Laser Original

Supersonic CO₂ Mixing Laser with N₂ Glow Discharge in Coaxial Nozzles

Kazuo MAENO*, Naohito YAMAGUCHI** and Yutaka HANAOKA*

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To investigate the characteristics of an electric discharge CO₂ mixing gasdynamic laser, a coaxial supersonic screen nozzle array is tentatively designed. Isentropic relations on gas flow are assumed for the design of conical and coaxial nozzles. A small signal gain coefficient and flow parameters were measured. Further numerical analysis on the small signal gain was made by assuming a steady, inviscid, quasi-one-dimensional vibrationally nonequilibrium flow, and a three-mode relaxational model with a constant discharge efficiency. Measured and calculated results show that this supersonic mixing CO₂ electric discharge laser can achieve high-gain operation.

Key Words : CO₂ laser, Mixing laser, Supersonic flow laser, Glow discharge.

Nomenclature

A : cross sectional area of nozzle

a : sound velocity

C_c : mass fraction of CO₂

C_N : mass fraction of N₂

c_p : specific heat with constant pressure

e : molecular energy per unit mass

G : small signal gain coefficient

H : enthalpy per unit mass of the gas

K_N : rate constant of translational-vibrational energy transfer for N₂

k : Boltzmann constant

M : Mach number

$\dot{m}$: mass flux

N : population density of molecules

p : pressure

Q : mass flux of gas

Q_{CN} : rate constant for intermolecular vibrational energy transfer

Q_v : vibrational partition function

R : gas constant

T : translational temperature of gas

T_3 : vibrational temperature of the upper laser level of CO₂

T_1 : vibrational temperature of the lower laser level of CO₂

u : gas velocity

W_d : discharged electric power

x : distance along the nozzle axis

γ : specific heat ratio = c_p/c_v

ϵ_i : i -th energy level

ρ : gas density

λ : wavelength of CO₂ laser ; 10.6×10^{-6} m

ν_c : collision frequency

* Department of Industrial Mechanical Engineering, Muroran Institute of Technology (27-1 Mizumoto, Muroran, Hokkaido 050)

** Laser Technology Group, Mechatronics Products Development, Ishikawajima-Harima Heavy Industries (1-1 Shin-Nakahara, Isogoku, Yokohama, Kanagawa 235)

- η : total discharge efficiency
 τ_{21} : the spontaneous radiative lifetime
 5.38sec
 θ : vibrational characteristic temperature
 ψ : molar fraction of the specified gas

Subscripts

- C : carbon dioxide
 N : nitrogen
 1 : symmetric vibrational mode of CO₂
 (lower laser level)
 3 : asymmetric vibrational mode of CO₂
 (upper laser level)
 3N : indicating energy transfer between
 asymmetric vibrational mode of CO₂
 and N₂

Superscripts

- s : stagnation conditions
 v : vibrational energy
 * : nozzle throat conditions (sonic conditions)

1. INTRODUCTION

There have been extensive literature on high power CO₂ lasers, such as CO₂ gasdynamic laser(GDL)^{1,2)}, thermally pumped CO₂ mixing gasdynamic laser(MGDL)^{3,4)}, axial-flow glow discharge, and transversely excited(2-D, 3-D) glow/silent discharge type lasers. Among which, a high performance gasdynamic laser can be realized by use of a large mass flux of population-inverted CO₂ gas, which is excited independently and highly without strong deactivation by collision with CO₂ or catalyst(e.g. He), and by low translational temperature of the laser medium. Although the finite rate of intermolecular vibrational-vibrational energy exchange CO₂ (001) and N₂ molecules gives severe limitation on the velocity of the laser medium, supersonic

flow can be regarded as one of the best methods for high power CO₂ lasers. For thermally pumped GDL, however, total enthalpy in the high temperature plenum is consumed mainly to yield the supersonic flow of room temperature.

This paper reports the lasing properties of a supersonic mixing CO₂ laser, which combined the glow discharge in the supersonic flow of N₂ from the moderate stagnation conditions. Although the concept of high speed flow mixing CO₂ glow discharge laser has previously been investigated in Refs. 5 and 6, their experimental facilities are not made for the supersonic discharge scheme but for the high subsonic velocity plenum discharge. In our experiment, a screen nozzle of coaxial array (6 row $\times$ 2 columns) was employed. The small signal gain coefficient was measured. The characteristic parameters of CO₂ mixing laser were also numerically determined for steady, inviscid, supersonic, quasi-one-dimensional, and vibrationally nonequilibrium flow. The numerical analysis is made by the assumptions of a three-mode model in the CO₂-N₂ vibrational system, instantaneous mixing, and a constant discharge efficiency in the supersonic N₂. The characteristics of this supersonic mixing laser are discussed by comparing the experimental data with the calculated values.

