HYDRODYNAMIC APPROACH TO IMPLOSION OF TARGET PLASMA BY LASER HEATING

Keishiro NIU and Takashi YABE
Department of Physical Engineering
Tokyo Institute of Technology
Ohokayama, Meguro-ku, Tokyo, Japan

§1. Introduction

So far calculations of implosion of a target pellet exposed to a laser light have been carried out in the United States¹⁻⁵ and in other countries. In order to give some theoretical informations to the groups which are performing experiments in Japan⁷⁻⁸, we are now making the same kind of calculations. Here we would like to report some results and make several serious points clear in order to improve the thermalization and the compression of ions in the target plasma.

§2. Fundamental Equations

The fundamental equations governing the implosion are the following six spherically symmetric hydrodynamic equations.

$$\text{The continuity equation : } \partial \rho / \partial t + 1/r^2 \cdot \partial \rho u r^2 / \partial r = 0. \quad (1)$$

The equation of motion :

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial}{\partial r} (p+q) - \frac{4}{3\rho r^2} \frac{\partial}{\partial r} (r^2 u \frac{\partial u}{\partial r}) - \frac{4u}{3\rho} \frac{\partial}{\partial r} \left(\frac{\mu}{r} \right) - \frac{4\mu u}{3\rho r^2}. \quad (2)$$

$$\text{The reaction equation : } \partial Y / \partial t + u \partial Y / \partial r = n(1-Y)^2 \langle \sigma v \rangle / 4. \quad (3)$$

$$\{ Y = (n_\alpha + n_n) / (n_D + n_T + n_\alpha + n_n) \}$$

$$\text{The equation for the laser power intensity : } 1/r^2 \cdot \partial r^2 P_L / \partial r + K_a P_L = 0. \quad (4)$$

The energy equation for the ion :

$$\partial T_i / \partial t + u \partial T_i / \partial r = -2m_i (p_i + q) / 3k\rho r^2 \cdot \partial r^2 u / \partial r - (T_i - T_e) \tau_{ei} + W_i + Q_i. \quad (5)$$

The energy equation for the electron :

$$\frac{\partial T_e}{\partial t} + u \frac{\partial T_e}{\partial r} = -\frac{2m_i p_e \partial r^2 u}{3k \rho r^2 \partial r} - \frac{2m_i c \partial r^2 P_L}{3k r^2 \rho \partial r} + \frac{T_i - T_e}{\tau_{ei}} + \frac{2m_i l}{3k \rho r^2 \partial r} \left(r^2 K_e \frac{\partial T_e}{\partial r} \right) + W_e + Q_e - A \rho T_e^{1/2}. \quad (6)$$

In eq.(2), q is the artificial viscosity term to which the expressions

$$q = c(\partial u / \partial r)^2 \text{ if } \partial u / \partial r < 0, \quad q = 0 \text{ if } \partial u / \partial r > 0 \quad (7)$$

are employed⁹. For the classical absorption in the under-dense region, the absorption coefficient K_a in eq.(4) is given by¹⁰

$$K_a = \lambda^2 n^2 / 5 \times 10^{27} (T_e / 1.16 \times 10^4)^{3/2} (1 - n/n_c)^{1/2}, \quad (8)$$

and for the anomalous absorption at and near the critical density regions, we use the following expression¹¹,

$$K_a = \text{smaller of } \left\{ \frac{4\gamma/c}{\gamma \kappa \lambda_D / 4c X^2 \cdot (X - 1/\kappa \lambda_D - 2)^4}, \quad \{X = (2P_L / c n_c K T_e)^{1/2}\} \right\} \quad (9)$$

where γ is the imaginary part of ω which satisfies the following equation for parametric instabilities¹²,

$$\omega^2 + 2i\omega\Gamma_j - \omega_1^2 = 2\pi\kappa^2 e^2 P_L \{1/(\omega + \delta + i\Gamma_2) - 1/(\omega - \delta + i\Gamma_2)\} / e m_i m_e \omega_2. \quad (10)$$

In eqs.(5) and (6), $W_{i,e}$ shows the energy which comes from reactions, and $Q_{i,e}$ is the dissipation energy caused by the viscosity.

After eqs.(1)-(6) are transformed to Lagrangian forms, they are approximated by the difference equations to be carried out numerical calculations.

