Laser Original**Double-Pulse One-Way Imaging by using Degenerate Four-Wave Mixing**

Naohiro TAN-NO* and Kenichi YOKOTO*

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The paper presents novel methods for one-way imaging by double-pulse transmission through a turbulent medium, using corrected wavefront generation due to degenerate four-wave mixing. The present methods are based on a principle that the mixing of a distorted object pulse with a pump pulse equivalently distorted through an identical turbulent medium produces corrected wavefront in a nonlinear medium in real-time. These methods require neither retroreflecting transmission of a phase-conjugate wave nor two fibers with the same length and quality. Two methods associated with the basic principle are presented and theoretically discussed with ability of arbitrary phase correction.

1. Introduction

The holographic technique of imaging through a distorting medium has been investigated in many cases¹⁻⁵). On the other hand, the studies of generation of optical time-reversed wavefront have received currently considerable attention⁶⁻¹²), because a phase-distorted object image can be reconstructed by the use of phase-conjugate wavefront.

The image reconstruction has been carried out through the retroreflecting propagation of phase-conjugate wave upon further propagation in an inhomogeneous, transmitting medium, while Yariv has proposed a novel method to undo the distortion of image transmitted by two multimode optical fibers⁷). Furthermore, Hellwarth has pointed out that in the case of technique using the degenerate four-wave mixing, neither a frequency shift nor phase-matching need play a role⁹). This ideal technique provides a new one-way coherent imaging through the distorting medium¹³⁻¹⁵). In this paper, we propose an application of the degenerate four-wave mixing, that leads to corrected wavefront generation in real-time due to

distortion-canceled phase-grating formation by double-pulse transmission through a turbulent medium. The present technique is based on a principle that the distortion of an object image is canceled out by virtue of the mixing of the object wave pulse with an equivalently distorted pumping pulse which passes through the identical turbulent medium. We present two methods in the paper in order to realize this principle. One of the method is to use two coincident pulses of which linearly polarized fields are orthogonal each other. Another method is to use two sequential pulses with arbitrary polarization. These two pulses, for either case, simultaneously enter a nonlinear medium at a small crossing angle. As a result, we can easily observe a corrected image in real time. Neither retroreflecting propagation nor two optical fibers, and nor the developing for a hologram are required for the present method. Accordingly, these pulse techniques present an advantage for one-way real-time transmission of optical information. In Sec. 2 we discuss the basic principle using the third-order nonlinear macroscopic polarization for the generation of corrected wavefront. In Sec. 3 we propose two methods based on the basic

* Department of Electronics, Yamagata University (4-3-16, Jonan, Yonezawa 992)

principle. In Sec. 4 we theoretically verify the correction for atmospheric degradation of optical beam transmission by using the present method.

2. Basic Principle

First, we examine corrected wavefront generation due to the third-order nonlinear macroscopic polarization, in order to realize the double-pulse one-way imaging. We assume the total field of degenerate waves taken as

$$E(r, t) = \frac{1}{2} \sum_{i=1}^4 \{ \varepsilon_i(r, t) \exp i[\varphi_i(r) - \omega t] + \text{c. c.} \}, \quad (1)$$

where $\varphi(r)_i - \omega t$ is the phase or eikonal which describes a wave of geometrical optics. This four-wave mixing involves two intense pump waves ε_1 and ε_2 of the same frequency of ω traveling in opposite direction and two signal waves ε_3 and ε_4 of the same frequency, which also travel in opposition to each other. Then, the third-order macroscopic polarization is written as follows^{9,16)}, using the third-order nonlinear susceptibility $\chi^{(3)}$,

$$P^{(3)}(r, t) = \varepsilon_0 \chi^{(3)} \text{Re} \left\{ \sum_{l=1}^4 \varepsilon_l e^{i\varphi_l(r)} \sum_{m=1}^4 \varepsilon_m e^{i\varphi_m(r)} \sum_{n=1}^4 \varepsilon_n^* e^{-i\varphi_n(r)} e^{-i\omega t} \right\}. \quad (2)$$

