Laser Original

Spontaneous Emission and Small-Signal Gain of a Low- γ , Waveguide-Mode Free-Electron Laser Operating in the Low-Gain Compton Regime

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Fundamental operation characteristics (a spontaneous emission spectrum and a small-signal gain) of a low- γ , waveguide-mode free-electron laser have been measured by using a good quality relativistic photo-electron beam. Measurements are carried out at microwave frequencies (25-40 GHz) in an X-band rectangular waveguide with a low energy (450-650 keV, $\gamma \sim 2$), low current (~ 1.5 A), low energy spread ($\Delta E/E \ll 10^{-4}$) relativistic photo-electron beam. The spontaneous emission spectrum measured agrees well with the homogeneously broadened spectrum of a TE_{10} waveguide mode. The small-signal gain spectrum measured is in good agreement with theoretical prediction calculated by assuming the operation in the low-gain Compton regime.

Key Words : Free electron laser, Spontaneous emission, Small-signal gain, Waveguide-mode, Relativistic photo-electron beam, Low-gain Compton regime.

1. Introduction

A free-electron laser (FEL) generates coherent electromagnetic radiation by converting the kinetic energy of a relativistic electron beam into the energy of electromagnetic wave. The physical mechanism involved in the operation of a FEL differs according to the properties of the electron beam used as the gain medium and the boundary condition of the electromagnetic radiation. A high- γ FEL using a high-energy electron accelerator such as a storage ring or a rf linac usually operates in the Compton regime in which the collective effects among the elec-

trons may be neglected. The spontaneous emission spectrum of a high- γ FEL in the free space has been widely investigated theoretically and experimentally.¹⁻³⁾ Madey⁴⁾ showed that the small-signal gain spectrum of a FEL operating in Compton regime is proportional to the derivative of the spontaneous emission spectrum, and it was demonstrated experimentally.⁵⁾

In a low- γ FEL, the spectrum of radiation is strongly affected by the presence of a waveguide, which is inevitably used for guiding the long-wavelength electromagnetic wave. The particle-wave matching condition is modified by the waveguide dispersion relation and the wave-

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guide modes that do not interact effectively with the wiggling electrons are filtered out. Haus and Islam,⁶⁾ and Amir *et al.*⁷⁾ analyzed theoretically the spontaneous emission spectrum in waveguide FELs and they discussed the differences and the similarities between the emission into the free space and the bounded space. Throop *et al.*⁸⁾ measured broad-band spectrum of high-power amplified spontaneous emission radiated from a high-current relativistic electron beam in a highly oversized waveguide. However, there has been no report on the measurement of the spontaneous emission spectrum in a low- γ , waveguide-mode FEL.

Most of low- γ FELs⁹⁾ use high-current electron beam and work in Raman regime in which the space charge effect among the electrons plays an important role in lasing mechanism. Fajans *et al.*¹⁰⁾ measured the amplification and the phase shift of an injected microwave in a Raman type FEL using a low-current electron beam.

A relativistic photo-electron beam (RPE),¹¹⁾ which is generated by accelerating the laser-induced photo-electrons, is a promising electron beam source for FEL research because its energy spread is extremely small ($\ll 10$ eV) and because it has the advantage of easy handling of the temporal structure and the current density. We have developed good quality RPE^{12,13)} for the application to FEL research. The good controllability of RPE makes it easy to measure the operation characteristics of a low- γ FEL.

In this paper, the spontaneous emission spectrum and the small-signal gain of a low- γ , waveguide-mode FEL are discussed. The spectral distribution of the spontaneous emission in a rectangular waveguide is derived by using the dyadic Green's function method^{7,14)} and the theoretical small-signal gain spectrum of a given waveguide mode is derived by using the classic-

al limit of Einstein's coefficient method¹⁵⁾ under the assumption of operation in the Compton regime. The experimental apparatus and the method of measurement are explained and the experimental results are compared with the theoretical predictions. The spontaneous emission spectrum measured is found to be homogeneously broadened and the small-signal gain spectrum measured shows that the FEL works in the Compton regime.

