \frac{\bar{T}^{3/2}}{\bar{n} \bar{Z}^2} (\text{sec}), \quad (35)$$

where Z is the effective charge of ions

7. electron cyclotron frequency, ω_{ce} :

$$\omega_{ce} = 1.8 \times 10^{13} \bar{B} (\text{sec}^{-1}), \quad (36)$$

and

$$\omega_{ce} \tau_e = 6.6 \times 10 \frac{\bar{T}^{3/2} \bar{B}}{\bar{n} \bar{Z}^2} \quad (37)$$

8. magnetic Reynold's number (ratio of the magnetic skin time and the Alfvén time), $S = a\mu_0 V_A/\eta$:

$$S = 3.8 \times 10^3 \frac{\bar{a} \bar{T}^{3/2} \bar{B}}{\bar{n}^{1/2} \bar{Z}^2}, \quad (38)$$

9. Ohmic heating time, $\tau_\Omega = \frac{2nT}{\eta J^2}$.

$$\tau_\Omega = 2.6 \times 10^{-4} \frac{\bar{n} \bar{a}^2 \bar{T}^{5/2}}{\bar{B}^2 \bar{Z}^2} (\text{sec}), \quad (39)$$

10. Bohm diffusion time, $\tau_B = \frac{16a^2 e B_c}{T_e}$

$$\tau_B = 1.6 \times 10^{-6} \frac{\bar{a}^2 \bar{B}}{\bar{B}} (\text{sec}), \quad (40)$$

11. Plasma resistivity, η :

$$\eta = 2 \times 10^{-8} \frac{\bar{Z}^2}{\bar{T}^{3/2}} (\Omega \text{ m}), \quad (41)$$

12. diffusion time of the magnetic field, $\tau_M =$

$$\tau_M = \frac{a^2 \mu_0}{\langle \eta \rangle (\lambda a)^2}$$

$$\tau_M = 6.3 \times 10^{-5} \frac{\bar{a}^2 \bar{T}^{3/2}}{\bar{Z}^2} \frac{1}{(\lambda a)^2} (\text{sec}), \quad (42)$$

where $\lambda a \approx 2.5$ for a reversed field pinch configuration and $\lambda a = 4.5$ for a spheromac configuration,

13. electron mean-free distance, $\lambda_m = v_T/v_{ei}$:

$$\frac{\lambda_m}{a} = 2 \times 10^{-2} \frac{\bar{T}^2}{\bar{n} Z^2 a}, \quad (43)$$

14. electron and ion heat confinement time based on classic perpendicular heat conductivity,

$$\tau_{Hj}^c = \frac{n a^2}{\kappa_{\perp j}} :$$

$$\tau_{He}^c = 3.3 \times 10^{-6} \frac{\bar{T}^{1/2} \bar{B}^2 \bar{a}}{\bar{n} Z^2} \quad (\text{sec}), \quad (44)$$

$$\tau_{Hi}^c = 6.4 \times 10^{-8} \frac{\bar{T}^{1/2} \bar{B}^2 \bar{a}^2}{\bar{n} Z^2} \quad (\text{sec}), \quad (45)$$

15. total plasma energy $E_i = 3 nTV$:

$$E_i = 4 \pi n T a^3 = 2 \times 10^3 \bar{n} \bar{T} \bar{a}^3 \quad (\text{Joule}), \quad (46)$$

for a spheromac

$$E_i = 6 \pi^2 a^3 n T = 1 \times 10^4 \bar{n} \bar{T} \bar{a}^3 \quad (\text{Joule}), \quad (47)$$

for an RFP,

16. α -particle confinement time, $\tau_{H\alpha} = \left(\frac{a}{\rho_\alpha}\right)^2 \tau_\alpha$:

$$\tau_{H\alpha} = 3.8 \times 10^{-6} \frac{\bar{B}^2 \bar{a}^2}{\bar{n}} \quad (\text{sec}), \quad (48)$$

17. Alcator scaling of energy confinement time, τ_A

$$= 1.3 \times 10^{-21} n a^3.$$

$$\tau_A = 1.3 \times 10^{-3} \bar{n} \bar{a}^2 \quad (\text{sec}), \quad (49)$$

18. expansion speed of the plasma, $u = (p/\rho_s)^{1/2}$:

$$u = 4.2 \times 10^6 \bar{n}^{1/2} \bar{T}^{1/2} \quad (\text{mm/sec}), \quad (50)$$

for a copper shell

$$u = 2.9 \times 10^6 \bar{n}^{1/2} \bar{T}^{1/2} \quad (\text{mm/sec}), \quad (51)$$

for a tungsten shell

$$19. \text{inertial confinement time, } \tau_{in} = \frac{a}{4u} \\ = \frac{a}{4} \left(\frac{\rho_s}{p} \right)^{1/2} :$$

$$\tau_{in} = 4.5 \times 10^{-8} \frac{\bar{a}}{\bar{n}^{1/2} \bar{T}^{1/2}} \quad (\text{sec}) \quad (52)$$

for a copper shell

$$\tau_{in} = 8 \times 10^{-8} \frac{\bar{a}}{\bar{n}^{1/2} \bar{T}^{1/2}} \quad (\text{sec}) \quad (53)$$

for a tungsten shell,

20. magnetic reconnection time,

$$\tau_R = \frac{a^2 \mu_0}{\eta} \frac{1}{S^{2/5}} \frac{1}{(\lambda a)^2} \\ = 2.3 \times 10^{-6} \frac{\bar{a}^{8/5} \bar{T}^{9/10} \bar{n}^{1/5}}{\bar{B}^{2/5} \bar{Z}^{6/5} (\lambda a)^2} \quad (54)$$