2. EXPERIMENTAL SETUP

A schematic diagram of the experimental setup is shown in Fig. 1. The nitrogen is supplied from the electro-magnetic valve and fed into the supersonic conical nozzles with near-atmospheric plenum conditions. Then it is vibrationally excited by the radial glow discharge in the supersonic region, and is mixed with the separately expanded supersonic CO₂ gas from the exit face of the coaxial nozzles. The dis-

Fig. 1. Experimental apparatus.

charge gap was 3mm in radial direction and 6mm in axial direction for each coaxial N_2 nozzle. Discharge power was supplied from a DC circuit (hand-made ; 1kV, 1A). Discharge voltage and current were measured. The laser cavity was made of the rectangular area of $34 \times 138\text{mm}^2$ with CaF_2 beam damping windows for gain measurements. A thermopile power meter and a hand-made CW CO_2 probe laser were used to measure the small signal gain coefficient. The stagnation pressure and the temperature in the nozzles, and the cavity pressure were also measured by the Bourdon's gauges and the CA thermocouples.

As the first approximation, ideal isentropic relations⁷⁾ on the gas flow were employed for the design of the conical and coaxial nozzles. If the stagnant condition is denoted by superscript "s" and the throat condition is marked by "*", then the following relations are given as

$$\frac{p^*}{p^s} = \left(\frac{T^*}{T^s} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}},$$

$$M^* = \frac{u^*}{a^*} = 1,$$

$$a = \sqrt{\gamma R T},$$

$$\dot{m} = \rho^* u^* A^*.$$

The throat conditions of the gas flow can be determined by these equations. Once the throat conditions of the gas flow and the nozzle area ratio are given, the Mach number is calculated by

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{(\gamma-1)M^2 + 2}{\gamma+1} \right]^{\frac{\gamma+1}{2(\gamma-1)}},$$

and by using the Mach number obtained the flow parameters were determined from the following equation,

$$\frac{p^s}{p} = \left(\frac{T^s}{T} \right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}}.$$

By assuming the instantaneous mixing of CO_2 and N_2 gases at the mixing position, one-dimensional conservation equations of mass, momentum, and energy give the mixed flow conditions⁴⁾. An example of flow parameters determined by these isentropic relations is shown in Table I, where the differences in pressure, temperature, and velocity between N_2 and CO_2 supersonic flows were observed. These differences mainly cause the pressure disturbances and increase in the entropy in mixed cavity flow in the present experiments. Thus, the estimated conditions of mixed gas in laser cavity are regarded as ideal approximation for designed specifications without including vibrational energy.

3. NUMERICAL ANALYSIS

For comparison with the experimental results, a numerical analysis of vibrational non-equilibrium supersonic flow with the glow discharge was performed by neglecting the collisional ionization and dissociation of N_2 molecules by electrons. By assuming the steady, in-

Table I Example of supersonic flow properties in nozzles and a cavity calculated from the ideal isentropic relations.

FLOW PROPERTIES IN EACH NOZZLE			
	N ₂	CO ₂	
Stagnation Pressure	101	101	kPa
Temperature	300	300	K
Throat Area	15.9	9.6	mm ²
Area ratio	5.33	2.94	
Exit Area	84.8	28.3	mm ²
Mach Number	3.25	2.50	
Pressure	1.9	5.8	kPa
Temperature	96	155	K
Velocity	635	480	m/s

FLOW PROPERTIES IN THE LASER CAVITY		
Area	4692	mm ²
N ₂ Mass Flux	45	g/s
CO ₂ Mass Flux	33	g/s
Pressure	860	Pa
Temperature	116	K
Flow Velocity	570	m/s
Mach Number	2.85	
Mole fraction	$\Psi_C : \Psi_N = 0.32 : 0.68$	

viscid, and quasi-one-dimensional flow, equations of mass, momentum, energy conservation, and state can be written⁴⁾. The energy equation for N₂ becomes, including the term of discharge energy divided by mass flux of nitrogen W_d/Q_N , as

$$\frac{dH}{dx} = \frac{W_d}{Q_N},$$

$$H = c_p T + e^v + \frac{u^2}{2},$$

where e^v indicates the total vibrational energy of the gas. Furthermore, by assuming the three-mode model in the CO₂-N₂ vibrational relaxation system, we have three vibrational rate equations⁴⁾. By introducing a discharge efficiency, the relaxation equation for N₂ vibrational energy becomes as :

$$\frac{de_N^v}{dx} = \frac{1}{u} \left(-\Delta e_N^v - \frac{C_c \theta_N}{C_N \theta_3} \Delta e_{3N}^v \right) + \frac{W_d \eta}{Q_N},$$

where

$$\Delta e_N^v = K_N \left\{ e_N^v - \frac{R_N \theta_N}{\exp\left(\frac{\theta_N}{T}\right) - 1} \right\},$$

$$\Delta e_{3N}^v = Q_{CN} \left[\left(e_3^v + R_c \theta_3 \right) \frac{e_N^v}{R_N \theta_N} \cdot \right. \\ \left. \exp\left(\frac{\theta_N - \theta_3}{T}\right) - \left(\frac{e_N^v}{R_N \theta_N} + 1 \right) e_3^v \right].$$