2. Theory

2.1 Electron Orbit

The single-particle equation of motion of an electron in the linearly polarized wiggler magnetic field (B_w) and the axial guiding magnetic field ($B_g e_z$) is given by

$$\frac{d}{dt} = (\gamma m \beta) = \frac{e}{c} \beta \times B, \quad B = B_g e_z + B_w \quad (1)$$

where $\gamma = (1 - \beta^2)^{-1/2}$, $\beta = V/c$, c is the light velocity, V , m , e are velocity, rest mass and magnitude of charge of an electron, respectively and e_z is the unit vector along the beam.

The distribution of an ideal wiggler magnetic field is represented by $B_w = B_w \sin(k_w z) e_x$ where $k_w = 2\pi / \lambda_w$, λ_w is the wiggler pitch and B_w is the peak amplitude of the wiggler magnetic field which is independent of x, y coordinate. An approximate solution of the equation of motion in the ideal wiggler magnetic field is

$$\beta_x = \frac{RK}{\gamma(1-R^2)} \cos(k_w z), \quad \beta_y = \frac{K}{\gamma} (1-R^2) \sin(k_w z), \quad \beta_z = \beta_y, \quad (2)$$

where $K = eB_w / mc^2 k_w$, $\omega_0 = k_w \beta c$, $\Omega_g = eB_g / \gamma mc$, $R = \Omega_g / \omega_0$. The average parallel speed, β_y is determined by the equation of energy conservation together with Eqs. (2). The term $1/$

$(1-R^2)$ represents the degree of resonance between the wiggling motion due to the wiggler magnetic field, and the cyclotron motion induced by the axial magnetic field. The transverse velocity increases at the expense of parallel velocity as the resonance parameter R approaches to 1 (magneto-resonance). The projection of electron trajectory on transverse plane (x - y plane) is an ellipse. The ratio of maximum excursion in x -direction to that in y -direction is R . The approximate solution, Eq. (2), is valid in the region sufficiently far from the magneto-resonance and for weak wiggler magnetic field.

A realistic wiggler has finite-length adiabatic region and transverse non-uniformity. The finite-length adiabatic region in the presence of axial guiding magnetic field induces the cyclotron motion whose amplitude differs according to the initial condition of each electron. Numerical calculation of the current-driven wiggler magnetic field shows that the transverse distribution of peak amplitude wiggler magnetic field, $B_w(x, y)$, is to be approximated by $B_w(x, y) = B_w \cosh(0.81k_w x)$. The effect of transverse non-uniformity of the wiggler magnetic field is drift motion¹⁶⁾ of off-axis electrons in radial direction. The drift motion also differs for different initial condition of the incoming electron. Consequently, the angular velocity spread of the electron beam increases and the radiation spectrum is broadened inhomogeneously.

2.2 Spontaneous Emission Spectrum

Let us calculate the spectral distribution of electromagnetic energy radiated by an electron passing through the combined axial guiding magnetic field and linearly polarized wiggler magnetic field in a perfectly conducting rectangular waveguide. The electron has a momentum $P = \gamma m \beta c$ where β is given by Eq. (2).

The electromagnetic energy propagating through a waveguide in the spectral range $\omega \sim \omega + d\omega$ is given by

$$\frac{dW}{d\omega} = \frac{c}{8\pi} \int_A (\mathbf{E}_\omega \times \mathbf{B}_\omega^*) \cdot \mathbf{e}_z dx dy, \quad (3)$$

where $\mathbf{B}_\omega = \Delta \times \mathbf{E}_\omega / ik$, $i = \sqrt{-1}$, $\mathbf{E}_\omega$ is the Fourier component of the electric field of the electromagnetic wave and $*$ represents the complex conjugate. The integration is performed over the waveguide cross section. The electromagnetic wave is excited by a current distribution, $\mathbf{J}(\mathbf{x}, t)$, and $\mathbf{E}_\omega$ is to be represented by