Having obtained these parameters, let us first study requirements on parameters to achieve the ignition condition. If the shell pressure remains lower than the plasma pressure before the shock wave passes through, that is, if the heating rate by photon deposition of the bremsstrahlung radiation or that by the α -particle from the plasma on the shell is not fast enough, the expansion speed of the fuel is decided by the piston speed given by Eq. (7'). The ignition condition for $u_d = 0$ is then obtained from Eqs. (6) and (52) or (53). For example, if a tungsten shell is used, the condition is given by

$$\bar{a} \sqrt{\bar{n}} = 0.28 \frac{\bar{T}^{3/2}}{\langle \sigma v \rangle} \quad (55)$$

where $\langle \sigma v \rangle [= 10^{16} \langle \sigma v \rangle \text{ in cm}^3/\text{sec}] = 1.1$ for 10 keV = 4.2 for 20 keV.

Equation (55) gives $(a\sqrt{n})$ scaling rather than (an) scaling in the usual inertial fusion. $(a\sqrt{n})$ becomes 8.1 at 10 keV and 6.0 at 20 keV.

This means that the radius of the plasma required in the ignition is on the order of several millimeters for the plasma density on the order of 10^{21} cm^{-3} . The input energy E_i required for 10 keV temperature plasma becomes approximately 10 MJ from Eq. (46). However if $u_d = -u$, the ignition occurs with arbitrarily small input energy so long as $T \geq 5 \text{ keV}$ is achieved.

The required input power p_i is given by E_i/t_i and becomes 72 TW, which is a level of power currently available in a large laser system.

The input energy is a very sensitive function of the plasma radius. Since the inertial confinement time is proportional to the ratio of the radius to the expansion speed, if the speed can be reduced, the plasma radius can be proportionally reduced. As can be seen in Eq. (7) the reduction of the piston speed is achievable by an increase of the shell pressure due to a pre-heating of the shell. Since the bremsstrahlung radiation carries energy which is fraction of the input energy, if it is deposited into a shell volume which is much smaller than the plasma volume, the shell pressure can become comparable or even higher than the plasma pressure which either slows down the expansion speed or even reverse the expansion direction. The α -particles of the fusion product can also play the similar role. (Since plasma mass density is smaller than the shell mass density, the imploding shock wave is not formed).

If the exploding speed is thus reduced, say, by a factor of two, the required input energy becomes approximately 1 MJ. To find how much reduction in input energy is possible requires experimental and computational search for various parameters.

Let us now derive the gain factor Q of the present scheme. Since the ignition condition derived here is not the breakeven conditions, the achieving of the ignition automatically warrants a gain of larger than unity. If we eliminate v_α from

Eqs. (3) and (4) and ignore D_α , κ , P_L and P_B , we have

$$\begin{aligned} \frac{d}{dt} \int_0^r \left[\frac{p}{\gamma-1} + E_\alpha n_\alpha \right] dV \\ = \int_0^r \frac{1}{4} n^2 E_\alpha < \sigma v > dV - 4\pi r^2 u p. \end{aligned} \quad (56)$$

If we note that $< \sigma v >$ is approximately proportional to T for $10 \text{ keV} < T < 20 \text{ keV}$, while u is proportional to $p^{1/2}$, Eq. (56) shows that once the breakeven condition is achieved at, say 10 keV the first term on the right hand side which is proportional to p^2 dominates over the second term which is proportional to $p^{3/2}$ so long as $p \sim r^{-2}$ and both n_α and p increases until the increase in $< \sigma v >$ saturate so that the expansion term dominates. This occurs when $T \approx 50 \text{ keV}$ whence $< \sigma v > \approx 9 \times 10^{-16} \text{ cm}^3/\text{sec}$. If we take advantage of this rapid α -particle heating, the gain can be enhanced by an installation of a solid fuel between the ignited gas fuel and the conducting shell as shown in Fig.2.

Fig. 2 Schematic diagram of the target. It consists of a gas fuel at $r < a$ used for the ignition, a solid fuel at $a \leq r < b$ used to increase the Q value of the reaction and an electrically conductive heavy metal shell at $b \leq r \leq c$ used to hold the magnetic field and plasma pressure.