The product of the fields in the right-hand side of eq. (2) can be expanded 64 terms of each component. Assuming a small cross-angle between the pumping beam and incident signal (object) beam, we obtain at least two interesting terms producing observable fields¹⁷⁾, as follows

$$P_c^{(3)} = \varepsilon_0 \chi^{(3)} \text{Re} \{ \varepsilon_1 e^{i\varphi_1(r)} \varepsilon_2 e^{i\varphi_2(r)} \varepsilon_4^* e^{-i\varphi_4(r)} e^{-i\omega t} \}, \quad (3)$$

$$P_n^{(3)} = \varepsilon_0 \chi^{(3)} \text{Re} \{ \varepsilon_1 e^{i\varphi_1(r)} \varepsilon_2^* e^{-i\varphi_2(r)} \varepsilon_4 e^{i\varphi_4(r)} e^{-i\omega t} \}. \quad (4)$$

The polarization of eqs. (3) and (4) and the total field of eq. (1) obey the Maxwell wave equation,

$$\nabla^2 E(r, t) - \varepsilon \mu \frac{\partial^2}{\partial t^2} E(r, t) = \mu \frac{\partial^2}{\partial t^2} P^{(3)}(r, t). \quad (5)$$

Thus we can see that the first polarization $P_c^{(3)}$ generates phase conjugate wavefront $\varepsilon_{3c} \exp(ik_{3c}r)$ and the second polarization $P_n^{(3)}$ generates non-conjugate wavefront $\varepsilon_{3n} \exp(ik_{3n}r)$, subject to the wave equation. The coupled equation of each field is derived as shown in a next section 3.

For plane waves $\varphi_i = k_i r$, the wave vector k_{3c} of the phase-conjugate wavefront and the k_{3n} of the nonconjugate wavefront are written as

$$k_{3c} = k_1 + k_2 - k_4, \quad (6a)$$

$$k_{3n} = k_1 - k_2 + k_4. \quad (6b)$$

Since we assumed two pump waves to travel in opposite direction, $k_1 + k_2 = 0$ is valid as shown in Fig. 1. Accordingly, $k_{3c} = -k_4$ is already known in the usual configuration of phase-conjugate wavefront generation, while phase-matching condition of $k_{3n} = 2k_1 + k_4$ is required for the nonconjugate wavefront generation. The reflecting beam of nonconjugate wavefront, which corresponds to a Bragg reflection from a phase grating based on standing wave of pump fields, can be observed at a symmetric position on the axis of the pump beams

Fig. 1 Wavevector geometry of phase conjugate wave E_{3c} and nonconjugate wave E_{3n} against to probe wave E_4 in degenerate four-wave mixing.

as illustrated in Fig. 1 for the small crossing angle θ satisfying approximately the phase-matching requirement. It is worthwhile to note, for the following discussion, that the polarization $P_n^{(3)}$ generates a normal image without wavefront inversion. At this stage, we consider what effect we could expect in the generation of wavefront involved in the nonlinear polarizations yielding the degenerate four-wave mixing.

We begin to consider the basic principle for the corrected wavefront generation. When one pump wave ε_2 and the object wave ε_4 , of which beams are collinear and equal diameters, propagate an inhomogeneous, transmitting medium, the respective phase in r -direction into the nonlinear medium result in $\varphi_2(r) = k_2 r + \phi_d(r)$ and $\varphi_4(r) = k_4 r + \phi_d(r) + \phi_s(r)$, where $\phi_d(r)$ is the phase distortion and the objective phase $\phi_s(r)$ is previously given by a real image. The macroscopic polarizations in the nonlinear medium produced by such the fields involve, as estimated from eqs. (3) to (5), interesting generative wavefronts as

$$\varphi_{3c}(r) = k_1 r + k_2 r + \phi_d(r) - k_4 r - \phi_d(r) - \phi_s(r), \quad (7a)$$

$$\varphi_{3n}(r) = k_1 r - k_2 r - \phi_d(r) + k_4 r + \phi_d(r) + \phi_s(r). \quad (7b)$$

When all beams are collinear, $\varphi_{3c}(r) = -k_4 r - \phi_s(r)$ and $\varphi_{3n}(r) = -k_4 r + \phi_s(r)$ are valid. The phase distortions are canceled out for each generation of wavefront. The reconstructed images with conjugate and nonconjugate wavefronts, however, are superposed on the same direction and thus we observe an interference fringe pattern instead of the corrected real image¹⁷⁾. For the configuration depicted in Fig. 1, on the other hand, $\varphi_{3c} = -k_4 r - \phi_s(r) + \delta(r, r')$ and $\varphi_{3n} = (2k_1 + k_4)r + \phi_s(r) - \delta(r, r')$, where $\delta(r, r') = \phi_d(r) - \phi_d(r')$ and r' the titled coordinate. The $\delta(r, r')$ means residual phase aberration which is not completely canceled

out because of wavefront mismatching between the pump wave and the object wave. If any angle between wave normals $n_2 = (c/\omega) \text{grad} \phi_d(r)$ and $n_4 = (c/\omega) \text{grad} \phi_d(r')$ is sufficiently small ($\delta(r, r') \ll \pi$) at every transverse points, the object image can be almost reconstructed.