$$\mathbf{E}_\omega(\mathbf{x}) = \frac{ik}{c} \int G_\omega(\mathbf{x}, \mathbf{x}') \cdot \mathbf{J}_\omega(\mathbf{x}') d^3x', \quad (4)$$

where $G_\omega(\mathbf{x}, \mathbf{x}')$ is a dyadic Green's function^{7,14)} appropriate for the boundary condition of the rectangular waveguide and $\mathbf{J}_\omega(\mathbf{x})$ is the Fourier transform of $\mathbf{J}_\omega(\mathbf{x}, t)$. The dyadic Green's function is expanded by the eigenvectors of the waveguide as following¹⁴⁾

$$G_\omega(\mathbf{x}, \mathbf{x}') = \sum \alpha_s \mathbf{E}_s(\mathbf{x}) \mathbf{E}_s^*(\mathbf{x}') \quad (5)$$

where $\mathbf{E}_s$ is an eigenvector of the waveguide and s is the mode index representing the TE_{pa} mode or the TM_{pq} mode. The coefficient s is determined by the orthonormal relation of the eigenvectors. The current density of an electron in the wiggler magnetic field is to be represented by

$$\mathbf{J}(\mathbf{x}, t) = -ce \beta(t) \delta[\mathbf{x} - \mathbf{x}_o(t)], \quad (6)$$

where $\mathbf{x}_o(t)$ is the position of the electron at time t and $\delta(\mathbf{x})$ is the Dirac δ -function. Substituting Eqs. (5) and (6) into Eq. (4),

$$\begin{aligned} \mathbf{E}_\omega(\mathbf{x}) = & -ce \sum \alpha_s \mathbf{E}_s(\mathbf{x}) \\ & \cdot \int \mathbf{E}_s^*[\mathbf{x}_o(t)] \cdot \beta(t) e^{i\omega t} dt. \end{aligned} \quad (7)$$

The scalar product in Eq. (7) represents the

“selection rule” of waveguide modes, *i.e.*, a particular waveguide mode of the radiation is coupled with a particular orbit of the electron. For example, if the wiggling motion is linear in y -direction ($\beta_y \neq 0$, $\beta_x = 0$), then any TE_{0q} ($q \neq 0$) mode can not be excited. It is straightforward to calculate the integration Eq. (3) with a help of Eqs. (2), (7) and the result, in the limit of small wiggling motion, is

$$\frac{dW}{d\omega} = \frac{\pi e^2 N^2 \lambda_w^2 K^2}{8 \beta_{\parallel} c \gamma^2 a^2 b^2} \left[\frac{1}{1-R^2} \right]^2 \cdot \Sigma (\Pi_{Epq} + \Pi_{Mpq} \left[\frac{\sin \phi}{\phi} \right]^2), \quad (8)$$

$$\begin{aligned} \Pi_{Epq} &= k(2 - \delta_p - \delta_q) (\epsilon_{oe} k_x^2 + \epsilon_{eo} k_y^2) \\ &\times [(1 - \delta_p)(1 + \delta_q) k_x^2 + (1 + \delta_p) \\ &\cdot (1 - \delta_q) k_y^2] / k_g k_c^2, \end{aligned} \quad (9)$$

$$\begin{aligned} \Pi_{Mpq} &= k_g(2 - \delta_p \delta_q) (\epsilon_{oe} k_x^2 + \epsilon_{eo} \\ &+ \epsilon_{eo} k_x^2 \Omega_g^2 / \omega_o^2) \\ &- [(1 + \delta_p)(1 - \delta_q) k_x^2 + (1 - \delta_p) \\ &\cdot (1 + \delta_q) k_y^2] / k k_c^2, \end{aligned} \quad (10)$$

$$\phi = [\omega_o - (1 - \beta_{\parallel}^*) \omega] N \lambda_w / 2 \beta_{\parallel} c \quad (11)$$