These ordinary differential equations are numerically integrated by the Runge-Kutta method. The initial conditions at the nozzle throat in the supersonic flow are estimated by assuming the vibrational equilibrium flow. In the discharge portion of the N₂ gas, the total efficiency η of the glow discharge energy to molecular vibrational energy is decided mainly by the positive column energy fraction (complementary energy rate to cathode fall consump-

tion) and the transfer rate of the discharge energy to N_2 molecular vibration which is affected by E/N . In this paper, this total efficiency η is roughly assumed to be constant of 40% and 45%. In determining these efficiency values, the positive column energy fraction was estimated from the experimental discharge characteristics to the measured cathode fall voltage of about 300V, and it had the range from 40% to 50%. Furthermore the transfer rate of the discharge energy to N_2 molecular vibration was assumed to have the range from 100% to 80% as our experimental values of the E/N were in the 4×10^{-20} to $7 \times 10^{-20} \text{Vm}^2$ range (See Ref. 8). At the mixing position, the instantaneous mixing assumption was also used to give the downstream cavity flow parameters.

After determining the parameters of the supersonic cavity flow, the population densities of the upper and lower levels of the lasing medium CO_2 were given by the vibrational temperatures from the following equations

$$N_3 = \frac{N_c}{Q_v} \exp\left(-\frac{\varepsilon_3}{kT_3}\right),$$

$$N_1 = \frac{N_c}{Q_v} \exp\left(-\frac{\varepsilon_1}{kT_1}\right),$$

where Q_v was the vibrational partition function, and was given by

$$Q_v = \left[1 - \exp\left(-\frac{\theta_1}{T}\right)\right]^{-1} \left[1 - \exp\left(-\frac{\theta_2}{T}\right)\right]^{-2} \times \left[1 - \exp\left(-\frac{\theta_3}{T}\right)\right]^{-1}$$

and in above equation the vibrational characteristic temperatures of CO_2 vibrational modes were as follows

$$\theta_1 = 1999\text{K}, \quad \theta_2 = 960\text{K}, \quad \theta_3 = 3383\text{K}.$$

By using these population densities and the translational temperature T of the mixed gas in

the cavity, the small signal gain coefficient G was calculated by the following relation for the $P(20)$ transition of the $10.6 \mu\text{m}$ band.

$$G = \frac{\lambda^2}{4\pi\tau_{21}\nu_c} (N_3 - N_1) \left(\frac{45.6}{T}\right) \exp\left(-\frac{234}{T}\right).$$

4. RESULTS AND DISCUSSION

Discharge properties in our experimental conditions are indicated in Fig. 2. This figure shows that steady glow discharge in the supersonic N_2 flow is confirmed by almost constant discharge voltage V_d and linearly increasing power W_d . The numerical calculation indicates that the pressure in the supersonic N_2 discharge section is about 15 Torr (2kPa), which is favorable to the glow discharge. In Ref. 6 the glow discharge was achieved at about 200 Torr subsonic plenum, and the translational temperature increase in the plenum was compensated by adding helium as a discharge stabilizer.

Fig. 2. Discharge characteristics in a supersonic N_2 flow.

Fig. 3. Distribution of a small signal gain coefficient along the cavity flow from the nozzle exit. $p_C^s = p_N^s = 76 \text{ kPa}$; $T_C^s = T_N^s = 290 \text{ K}$; $\Psi_C : \Psi_N = 0.32 : 0.68$; $W_d = 512 \text{ W}$; $\eta = 0.4$.

Fig. 4. Distribution of small signal gain coefficient along the cavity flow from nozzle exit; $p_C^s = p_N^s = 101 \text{ kPa}$, $T_C^s = T_N^s = 290 \text{ K}$; $\Psi_C : \Psi_N = 0.32 : 0.68$; $W_d = 518 \text{ W}$; $\eta = 0.4$.

Figures 3 and 4 represent the distributions of the small signal gain coefficient measured along the flow direction from the screen nozzle exit. The numerically calculated values are also presented with the dashed line. The gain distributions obtained are not high because of the small radial discharge gap of 3mm, as compared to the results quoted in Ref. 5, where 170mm discharge distance was employed in high speed nitrogen flow (the conditions for supersonic or subsonic flows were not qualified in their report). While, in our experiments the supersonic nitrogen discharge was maintained, and hence the positive gain was obtained in our laser scheme with electrically glow-pumped super-

sonic N₂ and pure CO₂ supersonic flow without helium or water vapour as a catalyst.