§3. Numerical Results

The numerical model of the plasma density of the target pellet is shown in Fig.1. At first, the laser power intensity P_L is assumed to be constant in time outside the pellet. A result is given in Fig.2. Results are checked by using the energy conservation. The errors involved in calculations are less than 5%. Figure 3 shows that the maximum values of n , T_i , T_e realized at the pellet center in the course of time and the total number N_{eu} of neutron yields versus the laser power density P_L . It is turned out that the most efficient compression and the most

efficient heating of ions occur at different values of P_L . Thus, there is the optimum value of P_L which leads us to the maximum nuclear reaction.

After the target plasma is irradiated by a laser light, a part of the plasma converges to the center of the pellet and another part expands into the vacuum. The ratio of the energy E_1 of the expanding part to the energy E_2 of the converging part is from 7 to 20 for various P_L . The majority of the laser energy absorbed in the target plasma escapes from the pellet and does not contribute directly to the fusion reaction. We can show from the plane one-dimensional similar solutions¹³ that

$$E_1 / E_2 = (m_2 n_2 / m_1 n_1)^{1/2}, \quad (11)$$

if the energy $E = E_1 + E_2$ is released at an instant on the boundary of the two different substances 1 and 2, provided that the boundary does not move. Therefore, we emphasize that the implosion is efficiently performed if the outside part of the critical density of the target plasma is replaced by a substance with a heavier particle mass. In the following calculations, we choose $m_1 = 10 m_2$.

The aspect of implosion changes according to the dependence of the laser power on time. We assume that dependence of the laser power $\dot{E}$ on time is given by

$$\dot{E} = \dot{E}_0 (1 - t / \tau)^{-2}, \quad (12)$$

where $\dot{E}_0$ and τ are constant. The fusion energy E_{out} emitted from the pellet is plotted against $\dot{E}_0$ in fig.4. For large pellets, larger $\dot{E}_0$ is efficient for increasing E_{out} . Figure 5 indicates that the optimum value of τ exists for the realization of the maximum N_{eu} . Figure 6 shows the fusion energy E_{out} versus the pellet size r_0 when $\dot{E}_0$ and τ take the optimum values, respectively.

§4. Stability of Implosion

In order to realize the thermonuclear fusion reaction in the target pellet by a laser heating, the implosion must take place in a stable manner. In §2, we assume that the motion is spherically symmetric. Now we check the stabilities of our results reported in the preceding section.

Here we consider the three-dimensional motion of the target plasma¹⁴. The fundamental equations in the polar coordinate system r, θ and ϕ are,

$$\partial \rho / \partial t + \text{div } \rho \vec{v} = 0, \quad (13)$$

$$\partial \vec{v} / \partial t + \text{grad } v^2 / 2 - \vec{v} \times \text{rot } \vec{v} = -1/\rho \cdot \text{grad } p, \quad (14)$$

$$3k\rho/2m_i \cdot DT_e/Dt = -\rho p_e \cdot D(1/\rho)/Dt + \nabla \cdot (K_e \nabla T_e) + K_a P_L + 3k\rho(T_i - T_e)/2m_i \tau_{ei}, \quad (15)$$

$$3k\rho/2m_i \cdot DT_i/Dt = -\rho p_i \cdot D(1/\rho)/Dt + 3k\rho(T_e - T_i)/2m_i \tau_{ei}, \quad (16)$$

In these equations, the viscosity of the fluid and the bremsstrahlung are neglected. In the course of the implosion, the fusion reaction does not occur since the ion temperature is not so high. The independent variable r, θ, ϕ and t are transformed to

$$r = \bar{r} + \tilde{r}(\bar{r}, \bar{\theta}, \bar{\phi}, \bar{t}), \quad \theta = \bar{\theta} \quad \phi = \bar{\phi} \quad t = \bar{t}. \quad (17)$$

The dependent variables, for example ρ , are splitted into two parts,

$$\rho(r, \theta, \phi, t) = \bar{\rho}(\bar{r}, \bar{t}) + \tilde{\rho}(\bar{r}, \bar{\theta}, \bar{\phi}, \bar{t}), \quad (18)$$

where $\bar{\rho}$ is the value obtained in the one-dimensional motion, and $\tilde{\rho}$ is the perturbation accompanied by the degree of freedom of the three dimensions. After we linearize eqs. (13) - (16) with regard to the perturbed quantities, the perturbed quantities are expanded in the spherical harmonics. For example,

$$\tilde{\rho} = \sum_{l,m} \tilde{\rho}_{lm}(\bar{r}, \bar{t}) Y_{lm}(\bar{\theta}, \bar{\phi}). \quad (19)$$

Then eq.(13) reduces to

$$D\tilde{\rho}/D\bar{t} + \bar{\rho}\tilde{\phi}_{lm} + \tilde{\phi}\bar{\rho}_{lm} = 0, \quad (20)$$

where

$$\tilde{\phi} = \text{div } \vec{v}.$$

We carry out numerical calculations for various l . A result is shown in Fig.7. The perturbation grows considerably. More detailed inspections must be done for the stability of implosion.