where $\beta^* = \beta_c / V_{ph}$, $k_x = p\pi/a$, $k_y = q\pi/b$, $k_c = 2\pi/\lambda_c$, $k_g = (k^2 - k_c^2)^{1/2}$, N is the number of wiggler periods, λ_c is the cutoff wavelength, V_{ph} is the phase velocity, and a, b are the inner dimensions of the rectangular waveguide. Dimensionless Π_{Epq} represents the degree of contribution of TE_{pq} mode to the spontaneous emission spectrum and Π_{Mpq} represents that of TM_{pq} mode. ϵ_{oe} is 1 only when $p = \text{odd}$ and $q = \text{even}$ simultaneously and is 0 otherwise. Note that only the modes that have (*odd, even*) or (*even, odd*) combination of (p, q) indices contribute to the spontaneous emission, which is a result of the selection rule. The radiation power increases to in-

finiteness as R approaches to 1 (magneto-resonance).

Though the spectrum is represented by the sum of all possible waveguide modes, only a finite number of modes that satisfy the matching condition ($\phi = 0$) contribute to the radiation power. The central frequency of a given waveguide mode is given by

$$\omega_c = \frac{\omega_o}{(1 - \beta_{\parallel} c / V_{ph})}, \quad (12)$$

where the effect of a waveguide comes in through the term c/V_{ph} which becomes 1 in the free space. With a simple manipulation of Eq. (12), we obtain two equations that the central frequency satisfy simultaneously;

$$\omega = \beta_{\parallel} c (k_w + k), \quad \omega^2 = c^2 (k_c^2 + k^2). \quad (13)$$

These represent the wiggler dispersion relation and the waveguide dispersion relation of the radiation, respectively. Figure. 1 shows the dispersion relations (Eq. (13)) for the radiation from a 526 keV electron in an X-band waveguide. The intersection of waveguide dispersion relation with the wiggler dispersion line defines the central frequency of the spontaneous emission. Note that only TE_{10} mode is excited.

Fig. 1 Waveguide dispersion relation and wiggler dispersion for the radiation from a 526 keV electron in an X-band waveguide. $B_w = 0.36$ kG, $B_g = 2.1$ kG, $a = 2.29$ cm, $b = 1.02$ cm, $\lambda_w = 5$ cm. Only TE_{10} mode is excited.

There are two branches of matching frequencies; the upper branch centered at 34.2 GHz (f_u) and the lower branch centered at 6.81 GHz (f_l).

The homogeneously broadened spectral width ($\Delta\omega_h \equiv \int (dW/d\omega) d\omega / (dW/d\omega)_{\omega=\omega_c}$) of a given waveguide mode is found to be

$$\Delta\omega_h = \omega_c / N, \quad (14)$$

which is inversely proportional to the flight time of the electron through the wiggler.

The spontaneous emission coefficient, $\eta_\omega(\mathbf{p})$, is defined as the time-average of the energy radiated by an electron having a momentum $\mathbf{p} (= \gamma m \boldsymbol{\beta} c)$ in the spectral range $\omega \sim \omega + d\omega$, i.e.,

$$\eta_\omega(\mathbf{p}) = \frac{1}{T} \frac{dW}{d\omega}, \quad (15)$$

where $T (= N\lambda_w / \beta_c)$ is the flight time of the electron through the wiggler. The instantaneous spontaneous emission power radiated by an ensemble of electrons is

$$P = \int \eta_\omega(\mathbf{p}) f(\mathbf{p}) d\mathbf{p} d\omega, \quad (16)$$

where $f(\mathbf{p})$ is the number of electrons having a momentum in the range $\mathbf{p} \sim \mathbf{p} + d\mathbf{p}$.

2.3 Small-Signal Gain Spectrum

The small-signal gain coefficient is derived from the spontaneous emission coefficient by using the Einstein's coefficient method¹⁵⁾. If the energy of an electron is sufficiently higher than the energy of a photon emitted by the electron ($E > \hbar\omega$), then the spontaneous emission coefficient (Eqs. (8), (15)) obtained from the classical electrodynamics may be used. When the electron beam is sufficiently tenuous the collective effects may be negligible. Then the small-signal gain coefficient of a given waveguide mode is given by

$$\Gamma_\omega = \frac{4\pi^3 c^2 N \lambda_w}{\hbar \omega^3} \int \eta_\omega(\mathbf{p}) [\mathbf{V}_p f(\mathbf{p}) \cdot \Delta\mathbf{p}] d^3p, \quad (17)$$

where $\Delta\mathbf{p} (|\Delta\mathbf{p}| = \hbar\omega/c)$ is the momentum change of an electron when it emits or absorbs a photon of energy $\hbar\omega$.