This condition at a local point is equivalent to that of the spherical wavefront generation^{1,3)}.

It can be concluded, in terms of holography¹⁻⁵⁾, that one pump field and an object field with the same phase distortion are to form an objective phase grating which has no phase distortion, and another pump field of plane wave generates corrected wavefront of an object image, instantaneously, that is, in real-time. The nonlinear medium provides a sort of spatial filter by means of the nonlinear coupling.

Accordingly, we can expect the corrected wavefront generation, provided that we employ double pulses that almost simultaneously propagate a turbulent medium as an object wave and one pumping, plane wave.

Two reconstructed images in the conjugate and nonconjugate version appear having different property from eqs. (7). The conjugate image shows a reverse-wavefront replica to an incident image, while the nonconjugate wave gives a real replica though the generation of incident image is limited by phase-matching constraint. For a three-dimensional object image the conjugate image needs an additional nonlinear medium in order to reverse the generated wave front, but the nonconjugate image is not necessary to reverse the wavefront. For a two-dimensional image, reconstruction can be achieved for the conjugate image without wavefront-reverse

3 Methods for one-way imaging

We, here, propose two methods of one-way imaging in order to perform the corrected wavefront generation based on the principle discussed

in the preceeding section. We take into account an application to correction for atmospheric degradation of optical beam transmission and signal generation associated with the parametric coupling equation involving only the phase conjugate field for simplicity.

Figure 2 illustrates a configuration of generation of corrected wavefront by utilizing linearly polarized waves for the double pulses, of which polarization are orthogonal each other. These pulses are an object wave and one pumping wave. They simultaneously propagate in the turbulent medium and are separated by a Glan-prism to enter a nonlinear medium at a small crossing angle θ , as depicted in Fig. 2. A delay line in Fig. 2 makes possible to adjust an optical delay time to the medium for coincident of the entrance pulses.

The degenerate four-wave mixing occurs in the medium by coincidence of another pumping field E_1 in the opposite direction against to the pumping field E_2 . As a result, the nonlinear medium acts as producing a phase conjugate wave E_3 with corrected wavefront based on the present principle.

Figure 3 shows another method of corrected wavefront generation for one-way imaging, by utilizing two sequential pulses of unpolarized fields. The sequential pulses of separation time T propagate in a turbulent medium and are simultaneously incident to the nonlinear medium via a beam splitter of which role is to compensate the separation time T along with a suitable optical path length. The degenerate four-wave mixing by the unpolarized fields of a transmitting object wave and a transmitting pumping wave does not

Fig. 2 A method using two orthogonal polarized beams as a pump pulse E_2 and an object pulse E_4 for corrected wavefront generation due to degenerate four-wave mixing in a nonlinear medium.

Fig. 3 Another method using two sequential short pulses as a pump pulse E_2 and an object pulse E_4 for corrected wavefront generation due to degenerate four-wave mixing in a nonlinear medium.

require a selection of polarization with respect to another pumping wave E_1 . However, a generating phase-conjugate wave E_3 depends on polarization of the pumping field E_1 . This fact is seen from the third-order nonlinear polarization in an isotropic nonlinear medium.

It is necessary for the separation time T to be sufficiently shorter than varying time of turbulence of atmosphere etc.. On the contrary, the proceeding method in Fig. 1 does not need such the condition, but it needs conservation of the polarization of transmitting pulses.

4. Arbitrary phase correction

We have already discussed a role of the third-order nonlinear polarization to produce the corrected wavefront. Here, we confirm our method for arbitrary phase correction, subject to the parametric coupled equation. The following analysis is applicable to both methods above discussed.

We assume that the pump fields E_1 , E_2 and the object field E_4 propagate to the r -direction and z -direction, respectively, in the nonlinear medium.