§4. Transport Coefficients

In §2, we use the classical values for the transport coefficients of the plasma. The numerical results of course depend very much on the transport coefficients. In

the classical forms, their values increase with the temperature. But in reality, the plasma is collisionless when the temperature is higher than 1 KeV.

Suppose a pellet of $r = 500$ (μ) radius. In the region $\Delta r = 10$ (μ) of the pellet surface, the laser energy of $E_L = 10^4$ (J) is assumed to be absorbed. The absorbed energy is shared with electrons in the high energy tail. If the internal energy U_n of these electrons is assumed to arrive at

$$U_n = m_e c^2 / 2 = 4.1 \times 10^{-14} \text{ (J)} , \quad (21)$$

where c is the light speed, then the total number n_h of electrons with such a high internal energy is given by

$$n_h = E_L / U_n = 2.4 \times 10^{17}. \quad (22)$$

Electrons which escape with the velocity c from the pellet surface where the density is critical, are pulled back through the electric field E induced by the charge separation. The maximum length λ in which the charge separation occurs can be estimated as follows,

$$\lambda = c^2 m_e / 2eE = 4\pi e c^2 m_e r / 2e^2 n_h = 0.1 \text{ (}\mu\text{)} . \quad (23)$$

This λ is much smaller in comparison with, for example, the thickness of the density variation in Fig.2. It is inevitable to obtain the reasonable values of the transport coefficients in order to suggest the suitable experimental conditions for obtaining the maximum fusion energy in the pellet by a laser heating. For this purpose, we are starting to calculate the following Vlasov equations and Poisson equation¹⁵,

$$\partial f_{i,e} / \partial t + u \partial f_{i,e} / \partial x \pm eE / m_{i,e} \cdot \partial f_{i,e} / \partial u = 0 , \quad (24)$$

$$dE / dx = e \int_{-\infty}^{+\infty} (f_i - f_e) du . \quad (25)$$

At the initial time, the ion is homogeneous, and $E = 0$ throughout the region. The electron has the low temperature in the negative x , and the electron has a high energy tail in the positive x . The electron temperature T_e versus the distance x are plotted for several times in Fig.8. The wave lengths seem to remain in a few Debye lengths. More detailed investigations are also required to know the exact values of the transport coefficient, that is, to know the dissipation mechanisms.

References

- 1 A.F. Haught and D.H. Polk : Phys. of Fluids 13 (1970) 2825.
- 2 M.S. Chu : Phys. of Fluids 15 (1972) 413.
- 3 E.B. Goldman : Plasma Physics 15 (1973) 289.
- 4 J.W. Shearer and J.J. Duderstadt : Nuclear Fusion 13 (1973) 401.
- 5 J.S. Clarke, J.N. Fisher and R.J. Mason : Phys. Rev. Letters 30 (1973) 89.
- 6 C. Fauquignon and F. Floux : Phys. of Fluids 13 (1970) 386.
- 7 C. Yamanaka et al : IPPJ 117 (1972).
- 8 C. Yamanaka et al : Phys. Rev. A6 (1972) 2335.
- 9 R.D. Richtmyer and K.W. Morton : "Difference Methods for Initial-Value Problems" (Interscience, New York, 1957).
- 10 L. Spitzer, Jr. : "Physics of Fully Ionized Gasses" (Interscience, New York, 1961).
- 11 W.L. Kruer and J.M. Dawson : Phys. of Fluids 15 (1972) 446.
- 12 K. Nishikawa : J. Phys. Soc. Japan 24 (1968) 916, 1152.
- 13 A. Sakurai : J. Phys. Soc. Japan 36 (1974) 610.
- 14 J.N. Shiau, E.B. Goldman and C.I. Weng : Phys. Rev. Letters 32 (1974) 352.
- 15 J.A. Byers and J. Killeen : "Methods in Computational Physics" (Academic press, New York London, 1970) vol.9, p.265.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.