The effect of momentum spread of the electron beam is represented clearly in Eq. (17). For example, if the distribution function $f(\mathbf{p})$ is independent of $\mathbf{p}$, then the small-signal gain coefficient becomes zero. If the electron beam is extremely cold, the distribution function is represented by

$$f(\mathbf{p}) = n \delta(\mathbf{p} - \mathbf{p}_0), \quad (18)$$

where $\mathbf{p}_0$ is the steady-state solution of the equation of motion (Eq. (2)) and n is the electron density. With some manipulations, the small-signal gain coefficient of a given waveguide mode becomes

$$\Gamma_\omega = - \frac{4\pi^3 n N^2 \lambda_w^2 F}{2\gamma m \alpha_{\parallel}} \frac{1}{V_{ph}} \frac{\partial \eta_\omega}{\partial \phi}, \quad (19)$$

where F is a filling factor (the ratio of the cross section of the electron beam to that of radiation). Note that the small-signal gain coefficient in a waveguide is proportional to the derivative of the spontaneous emission coefficient with respect to the tuning parameter, ϕ .

3. Experiments

Figure 2 shows the experimental setup. The RPE (450 ~ 650 keV, 8 ns at FWHM) is generated by accelerating electrostatically the photoelectrons produced by the 4-th harmonics (266 nm, 30 mJ, 8 ns) of a Q-switched Nd-YAG laser pulse incident on a Zn photo-cathode. The energy required for acceleration is supplied by a Marx generator. The acceleration tube is composed of 15 steps of grading rings, on which equally divided acceleration voltage is applied for the purpose of obtaining uniform acceleration voltage. Figure 3-(a) shows a typical waveform of the acceleration voltage and the 4-th

Fig. 2 Schematic diagram of the experimental setup. RPE, Relativistic Photo-Electron beam; GO, Gunn Oscillator; BPF, Band Pass Filter; D₁, D₂, 1N26 crystal detectors; I₁, I₂, Isolators; X-WG, X-band waveguide; Ka-WG₁, Ka-WG₂, Ka-band waveguides; T-WG₁, T-WG₂, Tapered waveguides. The waveguide configuration is shown in E-plane view. In the measurement of spontaneous emission spectrum, GO is set off. In the measurement of small-signal gain, GO is set on and BPF is removed.

Fig. 3 a) A typical waveform of the acceleration voltage and a Nd-YAG laser pulse in "add" mode of an oscilloscope; b) Expanded view of a); c) A typical waveform of the current of RPE; d) A typical waveform of the spontaneous emission.

harmonics of the Nd-YAG laser pulse in "add" mode of an oscilloscope and Figure 3-(b) is an expanded view. The acceleration voltage

changes due to RC drop of the discharge circuit by less than 0.1 % during 8 ns of the Nd-YAG laser pulse. The discharge circuit of the Marx

generator is shorted at time T_c by a laser-triggered spark gap. Figure 3-(c) shows a typical waveform of the current of RPE, which is measured by using a Faraday cup. The current density of RPE is limited to $\sim 2\text{ A/cm}^2$ by the space charge effect. The ratio of average angular velocity to parallel velocity ($\beta_\perp/\beta_\parallel$) is of the order of 10^{-3} and the relative energy spread ($\Delta E/E$) of the RPE is less than 10^{-4} . The current and the beam diameter of the RPE in the waveguide are 1.5 A and 0.5 cm, respectively. The pulsed linearly polarized wiggler magnetic field is generated by flowing current through an array of electromagnet windings. The wiggler magnetic field has a pitch of 5 cm, and the number of wiggler periods is 2 in the adiabatic region and is 17 in the interaction region.