The dependence of the gain coefficient on the helium molar fraction can be numerically calculated for this laser scheme, and is shown in Fig. 5 for the case of 45% discharge efficiency. The catalytic effect of helium is almost negative in this situation except in a low fractional range. In the conventional and subsonic CO₂ electric discharge laser, the catalyst helium is used for the diffusive and convective cooling of the mixture of CO₂-N₂-He from the water-cooled glass wall. This role is especially effective when the translational temperature of the laser gas mixture becomes relatively high by the Joule heating of glow discharge for the subsonic flows. For the supersonic expansion of laser gas mixture as shown in our results, the translational temperature is low enough to maintain the high population inversion, and the deactivating effect on the lower laser level of CO₂ by helium is not so remarkable as in the room temperature

Fig. 5. Estimated dependence of a gain coefficient on the helium molar fraction in a CO₂ nozzle flow. $p_C^s = p_N^s = 101 \text{ kPa}$; $T_C^s = T_N^s = 290 \text{ K}$; $W_d = 500 \text{ W}$; $\eta = 0.45$.

range. Thus in our laser scheme, the addition of helium in the plenum of CO_2 results in the diminished CO_2 molar fraction in the mixed supersonic cavity flow, which makes consequently a negative effect on the small signal gain coefficient by outrivaling the positive effect of cooling and deactivation by helium, as the fraction of vibrationally and separately excited nitrogen gas is very high. In Figs. 3 and 4, the positive gain without catalyst was supported by low translational temperature in mixed flow and high gas velocity as compared with the relaxation time of lower laser levels. Calculated results indicate the same tendency though the magnitude of the small signal gain for both cases is different. As mentioned in the preceding, the discharge energy efficiency η is assumed to be 40 and 45 percent in this analysis from the cathode fall and the appropriate vibrational energy transfer efficiency (i.e. appropriate E/N)⁸⁾.

The effect of the discharge power density (discharged power/ N_2 unit mass flux) to the small signal gain coefficient is shown in Fig. 6.

Fig. 6. Effect of the discharge power density (discharged power per N_2 unit mass flux) on a small signal gain coefficient. $p_C^s = p_N^s = 51\text{kPa}$, $T_C^s = T_N^s = 290\text{K}$; $\Psi_C : \Psi_N = 0.32 : 0.68$; $l = 6\text{mm}$; 190mm from the nozzle exit.

Fig. 7. Estimated effect of discharge power (and power density; discharged power per N_2 unit mass flux) on a small signal gain coefficient. $p_C^s = p_N^s = 51\text{kPa}$, $T_C^s = T_N^s = 290\text{K}$; $\eta = 0.4$.

The measured gain remains within relatively low values owing to the limitation of hand-made DC power supply (max. 1kW). The gain distribution in the figure is predicted to increase, in agreement with the calculated curve (solid line). Figure 7 represents the calculated result of the small signal gain coefficient for our coaxial and conical nozzles at several observing positions in the cavity. Numerical calculation indicates the maximum gain of about $3(1/\text{m})$ for this nozzle configuration. Though the experimental range is too narrow to extrapolate the value of the small signal gain to the higher order, it may be expected to achieve a high gain even in this coaxial short gap glow discharge scheme by increasing the power density of the input power up to the breakdown point of the normal glow discharge. From this figure it can be seen that the appropriate discharged power

density is in the 150 to 200J/g range without using the catalyst helium since the glow discharge was maintained. As indicated above, the estimated high gain is originated from low translational temperature and high mass flux of mixed laser gas in the supersonic expansion and efficient discharge.

In our experiments, high power density per unit volume of N₂ (about 80 W/cm³) can be obtained even in this low power configuration, which is two times larger than the value obtained for the conventional subsonic glow discharge CO₂ laser.

5. CONCLUSION

We have investigated the characteristics of the electric discharge CO₂ mixing gasdynamic laser having the supersonic screen nozzle constituted of the array of 2 × 6 coaxial and the conical nozzles with radial glow discharge. The small signal gain coefficient and flow parameters were measured. The conditions of laser medium flow was numerically estimated by

assuming the quasi-one-dimensional, steady, inviscid, and vibrationally nonequilibrium flow, and by using the three-mode relaxation model with appropriate constant efficiency of the glow discharge. The measured and calculated small signal gain coefficient show that the present scheme can produce the positive gain without any catalyst because of low translational temperature in supersonic expansion and effective activation of CO₂ vibrational mode in the supersonic glow discharge.

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