The respective wave vectors are k_1 , k_2 and K . The fields $\tilde{E}_2$ and $\tilde{E}_4$ denote the fields after propagating the turbulent medium as depicted in Fig. 2 and Fig. 3. These fields are assumed to be coherent monochromatic waves in initiative. The object field E_4 carrying image information and the pump field E_2 of constant amplitude are Fourier expanded as follows

$$\begin{aligned} \tilde{E}_4(x, y, z, t) = & \frac{1}{2} \int A_4(k_\perp, z) e^{i[k_\perp \cdot r + \phi_4(k_\perp)]} \\ & \times d^2 k_\perp e^{i(\omega t - kz)} + c. c., \end{aligned} \quad (8)$$

$$\begin{aligned} \tilde{E}_2(r, t) = & \frac{1}{2} A_2 \int e^{i[k_\perp \cdot r + \phi_2(k_\perp)]} d^2 k_\perp \\ & \times e^{i(\omega t - k_z r)} + c. c., \end{aligned} \quad (9)$$

where $A_4(k_\perp, z)$ is the amplitude of arbitrary wave vectors $k_\perp$ and A_2 the constant amplitude of pump field E_2 . The notation $\phi_d(k_\perp)$ designates the phase distortion of the wave vector $k_\perp$ due to the turbulent medium. On the other hand, the effect of nonlinear mixing is to produce a phase-conjugate wave E_3 which is taken as

$$\begin{aligned} E_3(x, y, z, t) = & \frac{1}{2} \int A_3(k_\perp, z) e^{i k_\perp \cdot r} d^2 k_\perp \\ & \times e^{i(\omega t + kz)} + c. c., \end{aligned} \quad (10)$$

and another pump field E_1 is written as

$$E_1(r, t) = \frac{1}{2} A_1 e^{i(\omega t - k_1 r)} + c. c. \quad (11)$$

The coupling of these fields eqs. (8) to (11) gives rise to degenerate four-wave mixing according to the nonlinear polarization of eq. (2). We derive coupled equations by combining the nonlinear polarization with the Maxwell equation. Here, we assume the slowly varying amplitude approximation, the rotating wave approximation and pump pulse nondepletion. Substituting the fields E_1 to E_4 into the scalar wave equation of eq.(5), we can obtain the following equations:

$$\begin{aligned} & \int [k_\perp^2 A_4^*(-k_\perp, z) - i2k \frac{\partial}{\partial z} A_4^*(-k_\perp, z)] \\ & \times e^{i[k_\perp \cdot r - \phi_4(k_\perp)]} d^2 k_\perp \\ & = \epsilon_0 \mu \omega^2 \chi^{(3)} A_1 A_2 \int A_3(k_\perp, z) e^{i k_\perp \cdot r} d^2 k_\perp \\ & \times \int e^{i[k_\perp \cdot r - \phi_2(k_\perp)]} d^2 k_\perp, \end{aligned} \quad (12a)$$

$$\begin{aligned} & \int [k_\perp^2 A_3(k_\perp, z) - i2k \frac{\partial}{\partial z} A_3(k_\perp, z)] e^{i k_\perp \cdot r} d^2 k_\perp \\ & = \epsilon_0 \mu \omega^2 \chi^{(3)} A_1 A_2 \int A_4^*(-k_\perp, z) \\ & \times e^{i[k_\perp \cdot r - \phi_2(k_\perp)]} d^2 k_\perp \int e^{i[k_\perp \cdot r + \phi_4(k_\perp)]} d^2 k_\perp. \end{aligned} \quad (12b)$$

Supposing that the cross angle θ between the pump beams and the object beam is sufficiently small and that both of the beams superpose each other in the thin medium, we can expand the products of Fourier components appeared in eqs.(12a, b) in terms of only the products of the same component of the wave vector $k_{\perp}$, ignoring the crossing terms between the different components. This situation is equivalent to an orthogonal condition for Fourier components.