The radiation ($\sim 34\text{ GHz}$) generated by the RPE is guided by a 120 cm-long, X-band waveguide (X-WG in Fig. 2, cross section; $2.29 \times 1.02\text{ cm}^2$) and is introduced into a Ka-band waveguide (Ka-WG₁, $0.71 \times 0.355\text{ cm}^2$) through a tapered waveguide (T-WG₁). Since the waveguides do not constitute a cavity and the pulse length of RPE is shorter than the round-trip time ($\sim 8.3\text{ ns}$) along the X-band waveguide, there is no possibility that the radiation travel back and forth through the waveguide and be amplified into high power. Furthermore, the isolator (I_2) forbids the radiation to be reflected into the X-band waveguide. The radiation is filtered by a variable-frequency band pass filter (BPF, $Q \sim 800$) and is detected by an IN26 crystal diode (D_1) having sufficiently fast response time ($< 1\text{ ns}$) and sufficiently high sensitivity. The operation characteristics of the crystal diode with respect to input microwave power is calibrated by using a cw microwave source (Gunn oscillator) and a calibrated variable attenuator. Figure 3-(d) shows a typical waveform of the spontaneous emission mea-

sured by the crystal diode. The energy of the RPE is fixed at 526 keV in every shot of the RPE and the central frequency of the BPF is changed in the range of $30 \sim 38\text{ GHz}$. The experimental conditions ($E = 526\text{ keV}$, $B_w = 0.36\text{ kG}$, $B_g = 2.1\text{ kG}$, $R = 0.552$) are selected to be far from the magneto-resonance region such that the self-amplification of spontaneous emission becomes small. According to the calculation, the instantaneous peak power of total spontaneous emission radiated by an electron beam of 1.5 Ampere is about $1\text{ l}\mu\text{W}$ when the radiation is filtered by the BPF. It is close to the threshold of detectability of the crystal diode.

For the measurement of small signal gain, a coherent cw microwave (20 mW, 34.4 GHz) from a Gunn oscillator is injected into the X-band waveguide (X-WG) through a Ka-band waveguide (Ka-WG₂) and a tapered waveguide (T-WG₂), and its amplification or absorption by the RPE is measured. Since only TE₁₀ mode of 34.4 GHz microwave can propagate through the Ka-band waveguide and the X-band waveguide is well-matched to the Ka-band waveguide, only TE₁₀ mode can be excited in the X-band waveguide. The frequency of the microwave is fixed at 34.4 GHz and the energy of the RPE is changed in the range of 480-580 keV.

4. Results and Discussions

4.1 Propagation of RPE

Figure 4 shows the transmission ratio of the RPE through the wiggler (the ratio of the current at the end of the wiggler to that at the entrance of the wiggler) as a function of the axial guiding magnetic field for various values of the wiggler magnetic field. The RPE current is measured by a Faraday cup located in the waveguide. B_{gr} is the value of axial guiding magnetic field for which the magneto-resonance para-

Fig. 4 Transmission ratio of RPE through the wiggler (the ratio of current at the end of the wiggler to that at the entrance of the wiggler).

Fig. 5 Spectrum of spontaneous emission power measured as a function of frequency (dots) and the energy radiated by an electron per unit frequency calculated theoretically (solid line). The vertical scale of the power measured is adjusted such that the vertical position of the dot for the peak power near 34.4 GHz coincide with the peak of the theoretical curve. The ellipse is the projection of the orbit of an electron on the transverse velocity plane (V_x - V_y plane).

meter, R , approaches to 1. The decrease of transmission ratio near the magneto-resonance region is due to the loss of RPE to the waveguide wall as the transverse excursion increases near the resonance region.