Since the solid fuel has a mass density much lower than the metal shell, the shock wave propagates through this region quickly and cools the central plasma adiabatically. Hence we cannot make the region b-a, too thick. A reasonable choice would be a ten percent thickness, i.e., $b-a = 0.1a$. Even with this thickness, this region can provide ten times larger fuel density on the average because the solid fuel has a density approximately 100 times larger than the assumed gas density of 10^{21} cm^{-3} . The gain factor Q can then be calculated from

$$Q = \frac{\frac{1}{4} E_n \langle \sigma v \rangle_{50} n_f^2 V_f \tau_{in}}{3 n T V_p} \quad (57)$$

where $\langle \sigma v \rangle_{50} = 9 \times 10^{-16} \text{ cm}^3/\text{sec}$. is the fusion cross section at 50 keV, n_f is the average full density (including the solid fuel), $V_p (= 4\pi a^3/3)$ is the plasma volume, $V_f (= 4\pi b^3/3)$ is the full volume and $E_n (= 14 \text{ MeV})$ is the neutron energy. If we take $b-a \approx 0.1a$, $n_f \approx 10^{22} \text{ cm}^{-3}$ with $n = 10^{21} \text{ cm}^{-3}$, $V_p \approx V_f$, while for $a = 8.1$, $T = 50$ and $n = 10$, τ_{in} becomes $2.9 \times 10^{-8} \text{ sec}$. for a tungsten shell without preheating. Then the gain factor Q becomes, from Eq. (57) $e.1 \times 10^2$, and the corresponding output energy becomes 3.1 GJ.

The thickness of the shell should be large enough to sustain the required inertial time. From Eq. (53) the inertial confinement time before ignition is $2.3 \times 10^{-7} \text{ sec}$. and is 2.9×10^{-8} after the ignition. The shock propagation speed u_s in the shell is given by¹³⁾

$$u_s = \left[\frac{4}{3} \frac{\rho}{\rho_s} \right]^{1/2} \\ = 1.15 u$$

Hence the shell thickness (c-b) of a tungsten shell is given from Eqs. (51) by

$$c - b \geq u_s \tau_{in} = 4.6 \text{ mm}.$$

Let us now study whether conditions required

on the magnetic field and heat confinement time are satisfied under this ignition condition. At 10 keV, the confinement time required for the ignition, is from Eq. (6), approximately 10^{-7} sec . Hence the lifetime of the magnetic field, as well as the heat conduction time should be larger than 10^{-7} sec .

The lifetime of the magnetic field τ_{MO} is given by Eqs. (16) and (42),

$$\tau_{MO} = 9.3 \times 10^{-7} \frac{\bar{a}^2 \bar{T}^{3/2}}{Z^2}$$

for a spheromac configuration. Hence with $T = 10$, $Z = 3$, τ_{MO} becomes larger than T_i if $a \geq 1$. The magnetic reconnection time which is the time needed to form a force-free magnetic configuration due to the tearing mode instability becomes approximately 10^{-7} sec for $T = 0.1$, $a = 10$, $Z = 3$ and $B = 1$. The actual reconnection time can be several times faster if a forced reconnection operates.

Now with respect to the heat confinement time, if we take τ_{Hi} in Eq. (45), $\tau_{Hi} = 1.1 \times 10^{-6} B^2$ for $T = 10$, $a = 8.1$ and $Z = 3$. Hence with $B \geq 1$ (1 MG) τ_{Hi} can be made larger than τ_{in} . If we use Bohm diffusion, Eq. (40) gives $\tau_B = 1.1 \times 10^{-5} B$, hence it requires even a smaller magnetic field. The α -particle confinement time $\tau_{H\alpha}$ is given by Eq. (48) which can very easily be made larger than τ_{in} with $B \approx 1$.

Hence the requirement on the necessary magnetic field comes primarily from Eq. (33) so than $\rho_\alpha/a < 1$. If $a = 8$, 1 MG again is acceptable. Thus the magnetic field required for the present scheme is within the level which have been already obtained in many experiments.

Finally, let us discuss the heating scheme. The Ohmic heating time becomes on the order of millisecond even for 10 MG magnetic field hence is too slow to depend on. Beside the total magnetic field energy is much too small to be dispensed for

the plasma energy. This means that the input energy must be put directly to the plasma from laser or particle beams.

IV. CONCLUSION

A new hybrid fusion scheme is introduced in which the laser generated magnetic field is utilized to insulate the plasma temperature from the shell and to reduce the sound speed in the conductive shell whose inertia sustains the plasma pressure. For an example of a gas fuel with a density of 10^1 cm^{-3} , surrounded by a solid fuel of $\sim 1 \text{ mm}$ thickness which are further surrounded by a tungsten spherical shell of 9 mm inner radius and 5 mm thickness gives an output energy of 3.1 GJ for an input energy of $\sim 10 \text{ MJ}$ and power of $\sim 72 \text{ TW}$.

The input energy and power may be reduced significantly (by an order of magnitude or more) if the preheating of the shell by the bremsstrahlung photons and/or α -particles is utilized, which slows down the expansion speed and induces the inward diffusion flux of cold fuel.

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