Accordingly, we can utilize the following relations:

$$\begin{aligned} \int A_3(k_{\perp}, z) e^{i k_{\perp} \cdot r} d^2 k_{\perp} & \int e^{i [k_{\perp} \cdot r - \phi_d(k_{\perp})]} d^2 k_{\perp} \\ & = \int A_3(k_{\perp}, z) e^{i [k_{\perp} \cdot r - \phi_d(k_{\perp})]} d^2 k_{\perp}, \quad (13a) \\ \int A_4^*(-k_{\perp}, z) e^{i [k_{\perp} \cdot r - \phi_d(k_{\perp})]} d^2 k_{\perp} \\ & \times \int e^{i [k_{\perp} \cdot r + \phi_d(k_{\perp})]} d^2 k_{\perp} = \int A_4^*(-k_{\perp}, z) \\ & \times e^{i k_{\perp} \cdot r} d^2 k_{\perp}. \quad (13b) \end{aligned}$$

It can be seen from eqs.(13a, b) that the arbitrary phase aberration $\phi_d(k_{\perp})$ is canceled out for each components $k_{\perp}$. After rearrangement, we obtain the coupled equations:

$$\begin{aligned} \frac{\partial}{\partial z} A_4^*(-k_{\perp}, z) & = -\frac{i}{4\pi} \lambda k^2 A_4^*(-k_{\perp}, z) \\ & + i \kappa A_3(k_{\perp}, z), \quad (14a) \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial z} A_3(k_{\perp}, z) & = -\frac{1}{4\pi} \lambda k^2 A_3(k_{\perp}, z) \\ & + i \kappa A_4^*(-k_{\perp}, z), \quad (14b) \end{aligned}$$

where $k = 2\pi/\lambda$, and $\kappa = (\epsilon_0 \mu \omega^2 \chi^{(3)}/2k) |A_1| |A_2|$. These coupled equations are the same formula introduced by Yariv et al.¹⁰⁾. Thus, their solutions are the same to eqs.(15) in Ref. 10. We rewrite the solutions:

$$\begin{aligned} A_4^*(-k_{\perp}, z) & = i e^{i(k - \lambda k_{\perp}^2/4\pi)z} A_4^*(-k_{\perp}, 0) \\ & \times \frac{\cos |\kappa| (z-L)}{\cos |\kappa| L}, \quad (15a) \end{aligned}$$

$$\begin{aligned} A_3(k_{\perp}, z) & = i e^{i(k - \lambda k_{\perp}^2/4\pi)z} \left(\frac{\kappa^*}{|\kappa|} \right) \frac{A_4^*(-k_{\perp}, 0)}{\cos |\kappa| L} \\ & \times \sin |\kappa| (z-L). \quad (15b) \end{aligned}$$

At the output for the conjugate wave A_3 at $z = 0$, we thus have

$$A_3(k_{\perp}, 0) = -i \left(\frac{\kappa^*}{|\kappa|} \tan |\kappa| L \right) A_4^*(-k_{\perp}, 0), \quad (16)$$

which shows that each $k_{\perp}$ component of the output beam A_3 couples directly to the $-k_{\perp}$ component of the input wave A_4 with no phase distortion by the atmospheric turbulence. Therefore we can observe the corrected wavefront of the input object wave for the case of the intensity-distributed, two dimensional image information. If we use the nonconjugate wavefront generation instead of the phase-conjugate component, we should succeed the three-dimensional image correction as mentioned in the section 2.

5. Concluding remarks

We discussed new methods for the restoration of object image transmitted through atmosphere or any other transmission. The proposed scheme requires neither the retroreflecting propagation of phase-conjugate wavefront nor two optical fibers with equal property. On the other hand, we need a well stabilized local oscillator in order to pump a nonlinear medium as the pumping wave E_1 , and an intense laser for the pumping wave E_2 transmitting through the turbulent medium. The present double-pulse one-way imaging shall be preformed in time range of nsec or

psec for communication. Recent development of laser oscillator easily overcomes these problems.

Since our methods are able to perform real-time restoration of distorted wavefront, phase distortion by temporal turbulence also is compensated by using suitable short pulse sequences. This method is easily applicable to multimode optical fiber one-way real-time image transmission.

In the present scheme of both phase-conjugate and nonconjugate wavefront generation, their behaviours should be simultaneously analyzed by more extended equations. This problem shall be reported our forthcoming paper.

In summary, we show that the degenerate four-wave mixing provides to generate corrected wavefront by using a phase grating formation without distortion in a nonlinear medium. This nonlinear processing of one-way imaging may reviewed in a general sence as a form of real-time holographic imagery and it is expected that it will find many other applications involving image processing and reconstruction in optics community and information.

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