4.2 Spontaneous Emission Spectrum

Figure 5 shows the measured spontaneous emission power filtered by the BPF as a function of the central frequency of the BPF (dots). The calculated energy radiated by a single electron in the spectral range $\omega \sim \omega + d\omega$ (Eqs. (8)-(11)) is represented by the solid curve. Each of dots is the average over more than 10 shots and the error bar represents the standard deviation. In every shots, the spectral power is normalized with respect to the total spontaneous emission power measured by D_2 (Fig. 2). The lower-branch radiation centered at 6.81 GHz (Fig. 1), if any, is filtered out by the Ka-band waveguide whose cutoff frequency is 21.13 GHz and only the upper-branch radiation is detected. The central frequency (34.4 GHz) and the spectral width (~ 2 GHz) measured experimentally agree well with those of homogeneously broadened spectrum predicted theoretically by Eqs. (12) and (14). As is previously mentioned, the finite-length adiabatic region induces electron cyclotron motion. Radiation whose central frequency is the Doppler-shifted cyclotron frequency does not appear since the waveguide dispersion curve does not intersect with the Doppler-shifted cyclotron mode.

Due to the finite pulse-width ($\tau_p \sim 8$ ns) of the RPE and various sources of inhomogeneity ($\Delta\omega_i$), the spectral width is broadened to be

$$(\Delta\omega)_t = [\Delta\omega_h]^2 + (2\pi/\tau_p)^2 + \sum (\Delta\omega_i)^2]^{1/2}. \quad (20)$$

The finite pulse-width induces 0.125 GHz ($=1/8$ ns) of homogeneous broadening. The spectral inhomogeneity due to the energy spread is $(\Delta\omega/\omega)_{e.s} = 2(1-1/\gamma)\Delta E/E < 10^{-4}$. The space charge effect causes another spread in axial component of energy¹⁷⁾; $(\Delta\gamma/\gamma)_{s.c} \sim \omega_p^2 r_b^2 / 4c^2$ where ω_p is the electron-plasma angular frequency (being 2

$\pi \times 0.01 \text{ GHz}$ in the present case) and r_b is the beam radius. The line broadening due to the space-charge effect is estimated to be $(\Delta\omega/\omega)_{s.c} \sim 10^{-4}$. The largest contribution to the spectral inhomogeneity is expected to come from the imperfectness of the adiabatic region and the transverse non-uniformity of the realistic wiggler magnetic field. The ellipse in Fig. 5 is a projection of the orbit of an electron in the realistic wiggler magnetic field on the transverse velocity plane ($V_x - V_y$ plane), which is obtained by solving numerically the equation of motion of an electron in the presence of the realistic wiggler and the axial guiding magnetic field. The realistic wiggler magnetic field causes deviation of electron orbit from a closed ellipse and the degree of deviation is represented as the width of the ellipse (see Fig. 5); $\Delta V_{\perp} \sim 0.02c$. The degree of deviation differs according to the initial condition of each electron. The cor-

responding inhomogeneous broadening is $(\Delta\omega/\omega)_w \sim 5 \times 10^{-3}$. Consequently, $(\Delta\omega/\omega)_t \sim 1/16.9 \sim 1/N = (\Delta\omega/\omega)_h$; that is the spectral width is very close to the homogeneously broadened spectral width.

Figure 6 shows (a) the total spontaneous emission power and (b) the central frequency of the spontaneous emission as a function of axial guiding magnetic field. The central frequencies are obtained by measuring the propagation time of the radiation through a 450 cm-long, Ka-band dispersive waveguide (see Fig. 2). The dispersive line is too short to work as a spectrum analyzer with desirable resolving power but is sufficiently long for the measurement of the central frequency since the pulse width of the spontaneous emission is short. Note that the central frequency of the spontaneous emission is "tunable" in the wide range of 25 GHz $\sim$ 40 GHz by changing the axial guiding magnetic field. As the experimental condition approaches to the magneto-resonance, transverse velocity increases at the expense of longitudinal velocity, and therefore the radiation power increases and the central frequency decreases near the magneto-resonance.

4.3 Small-Signal Gain Spectrum

Figure 7-(a) shows a typical waveform of amplification of the injected microwave signal by RPE-FEL and Figure 7-(b) shows the absorption of the injected microwave. Figure 8 shows the small-signal gain spectrum with respect to the energy of RPE. Each of dots represents the average of the small-signal gain coefficients ($\Gamma_w P_{out} = P_{in} e^{\Gamma_w}$ where P_{in} is the input power and P_{out} is the output power) over more than 10 shots and the error bar is the standard deviation. The positive gain means the amplification of the injected microwave by RPE and the negative gain means the absorption. The measured

Fig. 6 Variation, of (a) the total spontaneous emission power and (b) the central frequency as a function of axial guiding magnetic field.

Fig. 7 a) An amplified waveform of 34.2 GHz, 20 mW, cw microwave signal by RPE-FEL and b) absorbed waveform.

Fig. 8 Small-signal gain coefficient (Γ_ω , $P_{out} = P_{in} e^{\Gamma_\omega}$ where P_{in} is the input power and P_{out} is the output power) of 20 mW, 34.4 GHz microwave as a function of the energy of RPE.

Fig. 9 Variation in the small-signal gain coefficient as a function of the current of REP (dots). The solid line is a linear regression.

spectrum agrees well with the homogeneously broadened spectrum calculated from the single-particle, linear theory (Eq. (5)). The energy spread and the angular velocity spread not only induce inhomogeneous broadening of spectrum but also reduces the small signal gain. As is to be known from Fig. 8, there is no severe reduction of gain.

Figure 9 shows the small-signal gain coefficient as a function of the current of RPE. The current of RPE is varied by changing the intensity of the 4-th harmonics of Nd-YAG laser pulse incident on the photo-cathode. The dots represent the measured value of gain coefficient

Fig. 10 Variation in the small-signal gain coefficient as a function of the square of a wiggler magnetic field.

and the solid line is a linear regression. The gain coefficient is proportional to the current in the range of 0.3 ~ 1.5 A, which means that the FEL apparatus works in the low-gain Compton regime in the tenuous-beam limit. The plasma frequency corresponding to 1.5 A of 540 keV RPE current is 0.12 GHz, and it is much smaller than the frequency of the injected microwave. In a Raman FEL, the gain coefficient is proportional to $I^{1/4}$ where I is the current of electron beam¹⁸⁾. In the high-gain Compton regime, the gain coefficient is proportional to $I^{1/3}$.

Figure 10 shows the variation of small-signal gain coefficient as a function of the wiggler magnetic field. The error bar represents the standard deviation over 5 shots. The average parallel speed ($\beta_{\parallel}$) of a fixed-energy electron decreases and thus the matching condition changes as the wiggler magnetic field increases. Thus the energy of the electron beam must be increased as the wiggler magnetic field increases such that the average parallel speed and the tuning parameter ϕ (see Eq. (11)) is kept constant. The numerics represent the energy of RPE. Note that the gain coefficient is proportional to the square of the wiggler magnetic field in the limit of weak pumping ($B_w = 0.1 \sim 0.6$ kG). This is also a proof for the operation in the low-gain Compton regime. The gain coefficient of a Raman FEL is proportional to the wiggler magnetic field¹⁹⁾.

5. Conclusion

The spontaneous emission spectrum and the small-signal gain spectrum of a low- γ , waveguide-mode FEL using a good-quality RPE have been investigated theoretically and experimentally. The measured spontaneous emission spectrum agrees well with the homogeneously broadened spectrum of TE₁₀ waveguide mode centered at 34.3 GHz. A wide-range (25 ~ 40

GHz) tunability of the central frequency of the spontaneous emission is obtained by changing the axial guiding magnetic field. The small-signal gain spectrum is in good agreement with the homogeneously broadened gain spectrum calculated from the single-particle, linear theory in the low-gain Compton regime. This small signal gain coefficient of 34.4 GHz microwave is proportional to the current of electron beam in the range of 0.3 ~ 1.5 A and to the square of wiggler magnetic field in the limit of weak pumping (0.1 ~ 0.6 kG). The results may provide a guideline for the design and understanding of a low- γ waveguide-mode FEL. The results also prove the good applicability of the RPE to FEL research.

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